XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

OPTIMUM SELECTION OF INPUT SIGNALS FOR THE CALIBRATION OF MEASUREMENT DEVICES

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Abstract: A procedure for optimal selection of sample input signals to get the best calibration characteristics of measuring apparatus is proposed. As an example the problem of optimal selection of standart pressure setters when calibrating differential pressure measurer is solved.
Keywords: Calibration, measurement accuracy, least squares method, A-optimality criterion, differential pressure transducer.

1 INTRODUCTION

Accurate measurement is the basis of almost all engineering application, since uncertainity inherently exists in the nature of any measuring apparatus. The cost of a measuring apparatus, on the other hand, increases with its accuracy. Therefore low cost accurate measurement devices are one of the main goal of engineers. One way of reducing the cost of accuracy is the calibration process. Therefore this paper deals with the calibration of a low cost sensor with a high accurate one.

The method can be explained as follows. A sample signal whose characteristics are known priori is applied to both the low cost sensor and the high accurate sensor and outputs of both sensors are recorded. This experiment is repeated for a variety of input signals and the results are tabulated. The calibration characteristics can be evaluated from this table. Interpolation techniques should be used when this table does not have the required data. From the practical point of view, this characteristics should be in a polynomial form. The accuracy of this polynomial depends on the noise-free data which was used to obtain the characteristics. The reduce the effect of noise, excessive number of data should be used. However this requires more experiments and that will increse the cost. Thus the main question becomes the evaluation of accurate calibration characteristics with a few number of experimental data. On the other hand, though it is paradoxical, the application of equidistant sample signal to get the best calibration characteristic is erroneous [1].

Therefore this work deals with the problem of optimal selection of sample signal compositon with a view to get the best calibration characteristics (planning experiment problem)

2 PROBLEM STATEMENT

From practical considerations the calibration curve should be in a polynomial form as follows.

$$y_i = a_0 + a_1 p_i + a_2 p_i^2 + \ldots + a_m p_i^m \quad (1)$$

where y_i is the output of the low cost transducer and p_i are the outputs of the high precision transducer, $a_0, a_1, \ldots, a_m$ are the calibration curve coefficients. Measurement contains random noises in Gaussian form

$$z_i = y_i + \delta_i = a_0 + a_1 p_i + a_2 p_i^2 + \ldots + a_m p_i^m + \delta_i \quad (2)$$

where z_i is the measurement result, δ_i is meusurement error with zero mean and σ^2 variance. Let the calibration curve coefficients be denoted as $\theta = [a_0, a_1, \ldots, a_m]^T$

The coefficients in these polynomials were evaluated in [2] by the least squares method . The expressions used to make the evaluation had the form:

$$\hat{q} = (X^T X)^{-1}(X^T z) \quad ; \qquad D(\hat{q}) = (X^T X)^{-1} \sigma^2 \quad (3)$$

where $z^T = [z_1 , z_2 , ... , z_n]$ is the vector of the measurements;

$$X = \begin{bmatrix} 1 & p_1 & p_1^2 & . & . & . & p_1^m \\ 1 & p_2 & p_2^2 & . & . & . & p_2^m \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & p_n & p_n^2 & . & . & . & p_n^m \end{bmatrix} \quad (4)$$

is the matrix of the known coordinates (here, p_1 , p_2 ,..., p_n are values that are producible by the standard instruments).

The values of $p_1, p_2, ..., p_n$ the outputs of the high accuracy transducer, should be such that the polynomial characteristics given by (1) best approximates the real calibration characterisitics. Thus the problem can be stated as follows: Find the values of $p_1, p_2, ..., p_n$ such that the values of $a_1, a_2, ..., a_m$ are optimum in a sense that a given performance criterion is minimum.

As mentioned above the matrix D can be used as a measure of the error between the low cost transducer and the high precision transducer. A performance criteria for the minimum of the matrix D can be selected in several ways. A scalar measure of the matrix D,

the A optimality -the trace (sum of diagonal elements) criterion,
the D optimality -the generalize dispersion (determinant) criterion,
the E optimality-maximal eigenvalue of matrix criterion
sum of all the elements of the matrix and etc.

It should be noted that the choice of the scalar measure of covariance matrix estimation errors is founded by human experience and intuition. The mathematical simplicity and convenience of obtaining analytical results are of importance here. For this reason A optimality criterion is used in this work, i.e

$$\min_{p_i} [Tr(X^T X)^{-1} \sigma^2] \quad (5)$$

is sought. The values of p_1, p_2 ,..., p_n found by solving the above equations should be in the range 0... p_{max}. Otherwise the solution is invalid.

3 THE SOLUTION ALGORITHM

Let the objective function is denoted as:

$$f (p_1, p_2, ... , p_n)= Tr\{D(p_1, p_2, .., p_n)\} \quad (6)$$

As explained above the problem is a constrained optimization problem. The objective function is a multivariable, nonlinear, continuous and has derivative in the considered interval.

Assume that the minimum of $(p_1, p_2, ... , p_n)$ exists for the following values of $p_1, p_2, ..., p_n$
$p^* = [p_1^*, p_2^*, ... , p_n^*]^T$
In order that p^* is a minimum of D, the following conditions should be satisfied [3]

$$\nabla f(p^*) = 0 \quad (7)$$

$$\nabla^2 f(p^*) \text{ is semi posititive} \quad (8)$$

where ∇ denotes the gradiant. The extremum condition given by (7) can explicitly be written as:

$$\partial [TrD(p_1, p_2, ... ,p_n)] /\partial p_i =0, \quad (i = 1,2, ... ,n) \quad (9).$$

When the derivatives in (9) is calculated n algeabric equations with n unknowns are obtained.

$$Q_i (p_1, p_2, ... ,p_n) =0, \quad (i = 1,2, ... ,n). \quad (10)$$

where n denotes the number of measurements. As the value of n increases, the calculation of $Tr\{D(p_1, p_2, .., p_n)\}$and the derivatives given by (9) become difficult especially when n>3. For this reason computer programs that can process symbolic characters such as MAPLE and MATHEMATICA should be used together with numerical routines which solves nonlinear equations.

Numerical routines such as gradent descent algorithms [4] can be used. It should be noted that the sign of $\nabla^2 f(p)$ should be caalculated together with $\nabla f(p)$ in order to determine whether the values found corresponds a local minimum or a local maximum. Furthermore those values which makes $Tr\{D(p_1, p_2, .., p_n)\}$ minimum should be in the range $0\text{-}p_{max}$. The solution set which satisfy the above conditions can be used to calculate the polynomial coefficients given in (1). This polynomial best approximates the calibration characteristics between the low cost and high precision equipment and can be used to extract the accurate values from the outputs of the low cost transducer.

4 SIMULATION RESULTS

The method explained above is used for calibration of a differential pressure transducers. The transducer measurement range was 0 to 1.6 atm. and standard error was σ=0.0026. The characteristic polynomial was assumed is of the quadratic form, i.e

$$y= a_0 + a_1 p_i + a_2 p_i^2$$

It is also assumed that the measurement noise is of the gaussian with zero mean and variance σ^2.

Interval selection Criteria	Pressure Values			$Tr[D(\hat{q})]$
Optimum intervals	0	0.841	1.6	0.0393
Equal intervals	0	0.800	1.6	0.0395

Table 1. Optimum and equal intervals and corresponding $Tr[D(\hat{q})]$ values for n=3.

Interval selection Criteria	Pressure Values				$Tr[D(\hat{q})]$
Optimum intervals	0	0.161	0.8064	1.6	0.0335
Equal intervals	0	0.533	1.0667	1.6	0.0337

Table 2. Optimum and equal intervals and corresponding $Tr[D(\hat{q})]$ values for n=4.

Simulation is performed for n=3 and n=4. The method is used to obtain the optimum coefficients of the characteristic polynomial for a selected criteria. For simplicity A-optimality criterion is used for this work though any other criterion can also be used. Closed form algebraic equations is calculated to solve the equation given in (5). The software program MAPPLE [5] is used to find the trace and its gradient in symbolic form and MATLAB is used to find the optimum values of p_i , i=1,…,n. The optimum input values and corresponding $Tr[D(\hat{q})]$ values are tabulated in Table.1 and Table.2. For a comparison the optimality criterian values where pressure values p_i , i=1,…,n are equally spaced are also shown in the table. As can be seen clearly from the table the optimality criterian values where the coefficients are calculated by our method are smaller than that where p_i , i=1,…,n are equally spaced. As a result, the suggested method can be used to obtain the best (secilmis kriteriye gore) calibration characteristics polynomial. Simulation results are also performed for n>4. For n>7 optimality criterian values are much smaller with aur method than that where measurement values are equally spaced.

5 CONCLUSION

A procedure for optimal selection of sample input signals to get the best calibration characteristics of measuring apparatus was proposed. It was shown that application of equidistant sample signals does not provide the optimum calibration characteristics. The advantage of the proposed method was demonsrated on the calibration of a differential pressure sensor. An expensive sensor can be replaced by a cheap one with a good calibration process. When the cost of a high accurate sensor is in question the proposed method provides a practical solution.

Further work includes calculation of the optimal calibration characteristics where different optimality criterias are used.

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