XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

IMPEDANCE MEASUREMENT TECHNIQUE BASED ON SINE-FIT

A. Carullo, A. Vallan
Dipartimento di Elettronica – Politecnico di Torino
Corso Duca degli Abruzzi, 24 – 10129 – Torino - Italy

Abstract: An impedance measurement system that is based on a voltmeter with digitising capabilities and on a sine-fit processing is proposed in this paper. The system can be arranged by employing standard instruments that are commonly available in calibration laboratories for the measurement of other electrical quantities. Two different solutions are proposed that differ in the used standards. One solution employs a multifunction calibrator that feeds the unknown impedance and a digital multimeter that acquires the voltage signal across the impedance; the other solution employs the same digital multimeter and a standard resistor in order to estimate the current that flows through the unknown impedance. In both cases the acquired samples are processed according to a four-parameter sine-fit algorithm. The system the authors have arranged allows impedance measurements in the frequency range of 40 Hz to 5 kHz to be obtained with expected standard uncertainty of about 0.04 % on the amplitude and of a less than 0.06 º on the phase.
Keywords: Impedance measurement, Signal processing, Sine-fit algorithm.

1 INTRODUCTION

Impedance measurement plays a key role in many fields, such as the characterisation of AC current shunts, the characterisation of electrical machines, and the measurement of non-electrical quantities by means of transducers whose output is an electrical impedance. The traceability of such measurements requires the availability of systems to calibrate impedance standards.

The large diffusion of digital signal processors and digital instruments with interfacing capability has permitted to improve the performance of classic impedance measuring methods [1-3] and to implement new measuring techniques, which are based on digital signal processing [4-6]. The proposed technique, which takes advantage of digital instruments and digital signal processing, permits to carry out impedance measurements by employing instruments which are commonly employed in many calibration laboratories for the measurement of other electrical quantities.

Two measurement systems are proposed that both employ a Digital Multi Meter (DMM) with digitising capabilities and a Personal Computer (PC). The DMM acquires the voltage signal across the unknown impedance and the PC processes the acquired samples according to a sine-fit algorithm. Amplitude and phase of the impedance are estimated thanks to the knowledge of the current, which in one system is provided by a multifunction calibrator, while in the other is obtained by acquiring the voltage signal across a standard resistor.

2 MEASURING PRINCIPLE

2.1 The calibrator based technique

The first proposed technique (see figure 1.a) employs a calibrator, which acts as a traceable generator, in order to feed the unknown impedance $\boldsymbol{Z_x}$ with a known current and a DMM that samples the voltage signal $\boldsymbol{V_x}$ across the impedance. The calibrator triggers the DMM so that the acquisition starts simultaneously with the positive zero-crossing of the current. Both calibrator and DMM are connected to a PC by means of an IEEE-488 standard interface. The PC receives the samples acquired by the DMM and processes them according to a four-parameter sine-fit algorithm [7], which provides frequency (f), offset (V_0), and amplitude components V_P and V_Q of the signal, that are respectively in phase and in quadrature with the current.

Amplitude and phase of the unknown impedance are eventually obtained as:

$$|\boldsymbol{Z}_x| = \frac{\sqrt{V_P^2 + V_Q^2}}{I} \cdot K_C(f) \quad ; \quad j_x = \mathrm{arctg}\left(\frac{V_Q}{V_P}\right) - j_C(f) \tag{1}$$

where I is the current generated by the calibrator, while $K_C(f)$ and $j_C(f)$ are correction terms that take the systematic effects into account.

The most important systematic effects that affect the amplitude of the unknown impedance are due to the frequency response of the integrating Analog-to-Digital Converter (ADC) internal to the DMM and to the frequency response of the DMM input stage.

The effect of the integrating ADC on the amplitude of the estimated parameters can be expressed by means of the amplitude of the relevant frequency response:

$$\left|\boldsymbol{H}_{AD}(f)\right| = \frac{\sin(p \cdot f \cdot T_C)}{p \cdot f \cdot T_C} \tag{2}$$

where f is the frequency and T_C is the conversion time of the ADC internal to the DMM.

The frequency response $\boldsymbol{H}_i(f)$ of the DMM input stage has also to be taken into account, therefore the correction term $K_C(f)$ is computed as:

$$K_C(f) = \frac{1}{\left|\boldsymbol{H}_{AD}(f)\right| \cdot \left|\boldsymbol{H}_i(f)\right|} \tag{3}$$

The measured phase of the unknown impedance is affected by a phase shift between the signal fitted from the acquired samples and the actual voltage signal $\boldsymbol{V_x}$. Such a phase shift depends on the frequency responses $\boldsymbol{H}_{AD}(f)$ and $\boldsymbol{H}_i(f)$, on the phase delay between current and voltage signals at the calibrator outputs and on the DMM trigger latency time. All such effects can be taken into account by means of a correction table $j_C(f)$, which is built by measuring the phase of the voltage signal $\boldsymbol{V_R}$ acquired across a characterised resistor at different frequencies:

$$j_C(f) = \mathrm{arctg}\left(\frac{V_{QR}}{V_{PR}}\right) - j_R(f) \tag{4}$$

where j_R is the resistor phase and V_{PR} and V_{QR} are in-phase and in-quadrature components of $\boldsymbol{V_R}$.

The uncertainty specifications of commercial calibrators are certified only for limited reactive loads, hence the calibrator based technique is suitable for the measurement of impedances with a low reactive component, e.g. for the characterisation of AC resistive shunts.

2.2 The standard-resistor based technique

When the impedance that have to be measured exceeds the limit for which the traceability of the calibrator is certified, an alternative solution can be adopted. Such a solution (see figure 1.b) employs an AC current shunt and an ordinary current generator in the place of the calibrator. The generator triggers the DMM so that the acquisitions of the voltage across the current shunt and the unknown impedance start simultaneously with the positive zero-crossing of the current.

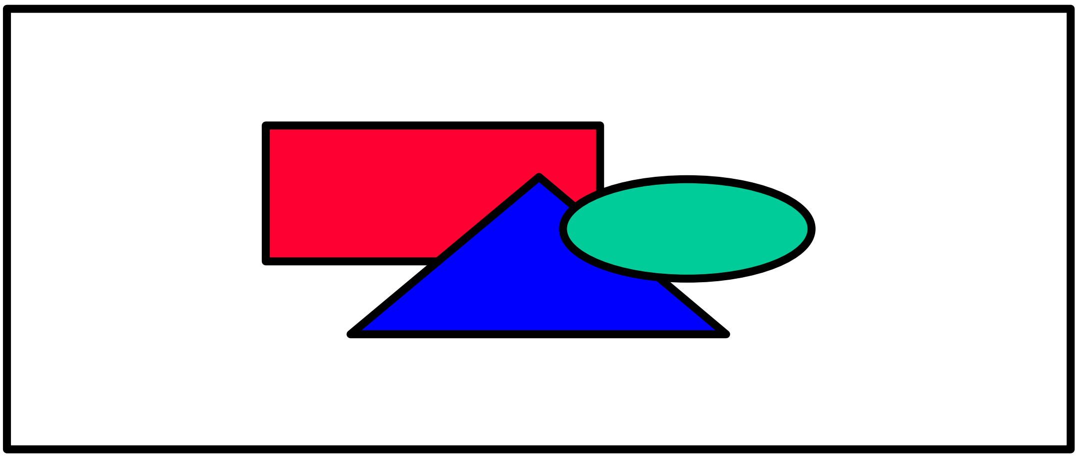

Figure 1. The measurement system based on the calibrator (a) and that based on the AC standard-resistor (b).

A two-step procedure allows the impedance measurement to be obtained. In the first step (switch in position 1) the voltage signal $\boldsymbol{V_s}$ across the current shunt $\boldsymbol{Z_s}$ is acquired and signal components in-phase (V_{PS}) and in-quadrature (V_{QS}) with respect to the current are estimated. In the second step (switch in position 2) the voltage signal across $\boldsymbol{Z_x}$ is acquired and in-phase (V_P) and in-quadrature (V_Q) components of the signal with respect to the current are estimated. Amplitude and phase of $\boldsymbol{Z_X}$ are eventually obtained as:

$$\left|\boldsymbol{Z_x}\right| = \sqrt{\frac{V_P^2 + V_Q^2}{V_{PS}^2 + V_{QS}^2}} \cdot \left|\boldsymbol{Z_S}\right| \quad ; \quad j_X = \mathrm{arctg}\left(\frac{V_Q}{V_P}\right) - \mathrm{arctg}\left(\frac{V_{QS}}{V_{PS}}\right) + j_S \tag{5}$$

where j_S is the phase of the current shunt.

One should note that in this case no corrections are required that take systematic effects into account, as equations (5) allow an implicit compensation for such effects to be obtained.

3 UNCERTAINTY ANALYSIS

3.1 Calibrator based technique

The amplitude of the unknown impedance is obtained by means of the first equation in (1), therefore the combined standard uncertainty can be estimated as [8]:

$$u_C\left(\left|\boldsymbol{Z_x}\right|\right) = \sqrt{\sum_{i=1}^{4} c_i^2 \cdot u^2(x_i)} \tag{6}$$

where c_i are the sensitivity coefficients of $|\boldsymbol{Z_X}|$ with respect to the input quantities I, V_P, V_Q and K_C, while $u(x_i)$ are their standard uncertainty. The standard uncertainty of K_C can be obtained by means of an equation similar to the (6), where the input quantities are f, T_C and $|\boldsymbol{H_i}|$.

The current uncertainty depends on the employed calibrator, while the uncertainty of the parameters V_P and V_Q depends on the DMM uncertainty. Thanks to the sine-fit algorithm, the parameter uncertainty is reduced in dependence of the correlation between the acquired samples, which strongly depends on the noise superimposed to the voltage signal. In the worst case (correlation coefficients equal to one), the uncertainty of V_P and V_Q is that of each sample.

The uncertainty of the impedance phase j_x depends, besides the uncertainty of the quantities present in the second equation in (1), on the skew of the acquisition starting-point. Such a skew is due to two main phenomena: the non-synchronisation between the time bases of calibrator and DMM and the presence of noise superimposed to the trigger signal. The effect of the former depends on the resolution of the DMM time-base, while that of the latter is related to noise amplitude and trigger-signal slope. The combined standard uncertainty of the measured phase can hence be obtained as:

$$u_C(j_X) = \sqrt{\sum_{i=1}^{3} c_i^2 \cdot u^2(x_i) + (2p \cdot f)^2 \cdot u^2(t_S)} \tag{7}$$

where c_i are the sensitivity coefficients of j_X with respect to the input quantities V_P, V_Q and j_C, $u(x_i)$ are their standard uncertainties, and t_S is the skew of the acquisition starting-point.

The standard uncertainty of j_C can be obtained by means of an equation similar to the (7), where the input quantities are (see equation 4) V_{PR}, V_{QR} and j_R.

3.2 Standard-resistor based technique

In the standard-resistor based technique, the amplitude of the unknown impedance is obtained by means of the first equation in (5), therefore the combined standard uncertainty is estimated as:

$$u_C\left(\left|\boldsymbol{Z_x}\right|\right) = \sqrt{\sum_{i=1}^{5} c_i^2 \cdot u^2(x_i)} \tag{8}$$

where c_i are the sensitivity coefficients of $|\boldsymbol{Z_X}|$ with respect to the input quantities V_P, V_Q, V_{PS}, V_{QS} and $|\boldsymbol{Z_S}|$, while $u(x_i)$ are their standard uncertainties.

From the second equation in (5), the combined standard uncertainty of the impedance phase is obtained as:

$$u_C(j_X) = \sqrt{\sum_{i=1}^{5} c_i^2 \cdot u^2(x_i) + 2 \cdot (2p \cdot f)^2 \cdot u^2(t_S)} \tag{9}$$

where c_i are the sensitivity coefficients of $\dot{J}_X$ with respect to the input quantities V_P, V_Q, V_{PS}, V_{QS} and $\dot{J}_S$, $u(x_i)$ are their standard uncertainties, and t_S is the skew of the acquisition starting-point. In this case, another possible uncertainty contribution is the drift of the current generator within the measurement time interval.

4 EXPERIMENTAL SET-UP

The two proposed techniques have been arranged by employing a Hewlett Packard 3458A DMM, which is able to digitise voltage signals with a maximum sampling rate of 100 kSample/s and a time resolution of 100 ns.

In both cases the conversion time of the ADC internal to the DMM has been set to 10 μs, in order to allow an amplitude resolution of 18 bits and an uncertainty of less than 0.005 % on each sample to be obtained. Such a choice limits the DMM sampling rate to 50 kSamples/s and corresponds to an effect of the ADC frequency response of about 0.4 % @ 5 kHz, as can be computed by equation (2).

The DMM specifications state that the bandwidth of the DCV input path is limited to 80 kHz when the 0.1 V range is selected and to 150 kHz when the 1 V or 10 V are selected. Unfortunately, no further information is available that allows the frequency response $\boldsymbol{H}_i(f)$ of the DMM to be estimated. In order to overcome the problem, the amplitude of the frequency response of the DMM is estimated in the frequency range of 40 Hz to 5 kHz by feeding the DMM with a known voltage and by comparing such a voltage with that obtained with the sine-fit algorithm.

If instruments more accurate than the HP-3458A DMM are not available, it is possible to built the correction table by measuring the applied voltage by means of the DMM itself set in AC mode. In this case the uncertainty of the measured AC voltage, which is of about 0.01 % in the frequency range of 40 Hz to 5 kHz, becomes the main uncertainty contribution in the determination of the amplitude correction-table. For this reason an equivalent procedure can be implemented by directly measuring the root mean square value of the voltage with the DMM set in AC mode and by applying the sine-fit based procedure only for the phase determination. However, for generality reasons the following analysis refers to the calibrator based technique as described in the previous sections.

A Fluke 5520A multifunction calibrator has been used to implement the calibrator based technique. The calibrator has two couples of output terminals, which are referred to as normal and auxiliary output terminals. The auxiliary output terminals, which can provide sinusoidal currents with expanded standard uncertainty (coverage factor k = 2.73) of less than 0.05 % in the frequency range of 40 Hz to 1 kHz and of less than 0.1 % in the frequency range of 1 kHz to 5 kHz, feed the unknown impedance. The normal output terminals, which provide voltage signals, trigger the DMM by means of a square signal in phase with the current signal. The rise time of the square signal is of less than 1 μs, while the noise amplitude is lower than 0.1 % of the signal amplitude.

The skew of the acquisition starting-point, which is mainly due to the resolution of the DMM time-base, can be considered a random variable with average value equal to zero and probability density function uniformly distributed between – 50 ns and + 50 ns.

The resistor employed to build the phase correction table has been characterised with a phase uncertainty of about 0.05 º.

Table 1 shows the standard uncertainty of the input quantities that are involved in the determination of amplitude and phase of the unknown impedance by employing the calibrator based technique. In the table, $u(v_i)$ is the standard uncertainty of the acquired samples and hence that of the parameters V_P, V_Q, V_{PR} and V_{QR}. By using such uncertainties, a relative standard uncertainty of less than 0.04 % on the impedance amplitude and a standard uncertainty of less than 0.06 º on the phase are expected in the frequency range of 40 Hz to 5 kHz.

The main uncertainty contributions are the current uncertainty of the calibrator for the impedance amplitude and the skew of the acquisition starting-point for the impedance phase.

The standard-resistor based technique has been arranged by employing a set of fives AC current shunts (0.1 Ω, 1 Ω, 10 Ω, 100 Ω and 1 kΩ), whose standard uncertainty is of about 0.02% for the amplitude and of about 0.03 º for the phase in the frequency range of 40 Hz to 5 kHz.

The current generator is made up of a signal generator that feeds a low-noise low-distortion current amplifier. The signal generator triggers the DMM by means of a TTL square wave synchronised to the main function output, which is available at the “sync” output of the signal generator. The rise time of the trigger signal is of less than 30 ns and the noise amplitude is lower than 0.5 % of the signal amplitude. In this condition, the skew of the acquisition starting-point is that estimated for the calibrator based technique. The drift of the current generator within the measurement time interval (few seconds) has a negligible effect on the overall uncertainty.

Table 1. Quantities involved in the estimation of amplitude and phase impedance and corresponding standard uncertainty.

Quantity (x_i)	Uncertainty	Distribution	$u(x_i)$
I	0.05 % (k=2.73); 40 Hz < f < 1 kHz 0.1 % (k=2.73); 1 kHz < f < 5 kHz		0.018 % 0.036 %
v_i	0.005 %	Rectangular	0.003 %
T_C	50 ns	Rectangular	30 ns
$\|H_i(f)\|$	0.01 %	Rectangular	0.006 %
t_S	50 ns	Rectangular	30 ns
j_R	0.05 °	Rectangular	0.03 °

The expected standard uncertainty of impedance amplitude and phase, which are estimated by means of equations (8) and (9), are respectively of less than 0.03 % and of less than 0.06 ° in the frequency range of 40 Hz to 5 kHz. In this case, the uncertainty of the impedance amplitude is mainly related to the calibration uncertainty of the current shunts, while the skew of the acquisition starting-point is the main contribution of the phase uncertainty.

5 EXPERIMENTAL RESULTS

The phase correction table $\varphi_C(f)$ (see the second equation in 1) has been built by measuring the phase of the voltage signal acquired across a characterised resistor at different points in the frequency range of 40 Hz to 5 kHz and for the different ranges of the DMM and the calibrator.

Each value of the phase correction table has been estimated by averaging a hundred measured values, in order to make the skew uncertainty contribution negligible with respect to the phase uncertainty of the resistor used for building the phase correction table.

Maximum values of about 3 ° and experimental standard deviation of the average of the measured phase of about 0.007 ° @ 5 kHz have been obtained during the building of the phase correction table.

The amplitude $|H_i(f)|$ of the DMM frequency response has been estimated by feeding the DMM with the calibrator and comparing the voltage obtained with the sine-fit based procedure with that measured with the DMM set in AC mode. Maximum deviation of about 0.1 % @ 5 kHz have been obtained with uncertainty of almost 0.01 %.

An experimental preliminary characterisation of the proposed measurement systems has been then performed by comparison with a commercial low-frequency impedance analyser (HP-4192A), whose cost is of about 15000 Euro. Amplitude and phase basic uncertainties of the impedance analyser are respectively of 0.15 % and 0.08 °.

For both techniques, each measured value is obtained by acquiring a hundred voltage waveforms and by averaging the corresponding measurements, in order to make the skew contribution negligible.

Figure 2 shows, as an example, the results obtained during the calibration of two different AC standard resistors (nominal value of 1 Ω and 100 Ω) in the frequency range of 50 Hz to 5 kHz. The dots represent the measurements carried out with the calibrator based technique, while the line segments are the uncertainty ranges of the impedance analyser, which are centred with respect to its

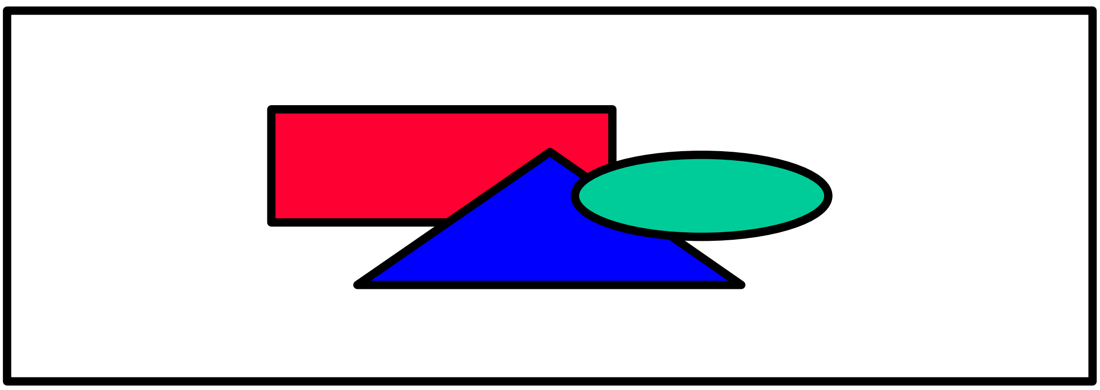

Figure 2. Calibration of two AC standard resistors (100 Ω e 1 Ω): measurements obtained with the calibrator based technique (dots) and corresponding uncertainty ranges of the impedance analyser (line segments).

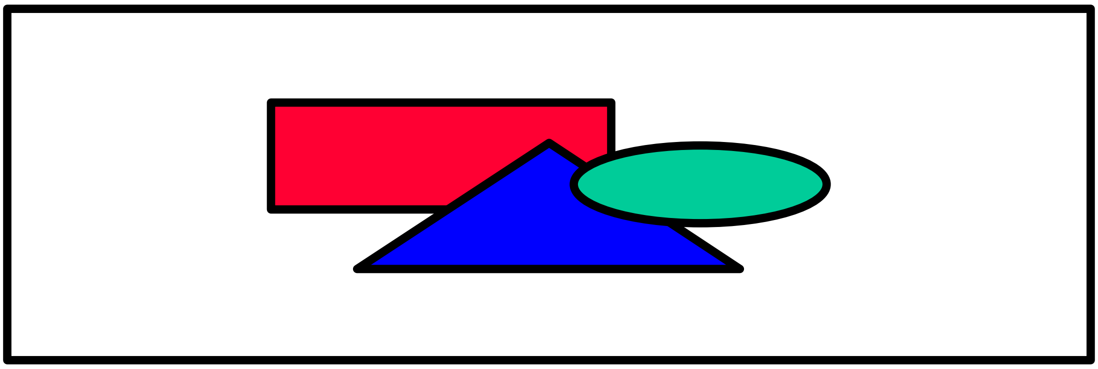

Figure 3. Calibration of an impedance (100 Ω resistor in series with 10 mH inductor) with the standard-resistor based technique: amplitude and phase behaviours and differences with respect to the impedance analyser measurements.

measured values. A maximum difference of about 0.05 % for the amplitude and of about 0.05 ° for the phase is obtained in both cases.

Figure 3 shows the measurement performed with the standard-resistor based technique of an impedance that is made up of a 100 Ω resistor in series with a 10 mH inductor. Amplitude and phase differences of about 0.05 % and of a few tenths of degrees have been obtained in the frequency range of 50 Hz to 5 kHz.

The obtained results show that the differences between the measurements carried out with the proposed system and the impedance analyser are of the same order of the expected uncertainty. Unfortunately such differences are also of the same order of the uncertainty stated in the impedance analyser specifications, thus highlighting that this instrument is not suitable to act as a reference.

6 CONCLUSIONS

An impedance measurement technique has been proposed in this paper that can be arranged by employing instruments commonly available in calibration laboratories for the measurement of other electrical quantities. The measurement system the authors have arranged allows impedance measurements in the frequency range of 40 Hz to 5 kHz to be obtained with amplitude and phase standard uncertainties that are expected to be of about 0.03 % and of about 0.06 °. A preliminary characterisation of the proposed measurement systems, which has been performed by comparison with a commercial impedance analyser, have shown a good agreement between the obtained and the expected results. Unfortunately, the employed impedance analyser is not suitable for a metrological characterisation of the system, since its uncertainty is not low enough. The authors are hence planning a metrological characterisation of the proposed system by using more accurate standard instruments, in order to state its actual uncertainty.

REFERENCES

[1] W. Helbach, P. Marczinowski, G. Trenkler, High-precision automatic digital ac bridge, *IEEE Trans. on IM* **IM-32**(1) (1983) 159-162.
[2] R.D. Cutkosky, An automatic high-precision audiofrequency capacitance bridge, *IEEE Trans. on IM* **IM-34**(3) (1985) 383-392.
[3] B.C. Waltrip, N.M. Oldham, Digital impedance bridge, *IEEE Trans. on IM* **44**(2) (1995) 436-439.
[4] L. Angrisani, A. Baccigalupi, A. Pietrosanto, A Digital Signal-Processing Instrument for Impedance Measurement, *IEEE Trans.on IM* **45**(6) (1996) 930-934.
[5] L. Angrisani, A. Baccigalupi, A. Pietrosanto, A VXI instrument for real-time tracking of impedances, *Measurement* **20**(4) (1997) 277-285.
[6] J.G. Liu, U. Frühauf, A. Schönecker, Accuracy improvement of impedance measurement by using the self-calibration, *Measurement* **25** (1999) 213-225.
[7] J.P. Deyst, T.M. Souders, O.M. Solomon Jr., Bounds on Least-Squares Four-Parameter Sine-Fit Errors due to Harmonic Distortion and Noise, *IEEE Trans.on IM* **44**(3) (1995) 637-642.
[8] International Organization for Standardization, Guide to the Expression of Uncertainty in Measurement.

AUTHORS: Dipartimento di Elettronica, Politecnico di Torino, corso Duca degli Abruzzi, 24 – 10129 Torino, ITALY, Phone: +39-011-564-4114; Fax: +39-011-564-4099, E-mail: carullo@polito.it