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METHODS OF FINDING ACTUAL SIGNAL PERIOD TIME

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Abstract: An overview of methods of actual signal period time (and instantaneous signal frequency) measurement is presented, and the methods are compared from the point of view of achievable accuracy, sensitivity to noise, higher order harmonic components and computational complexity. Importance of T (or f) estimation is pointed out.
Keywords: signal period measurement, instantaneous signal frequency measurement, nonstationary signal analysis

1 INTRODUCTION

Knowledge of signal frequency (or signal period time) is very important in several fields of measurement. Examples of these are: classical sampling measurements – RMS values of voltage, current and active power [1-3], leakage reduction using interpolation and resampling in the time domain [3-7], and power line transients and quality analysis (instantaneous frequency and associated parameters' estimation, real-time measurement) [5-11].

Our attention is paid mainly to power line transient waveforms, typical for global and local power - line systems – quasi-periodical lightly distorted harmonic signals with relatively slow changes in frequency (in the order of 1Hz/s), possibly corrupted with noise. Keeping frequency close to its nominal value is important, since resonance of the steam turbine blades caused by turbine operation in the vicinity of its nominal rotating speed could damage the blades [11]. Also the series mode rejection of laboratory digital multimeters using the integrating ADCs can be increased when power-line frequency is known.

Methods suitable for nonharmonic signals changing their frequency and/or amplitude in much broader limits than signals of power-line systems are described in [13-15].

Our intention is to compare eight various methods of instantaneous signal frequency measurement published in recent years. A brief description of the individual method principles is given and results of computer simulations of these methods are reported. Several simulations were verified by measurements on physical signal.

2 FREQUENCY DEFINITIONS

Frequency f is number of occurrences of a *periodical phenomenon* in one second. It can be found also from the time between successive occurrences of the periodical phenomenon (signal *period time* T) as $f=1/T$. Finding f as $1/T$ is called *indirect frequency measurement*. Frequency is measured directly or indirectly most frequently by universal counters. Direct measurement of frequency around 50 Hz lasts about 1s or 10s and its measurement uncertainty is 2% or 0.2%. Indirect measurement needs much shorter time (about 20 ms) and has much lower uncertainty. Troubles may occur when measuring signals with noise or harmonic distortion, with possible several zero crossings in one period.

Instantaneous frequency of *nonstationary* harmonic *signal* $u(t)=U_m(t).\sin(\omega(t)+\varphi(t))$ is defined as $f_i=(1/(2\pi))(\omega(t)+d\varphi(t)/dt)$. For three-phase systems ω_i is the instantaneous angular velocity of rotation of the space phasor of the system.

After completion of the original (real) signal x(t) by an imaginary part y(t) found by Hilbert transformation and forming so an *analytical signal* $\mathbf{z}(t)=x(t)+j.y(t)=z(t)e^{j\varphi(t)}$, the instantaneous frequency can be defined as $\omega_i(t)=d[\varphi(t)]/dt$ in rad/s [12] or as $f_i(t)=(1/(2\pi))\ d[\varphi(t)]/dt$ in Hz.

3 OVERVIEW OF METHODS AND THEIR PROPERTIES

We have simulated the below described eight methods in MATLAB. Function *chirp* producing a cosine function with $\varphi=0$, unit amplitude and frequency increasing linearly from 49 Hz to 51 Hz with the slope 1, 3 or 5 Hz/s was used as the input signal. The analyzed methods were applied on 1) pure cosine, 2) cosine with higher order harmonic components corresponding to the harmonic content in power systems in the Czech Republic (see caption of Fig.3), 3) cosine with added noise (Gaussian, zero mean white) with selectable SNR (30dB to 70 dB). In the simulations we used sampling frequency f_S 1 kHz and 10kHz, total numbers of samples taken were from 700 op to 10000, numbers of samples used for finding one value of the instantaneous signal frequency f_i depend on the used method. When inspecting the influence of noise, we used f_s=1kHz, 1000 samples and 50 repetitions, only for methods 3.1 and 3.5 (see paragraphs below) 10000 samples, 50 repetitions and smling at f_s=10kHz were used. We have computed average relative error and the maximum error bound (3σ).

3.1 Classical zero-crossing (or level-crossing) method

Time between two successive zero crossings (or level crossings) with the same sign of slope is measured. This method is the basis of indirect frequency measurement regime of universal counters. To increase the accuracy of finding zero crossings, linear interpolation between two samples with opposite signs (before and after zero crossing) and higher sampling frequency is used. At least one complete signal period must be sampled to find the f_i value. Errors see Fig.1 and Fig.3 (for f_S=10 kHz).

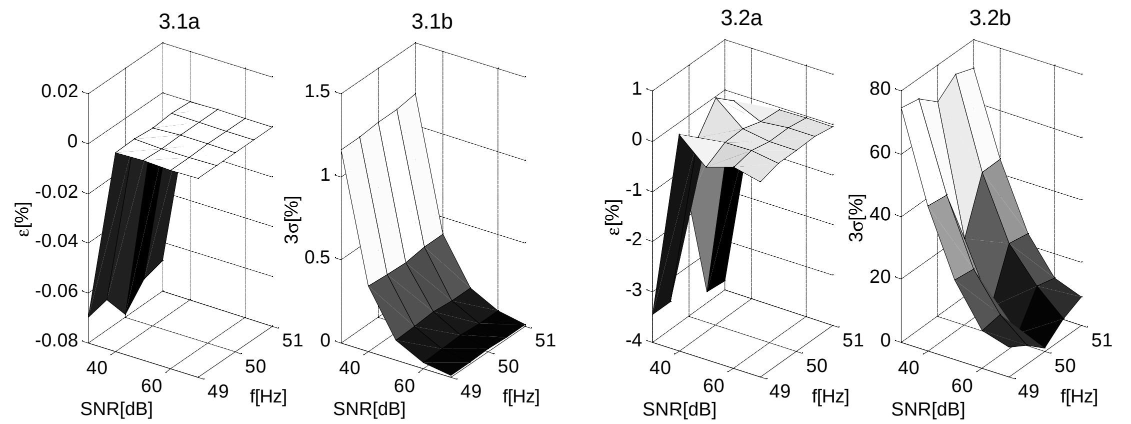

Fig.1 Averaged relative errors (a) and maximum error bounds (3σ) (b) of frequency for methods 3.1 (f_S=10 kHz) and 3.2 (f_S=1 kHz) as a function of the SNR and of the frequency.

3.2 Frequency of a sine wave computed from three successive samples [8]

Values of three successive samples of uniformly sampled signal sine wave x1, x2 and x3 (sampling interval T) are used for computation of signal frequency ($f=(1/(2\pi)).\cos^{-1}((x1+x3)/(2.x2))$. Recommendations concerning the position of samples in the signal period are given for preventing excessive noise caused by signal quantization in the ADC (there should be |x2|>0.05 ADC full scale according to [8]). Fast step response (of about 3.5 ms) is achieved by using a hardware multiplier. Three successive samples are necessary to find the f_i. Errors are given in Fig.1 and Fig.3 (for f_S=1kHz).

3.3 An iterative comparison of sinusoidal signal with a mathematical model of this signal [9]

Comparing real input voltage with a mathematical model $v(t)=A\sin(\omega t)+B\cos(\omega t)$ using recursive (iteration) technique. The signal is supposed to be pure sinusoid with frequency not changing during the data window, filtration of signal by a (non described) LP filter introducing a delay of 2.6 ms is supposed to get rid of higher-order harmonics. Sampling frequency in [9] was 900 Hz. Signal model parameters A, B and ω are found by minimizing the total square error between N (=13) real signal and model signal samples taken in the same times with the sampling interval T. This leads to the solution of three linear equations with unknowns ΔA, ΔB and $\Delta\omega$ and to the updating of parameters A, B and ω in each iteration step. A formula for finding suitable initial value of ω_0 using the first five samples is given, which makes only three iteration steps sufficient. Results of our simulation show Fig.2 and Fig.3 (for sampling frequency f_S=1 kHz).

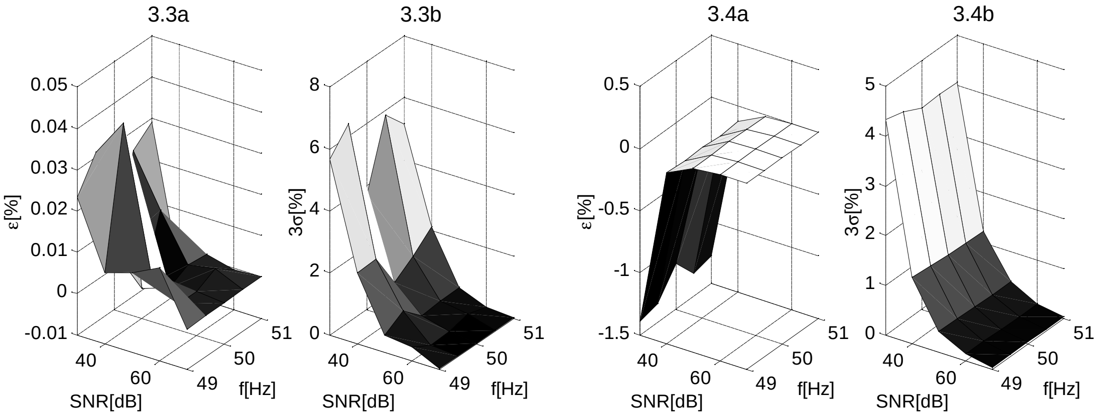

Fig.2 Averaged relative errors (a) and maximum error bounds (3σ) (b) of frequency for methods 3.3 and 3.4 as a function of the SNR and of the frequency (for f_S=10 kHz).

3.4 Using also the 2^{nd} derivative of signal for instantaneous frequency estimate [10]

Two different formulae are given for frequency estimation, depending of the presence of noise. Derivatives are computed using differences and either 3 or 7 samples are used in the 2^{nd} derivative computation. 20 to 40 samples for signal period are necessary for lowest errors using 7-points for the 2^{nd} derivative computation if signal is free from noise**.** We have used 30 samples**.** For signal without noise 7 samples are sufficient for error of 0.5%. For signal with noise, 2 or 4 signal periods have to be sampled. Repeated smoothing of signal with an LP filter (weighted MA filter) with L=5 is used. Frequency is found from two formulae using division by signal samples, therefore points with signal values < $0.1U_{max}$ must be omitted. Simulation results show Fig.2 and Fig.3 (for f_S=1 kHz).

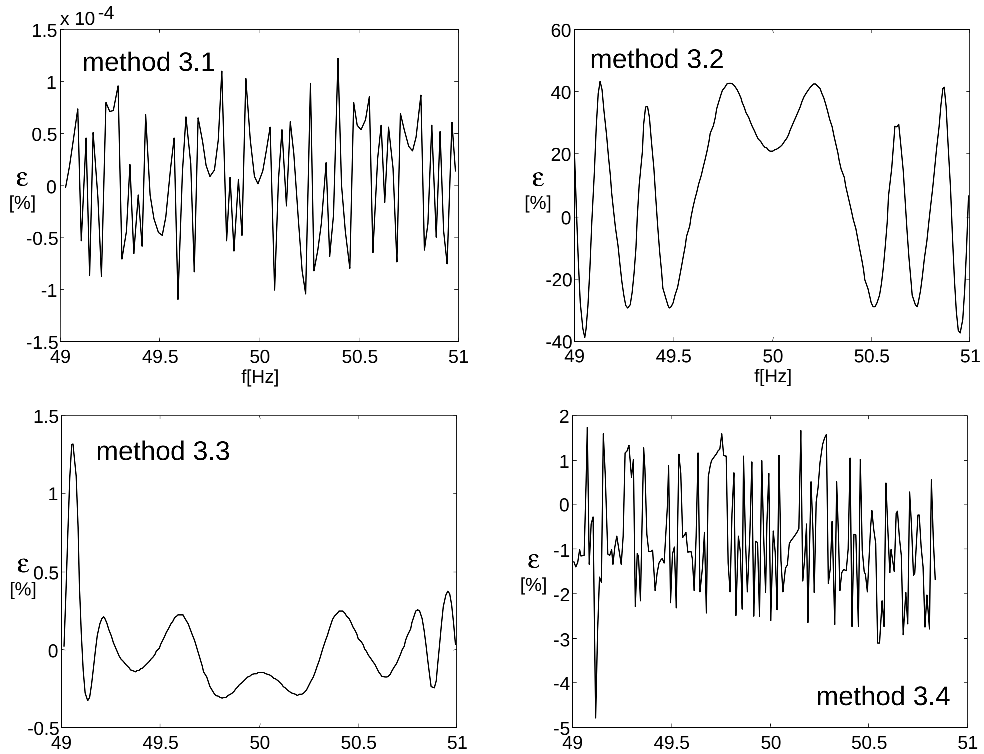

Fig.3 Relative errors of instantaneuos frequency (in %) for methods 3.1 to 3.4 by the cosinusoidal input signal with frequency sweep 1 Hz/s and with higher order harmonics corresponding to the waveform of voltage in power-line system in the Czech Republic (only further enumerated harmonics present with given average ratios to the fundamental harmonic component: 3^{rd} :0.5%, 5^{th}: 2%, 7^{th} : 1%, 9^{th}: 0.25% and 11^{th} : 0.25%).

3.5 Using zero crossing method after signal integration (IZC method) [5]-[7]

Better properties of the zero crossing method (3.1) are often achieved if the signal is integrated before being processed by zero-crossing method, see Fig.4 and Fig.7, f_S=10 kHz. About two-times lower errors can be achieved by using a triangular sliding window applied on the signal.

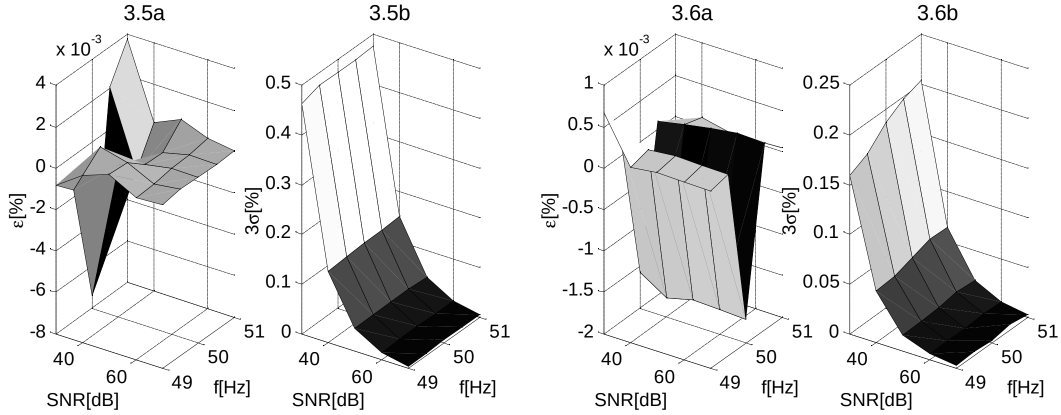

Fig.4 Averaged relative errors (a) and maximum error bounds (3σ) (b) of frequency for methods 3.5 (f_S=10 kHz) and 3.6 (f_S=1 kHz) as a function of the SNR and of the frequency.

3.6 FFT using windowing and interpolation in frequency domain [2], [3]

This method is not designed for the application at nonstationary signals, but it can be successfully applied for quazistationary signals of the type investigated in this paper, as follows from Fig.4 and Fig.7 (f_S= 1 kHz, window length: 128 samples). Errors are lower for window length 256 samples.

3.7 STFT including windowing and interpolation [13]

This method is suitable for quazistationary signals, so it can be used in the investigated case. Only in case of very abrupt changes in frequency (much faster than 1 Hz/s) errors would be high. An interpolation method described in [15] developed for linear frequency change is implemented in [13]. The method was implemented on the ADSP-2181 fixed-point signal processor and was also simulated in MATLAB. Simulation results show Fig.6 and Fig.7 (f_S= 1 kHz, window length: 128 samples).

Fig.6 shows errors for stationary signal as for other methods. For signal with frequency sweep, the errors are about five times lower, because of incorporated interpolation for frequency sweep.

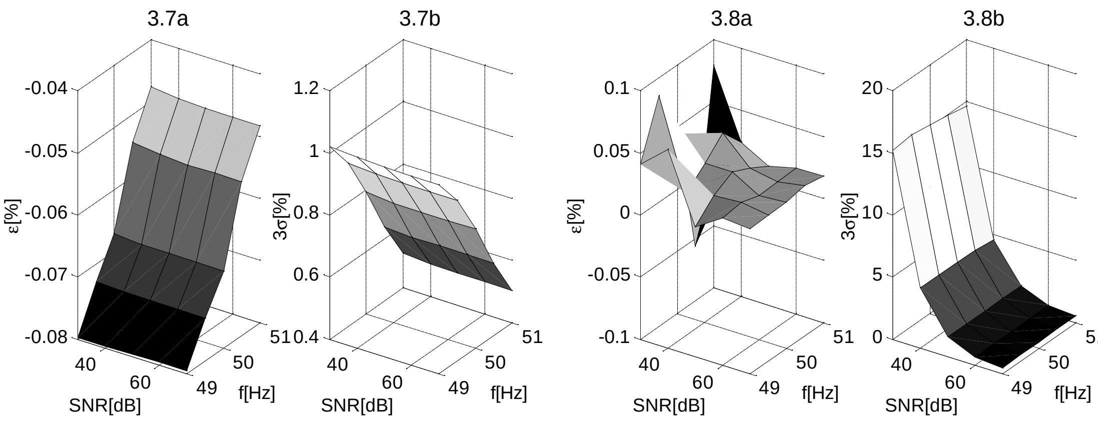

Fig.6 Averaged relative errors (a) and maximum error bounds (3σ) (b) of frequency for methods 3.7 and 3.8 as a function of the SNR and of the frequency (f_S= 1 kHz, window length for 3.7 was 128).

3.8 Function “instfreq” from the Time-Frequency Toolbox described in [14]

It uses an analytic signal as the input. Since there is a MATLAB Data Acquisition toolbox available recently, this function can be used not only for simulation, but also for measurement. The principle of the used method is unfortunately not explained in [14]. Simulation results present Fig.6 and Fig.7, both for sampling frequency f_S= 1 kHz.

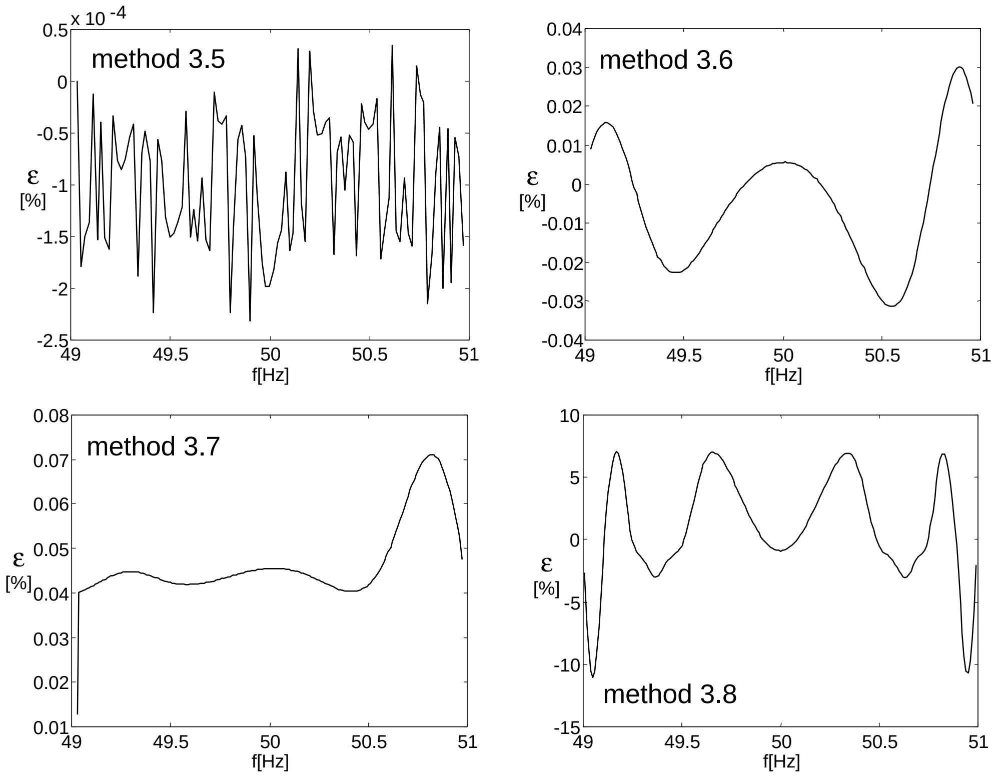

Fig.7 Relative errors of instantaneous frequency (in %) of the cosinusoidal signal with frequency sweep 1 Hz/s and with higher order harmonics as described in the caption of Fig.3, for methods 3.5, 3.6, 3.7 and 3.8.

3.9 Other methods usable for instantaneous frequency measurement, not simulated in this paper [11], [12]

Orthogonal filters are used to extract the real and imaginary parts of the signal and finding the angular velocity of rotation of the voltage phasor in [11]. Wigner-Ville distribution (WVD) and STFT are investigated for instantaneous signal spectrum measurements in [12]. The instantaneous frequency is found as the frequency bin of the maximum value of the spectrogram $\theta_m(n)$ for given time, multiplied by the frequency resolution Δf, i.e. $f_i = \theta_m(n)\ \Delta f$. STFT should be preferred in all cases where the window length is less than the duration of signal frequency change (as it is by signals inspected in this paper). The STFT is simulated as the method 3.7 in this paper.

4 CONCLUSION

From the comparative analysis and from additional simulations in C-language of the methods 3.1 and 3.5 we can come to following conclusions:

- All the investigated methods work very well if the analyzed signal is a sinusoid without distortion and without noise. Errors in the order of 0.001% or 0.01% were found even for fastest investigated frequency sweep (5Hz/s).
- The least sensitive methods to the *harmonic distortion* are methods 3.5 and 3.1, (for sampling frequency 10 kHz errors are in the order 0.001%, higher errors are for lower sampling frequency). Very good are also methods 3.6 and 3.7 (errors in the order of 0.01%, but lower for higher window length). Errors of methods 3.4, 3.8 and 3.3 are in the order of 1%, and unusable for distorted signals is method 3.2 (errors are in the order of 10% or higher). Method 3.3 also gives sometimes wrong output in this case (isolated errors in the order of 10% or higher). Preliminary filtration of signal before processing therefore is necessary for methods 3.2 and 3.3.
- Results from investigation of the *influence of the additional noise* (Gaussian, zero mean, white) have lead to following maximum error bounds (3σ) for 1000 samples, 50 experiment repetitions and 30 dB SNR (method: 3σ): 3.6: 0.17%, 3.5: 0.47%, 3.7: 0.8%, 3.1: 1.2%, 3.4: 4.3%, 3.3: 5%, 3.8: 15% and 3.2: 70%. Since this investigation was performed for stationary signal, each error is found by statistical processing. Approximate levels of errors for lower values of SNR (more likely to be found in practice) can be found form Figs.1, 2, 4 and 6. As mentioned above, method 3.7

gives lower errors for nonzero frequency sweep (since interpolation respecting this sweep is incorporated in the algorithm).

- We have measured also the computation time necessary for finding one frequency value by all the simulated methods. When using MATLAB 5.3 and a PC with the CPU Celeron 418 MHz, this time was below 5 ms for all methods. Since the computation time is dependent on programmer's skill, more detailed data are not presented here. By implementations using digital signal processors the relations may be different. Several methods were implemented in real-time, using various digital signal processors. The method described in [11] using floating point TMS32030 DSP achieved errors 0.05%, 0.2% and 0.5% for SNR=70 dB, 50 dB and 30 dB. The method 3.5 using ADSP-21602 floating point DSP and a standard DAS-board for PC achieved errors of 0.003%, 0.03% and 0.3% (2σ) for SNR 70 dB, 50 dB and 30 dB [5-7], which correspond well to Fig.4, where we use the 3σ bound. Sampling frequencies were 6.4 kHz and 12.8 kHz, processing speed was 20 ms in tracking mode. Method 3.7 was implemented on ADSP-2181 fixed point DSP [13].

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