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STATISTICAL APPROACHES FOR SEPARATING THE RESPONSIBILITY FOR HARMONIC LOSSES

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Abstract. In practical situations, the voltage and the current at any supply terminal are not steady-state quantities, since the customers' load operation and the equivalent impedance of the network vary randomly in time. A search is active for on-line investigation methods able to provide the separation of the responsibility for the electric-power quality degradation. Under nonstationary conditions the methods proposed in the literature require the processing of a great amount of data. In this connection, a statistical approach is recalled in this paper which is aimed to gauge information about the responsibility for the harmonic losses by processing only a limited amount of data. The results of some experimental work show the good performance of the proposed approach.

Keywords: Electrical Quantities; Digital Signal Processing; Statistical Evaluation of Measurement Results.

1 INTRODUCTION

Electricity can be treated as a processed material, having immediate delivery in continuous flow, which derives from a continuous process [1]. The electricity quality may also depend on phenomena occurring outside the continuous process, such as the interaction between the end-use processes and the continuous-process hardware. The distortion of the voltage supplied by public distribution networks, along with the increase of losses in distribution transformers, phase and neutral conductors, are well-known effects of this interaction. The distribution network behaves like a "healthy carrier", so that the current and voltage harmonics measured in a given metering point may arise from the operation of the loads of both the monitored customer and the other customers connected to the same point of common coupling. The reliable operation of the customer loads can be adversely affected by the voltage distortion and this may turn into customer unsatisfaction. Even if harmonics are a minor cause of customer complaints, they give rise to power loss increase which may require re-sizing of hardware components of the continuous process. This turns into increased electricity costs that are ultimately borne by all customers.

The present transition of the electricity market from monopolistic situations to competition emphasizes the need for measurement systems implementing on-line investigation methods able to provide information about the traceability in a product sense (see ISO Std. [2]), of the power quality issues. Several measurement methods have been recently proposed to share the responsibility for harmonic distortion between the customer and the utility. They substantially focus on two different issues: the sign analysis of the harmonic active powers and the conformity between the voltage and current waveforms [3]. However, under nonstationary conditions they require the on-line processing of an enormous amount of data.

A statistical method for the evaluation of the responsibility for the harmonic power losses is recalled in this paper. It needs for processing a limited amount of data even in the presence of randomly modulated harmonic distortion. The paper reports the results of some experimental work showing the good performance of the considered approach.

2 THEORETICAL BACKGROUND

Due to the random variation in time of both the load operation and the equivalent impedance of the supply network, the current $i(t)$ and the voltage $u(t)$ in a given metering point are not steady-state quantities and can be treated as stochastic processes. From this point of view, they can be expressed by means of a deterministic sinusoidal carrier $c(t) = C_1 \sin(2\pi/T)t$ (where T is the carrier period) and a modulating random component $m(t)$:

$$i(t) = \sqrt{2} \cdot C_{i,1} \sin(2\pi/T)t \cdot m_i(t) \qquad (1)$$

$$u(t) = \sqrt{2} \cdot C_{u,1} \sin(2\pi/T)t \cdot m_u(t)\,. \tag{2}$$

In (1) and (2) the suffix 1 refers to the industrial frequency. Moreover, we assume that the billing interval T_β can be sub-divided into a suitable number $T_\beta/(nT)$ of intervals (n is an integer) in which the modulating functions can be considered constants. Let us denote with the integer s the generic sub-interval. The theory and the algorithms valid for periodic operation can be therefore applied for the signal characterization within each sub-interval.
The harmonic analysis of the voltage and current signals in each sub-interval s allows for determining the sign of the active power $P_{s,r}$ associated to the voltage, $u_{s,r}(t)$, and current, $i_{s,r}(t)$, components of harmonic order r. This way, the line current $i_s(t)$ can be decomposed into two orthogonal components [3]:

$$i_s(t) = i_s^{(S)}(t) + i_s^{(L)}(t)\,. \tag{3}$$

In (3) the quantity:

$$i_s^{(S)}(t) \stackrel{\Delta}{=} i_{s,1}(t) + \sum_{r \neq 1 | P_{s,r} \geq 0}^{N} i_{s,r}(t) \tag{4}$$

is considered the current component responsible for the losses at the load side, whilst:

$$i_s^{(L)}(t) \stackrel{\Delta}{=} \sum_{r | P_{s,r} < 0}^{N} i_{s,r}(t) \tag{5}$$

is considered the current component responsible for the harmonic losses at the supply side caused by the operation of the monitored-customer distorting loads. In (4), $i_{s,1}(t)$ is the current component at industrial frequency.

According to the approach recalled, the separation of the responsibility for the harmonic losses can be performed by measuring the current rms values $I_s^{(S)}$ and $I_s^{(L)}$ for $s \in [1; T_\beta/(nT)]$.

The harmonic analysis of $i_s(t)$ and $u_s(t)$ within each s-th sub-interval, along with the sign analysis of $P_{s,r}$ for any $r \neq 1$, does not turn into strict requirements for the measurement system.
It will be shown in the following Section 3 that the estimate of both the current rms values $I_s^{(S)}$ and $I_s^{(L)}$ can be attained by properly processing a more limited amount of data if a sub-set of the above observation sub-intervals is randomly chosen.

3 THE PROPOSED APPROACH

Let us consider the stochastic process $x(t)$ representing, in the discrete time representation, the array of the rms values $I_s^{(S)}$ and $I_s^{(L)}$ of $i_s^{(S)}$ and $i_s^{(L)}$ respectively, measured within each interval s over T_β. For a better description of the theoretical arguments that follow, we consider x as a continuous-time stochastic process.
Let us consider the periodic repetition x_p of x:

$$x_p(t) = \sum_n x(t) * \delta(t - nT_\beta)\,, \tag{6}$$

where the symbol $*$ denotes the convolution function. The signal $x_p(t)$ is periodic with period T_β and can be characterized by using the Fourier series:

$$x_p(t) = \sum_{r=-M}^{+M} \underline{X}_{p,r} e^{i r \omega_0 t} \tag{7}$$

In (7), it is $\omega_0 = 2\pi/T_\beta$; moreover M is the maximum harmonic order of x_p and, finally, the r-th spectral component is given by:

$$\underline{X}_{p,r} = \frac{1}{\sqrt{2}T_\beta} \int_{-T_\beta/2}^{T_\beta/2} x_p(t) e^{-j r \omega_0 t} dt\,. \tag{8}$$

Let us split the generic complex rms value $\underline{X}_{p,r}$ into two contributions: a complex constant $\underline{A}_{p,r}$ and a random complex variable $\underline{\varepsilon}_{p,r}$:

$$\underline{X}_{p,r} = \underline{A}_{p,r} + \underline{\varepsilon}_{p,r}\,. \tag{9}$$

The generic random variable $\underline{\varepsilon}_{p,r}$ has zero expected value, i.e. $M\{\underline{\varepsilon}_{p,r}\} = 0$ and variance $Var\{\underline{\varepsilon}_{p,r} \cdot \underline{\varepsilon}^*_{p,r}\} = \sigma_r^2$. The symbol $(\cdot)^*$ denotes the complex conjugate of $(\cdot)$. $\underline{\varepsilon}_{p,r}$ is assumed to be an orthogonal noncorrelated random variable (i.e., $M\left\{\underline{\varepsilon}_{p,n}\underline{\varepsilon}^{*}_{p,m}\right\}=0$ for any $n \neq m$).

The expected value of $\underline{X}_{p,r}$ and its variance, respectively, are given by:

$$M\left\{\underline{X}_{p,r}\right\}=M\left\{\frac{1}{T_\beta}\int_{-T_\beta/2}^{T_\beta/2} x_p(t)e^{-jr\omega_0 t}dt\right\}=\frac{1}{T_\beta}\int_{-T_\beta/2}^{T_\beta/2}\sum_{n=-M}^{+M} M\left\{\underline{A}_{p,r}+\underline{\varepsilon}_{p,r}\right\}e^{jn\omega_0 t}\cdot e^{-jr\omega_0 t}dt=\underline{A}_{p,r} \tag{10}$$

$$Var\left\{\underline{X}_{p,r}\right\}=M\left\{\left|\underline{X}_{p,r}-M\left\{\underline{X}_{p,r}\right\}\right|^2\right\}=M\left\{\left|\underline{A}_{p,r}+\underline{\varepsilon}_{p,r}-\underline{A}_{p,r}\right|^2\right\}=\sigma_r^2 . \tag{11}$$

3.1 Estimate of the spectral components of *x*(t)

In order to estimate the rms values $I_s^{(S)}$ and $I_s^{(L)}$ over T_β we define a random sampling technique in which t_k represents the initial instant of the signal acquisition within the *k*-th sub-interval. If we denote with x_k a random variable, uniformly distributed in the interval [-*a*,*a*] with *a*=1/2, t_k is given by the following relationship:

$$t_k=(k+x_k)T_C. \tag{12}$$

The current and voltage signals are therefore acquired for *nT* periods starting from the random instants t_k. In (12) it is *k*=1,2,… and T_C is the mean sampling time, usually much greater than *T*.

It was verified, [4], that this sampling strategy and the associated filtering algorithms do not introduce any bandwidth limitation when used to characterize a finite bandwidth signal.

In this connection, let us call $x(t_k)$ the value of *x* in the *k*-th sub-interval.

The estimate $\underline{\tilde{X}}_{p,r}$ of the *r*-th spectral component $\underline{X}_{p,r}$ is given by [5,6]:

$$\underline{\tilde{X}}_{p,r}=\frac{1}{N}\sum_{k=0}^{N-1}x(t_k)e^{-j\omega_0 r t_k} , \tag{13}$$

where *N* is the number of the samples acquired at the instants $t_k=(k+x_k)T_C$. In order to characterize the estimate (13), let us evaluate the statistical parameters of $\underline{\tilde{X}}_{p,r}$, i.e. its expected value $M\left\{\underline{\tilde{X}}_{p,r}\right\}$ and variance $Var\left\{\underline{\tilde{X}}_{p,r}\right\}$. We show in Appendix 6.A that the asymptotic expected value is:

$$\underset{\rightarrow}{M}\left\{\underline{\tilde{X}}_{p,r}\right\}=\underline{A}_{p,r} , \tag{14}$$

i.e., the asymptotic bias is null. It can also be shown (Appendix 6.B) that the asymptotic variance is:

$$\underset{\rightarrow}{Var}\left\{\underline{\tilde{X}}_{p,r}\right\}=\underset{\rightarrow}{M}\left\{\left|\underline{\tilde{X}}_{p,r}-M\left\{\underline{\tilde{X}}_{p,r}\right\}\right|^2\right\}=\sigma_r^2 \tag{15}$$

which coincides with (11). Hence, the estimate of the spectral components of *x* is correct, consistent and efficient.

Fig. 1 is the plot of $\underset{\rightarrow}{Var}\left\{\underline{\tilde{X}}_{p,r}\right\}$ vs. the number *N* of periods acquired, in the case of σ_r^2=0.1 volt^2.

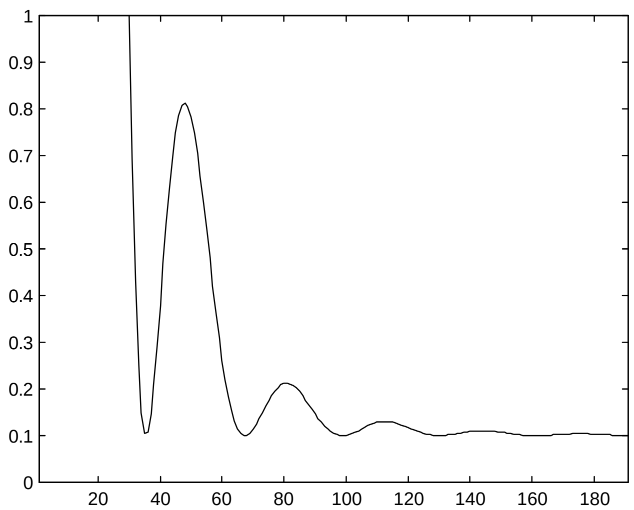

Fig. 1 $\underset{\rightarrow}{Var}\left\{\underline{\tilde{X}}_{p,r}\right\}$ vs. the number *N* of periods acquired

The inverse of the Fourier transform can than be computed to gauge the estimate $\tilde{x}_p$ of x_p, which coincides with x within T_β, [7].

4 EXPERIMENTAL WORK

The approach described in the previous Section was applied for estimating the trend over T_β of both $I^{(S)}$ and $I^{(L)}$ by evaluating $I_s^{(S)}$ and $I_s^{(L)}$ in a low number of sub-intervals randomly chosen according to the technique (12) from the whole set of the $T_\beta/(nT)$ sub-intervals.

Some simulations were carried out in order to verify the proposed approach capabilities and performance. In this connection, voltage and current waveforms, with 50-Hz fundamental frequency and harmonic components with amplitudes and phases randomly variable around constant default values, were generated. It was assumed that all the random effects can be considered constant factors inside each couple of periods of $i(t)$, but changeable from any couple of periods to the following ones.

$i(t)$ and $u(t)$ were synchronously sampled within a couple of periods starting from each t_k and belonging to $\{k\}$, a sub-set of $\{s\}=[1;T_\beta/2T]$, so that the array of the rms values $I_k^{(S)}$ and $I_k^{(L)}$ can be set up. The DFT algorithm is than applied to these two arrays to evaluate the spectral components. The inverse Fourier transform is finally applied to find out the estimates $\tilde{I}^{(S)}$ and $\tilde{I}^{(L)}$ of $I^{(S)}$ and $I^{(L)}$, respectively.

Fig. 2 shows the scopes relevant to the above estimates. The upper side diagrams are relevant to $I^{(S)}$ and $I^{(L)}$ computed for each couple of periods of $i(t)$, whilst the lower side scopes are relevant to the estimates $\tilde{I}^{(S)}$ and $\tilde{I}^{(L)}$ over T_β. In this example the mean sampling time T_C was 4s and the number N of starting sampling instants was 100 over an observation interval T_β of 4 minutes, that is 12000 periods of $i(t)$.

5 CONCLUSIONS

In practical situations, the voltage and current waveforms at any supply terminal are randomly modulated quantities because of the interaction of the distorted currents, drawn by the customers' loads, with the equivalent impedance of the supply network. Evaluating the trend vs. time of the rms current components $I^{(S)}$ and $I^{(L)}$, associated to the harmonic losses at the load and the supply side, respectively, needs for processing a very great amount of data. A statistical approach has been proposed which is based on both a stochastic process model for the quantities $I_s^{(S)}$ and $I_s^{(L)}$ and a random sampling strategy. Both the theoretical analysis of the statistical properties of the monitored parameters $I^{(S)}$ and $I^{(L)}$, and the results of some experimental work, showed good performance of the approach proposed in the paper since the parameter estimates are correct, consistent and efficient.

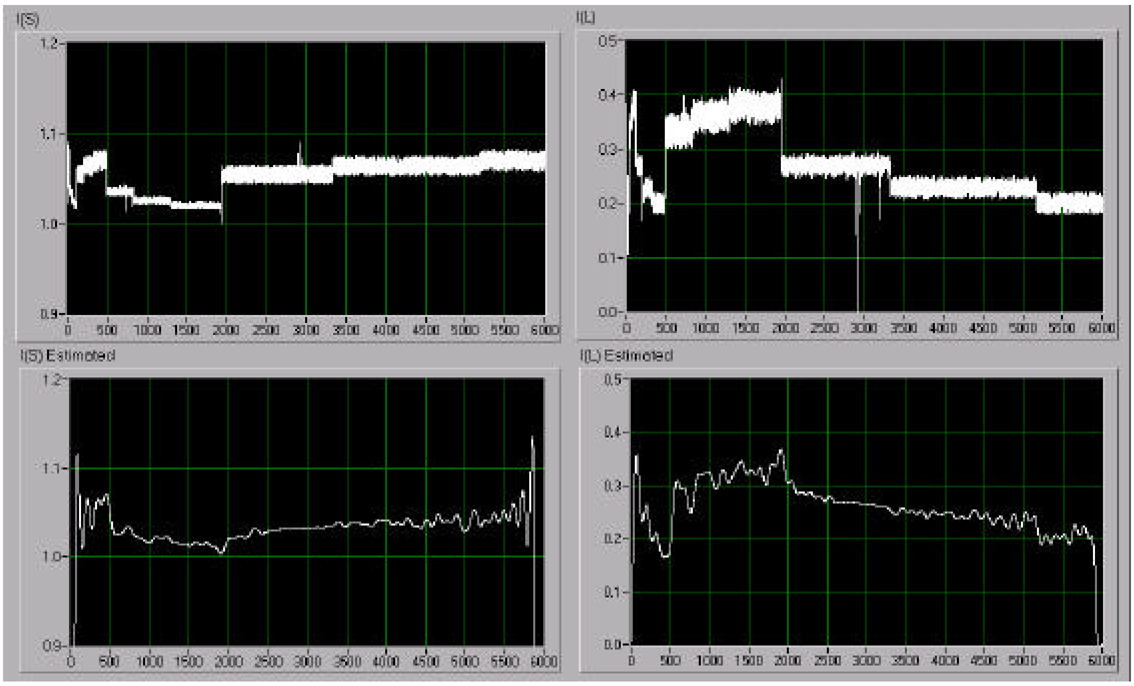

Fig. 2 Plots of the actual rms values $I^{(S)}$ and $I^{(L)}$ vs. time (upper scopes) and of their estimates (lower scopes)

6 APPENDIX:
THE STATISTICAL PROPERTIES OF THE MONITORED PARAMETERS

6.A Expected value of $\underline{\tilde{X}}_r$

Equation (13) can be re-written as follows:

$$\underline{\tilde{X}}_r = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{n-M}^{+M}\underline{X}_r e^{jn\omega_0 t_k} e^{-jr\omega_0 t_k} = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{n-M}^{+M}\underline{X}_r e^{j(n-r)\omega_0 t_k} = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{n-M}^{+M}\underline{X}_r e^{j(n-r)\omega_0[(k+x_k)T_C]}. \quad \text{(A-1)}$$

By exploiting the independence between x_k and $\underline{\varepsilon}_r$, we can write:

$$M\{\underline{\tilde{X}}_r\} = M\left\{\frac{1}{N}\sum_{k=0}^{N-1}\sum_{n-M}^{+M}\underline{X}_r e^{j(n-r)\omega_0[(k+x_k)T_C]}\right\} = M\left\{\frac{1}{N}\sum_{k=0}^{N-1}\sum_{n-M}^{+M}(\underline{A}_r+\underline{\varepsilon}_r)e^{j(n-r)\omega_0[(k+x_k)T_C]}\right\} =$$
$$= \frac{1}{N}\sum_{k=0}^{N-1}\sum_{n-M}^{+M}M\{(\underline{A}_r+\underline{\varepsilon}_r)\}e^{j(n-r)\omega_0 kT_C}M\{e^{j(n-r)\omega_0 x_k T_C}\}. \quad \text{(A-2)}$$

If we remember that x_k is a random variable uniformly distributed over [-*a*,*a*], the expected value $M\{e^{j(n-r)\omega_0 x_k T_C}\}$ is:

$$M\{e^{j(n-r)\omega_0 x_k T_C}\} = \frac{1}{2a}\int_{-a}^{+a} e^{j(n-r)\omega_0 x T_C}dx = \frac{1}{2a}\frac{1}{j(n-r)\omega_0 T_C}\left[e^{j(n-r)\omega_0 x T_C}\right]_{-a}^{+a} =$$
$$= \frac{\sin((n-r)\omega_0 a T_C)}{a(n-r)\omega_0 T_C} = \frac{\sin((n-r)\pi T_C/T_\beta)}{(n-r)\pi T_C/T_\beta} = \operatorname{sinc}((n-r)T_C/T_\beta). \quad \text{(A-3)}$$

Equation (A-2) becomes:

$$M\{\underline{\tilde{X}}_r\} = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{n-M}^{+M}\underline{A}_r e^{j(n-r)\omega_0 kT_C}\operatorname{sinc}((n-r)T_C/T_\beta) = \frac{1}{N}\sum_{n=-M}^{+M}\underline{A}_r \operatorname{sinc}((n-r)T_C/T_\beta)\sum_{k=0}^{N-1}e^{j(n-r)\omega_0 kT_C} \quad \text{(A-4)}$$

The last sum results in a geometrical progression. Hence, its value is:

$$\frac{1}{N}\sum_{k=0}^{N-1}e^{j(n-r)\omega_0 kT_C} = \frac{1}{N}\frac{e^{j(n-r)\omega_0 NT_C}-1}{e^{j(n-r)\omega_0 T_C}-1} = \frac{1}{N}e^{j(n-r)\omega_0\frac{(N-1)}{2}T_C}\frac{e^{\frac{j(n-r)\omega_0 NT_C}{2}}-e^{\frac{j(n-r)\omega_0 NT_C}{2}}}{e^{\frac{j(n-r)\omega_0 T_C}{2}}-e^{\frac{j(n-r)\omega_0 T_C}{2}}} =$$
$$= e^{j(n-r)\pi\frac{(N-1)}{T_\beta}T_C}\frac{\operatorname{sinc}((n-r)NT_C/T_\beta)}{\operatorname{sinc}((n-r)T_C/T_\beta)}. \quad \text{(A-5)}$$

Finally (A-4) becomes:

$$M\{\underline{\tilde{X}}_r\} = \sum_{n=-M}^{+M}\underline{A}_r \operatorname{sinc}((n-r)T_C/T_\beta)e^{j(n-r)\pi\frac{(N-1)}{T_\beta}T_C}\frac{\operatorname{sinc}((n-r)NT_C/T_\beta)}{\operatorname{sinc}((n-r)T_C/T_\beta)} \quad \text{(A-6)}$$

If $n=r$ it is $M\{\underline{\tilde{X}}_r\} = \underline{A}_r$ whilst if $n\neq r$ it is $\underset{\to}{M}\{\underline{\tilde{X}}_r\} = 0$. Hence, we can write that $\underset{\to}{M}\{\underline{\tilde{X}}_r\} = \underline{A}_r$, which coincides with (14).

6.B Variance of $\underline{\tilde{X}}_r$

The variance of the estimate of the spectral component $\underline{\tilde{X}}_r$ is:

$$Var\{\underline{\tilde{X}}_r\} = M\left\{\underline{\tilde{X}}_r \cdot \underline{\tilde{X}}_r^*\right\} - M\{\underline{\tilde{X}}_r\}M\left\{\underline{\tilde{X}}_r^*\right\}. \quad \text{(B-1)}$$

Let us consider the first term on the right side of (B-1). It is:

$$M\left\{\underline{\tilde{X}}_r \cdot \underline{\tilde{X}}_r^*\right\} = \frac{1}{N^2}\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}M\{u(t_{k_1})u(t_{k_1})e^{-jr\omega_0 t_{k_1}}e^{jr\omega_0 t_{k_2}}\} =$$
$$= \frac{1}{N^2}M\left\{\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\sum_{n_1=-M}^{+M}\sum_{n_2=-M}^{M}(\underline{A}_{n_1}+\underline{\varepsilon}_{n_1})(\underline{A}_{n_2}^*+\underline{\varepsilon}_{n_2}^*)e^{-j(n_1-r)\omega_0 t_{k_1}}e^{j(r-n_2)\omega_0 t_{k_2}}\right\}$$

If we recall (12) and remember that $\underline{\varepsilon}_{p,r}$ and x_k are independent random variables, we can write:

$$\frac{1}{N^2}\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\sum_{n_1=-M}^{+M}\sum_{n_2=-M}^{M}\underline{A}_{n_1}\underline{A}_{n2}^{*}e^{-j(n_1-r)\omega_0 k_1 T_C}e^{j(r-n_2)\omega_0 k_2 T_C}M\left\{e^{-j(n_1-r)\omega_0 x_{k_1}T_C}\right\}M\left\{e^{-j(r-n_2)\omega_0 x_{k_2}T_C}\right\}+ + \sum_{n_1=-M}^{+M}\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\sigma_{n_1}^2 e^{-j(n_1-r)\omega_0 k_1 T_C}e^{j(r-n_1)\omega_0 k_2 T_C}M\left\{e^{-j(n_1-r)\omega_0 x_{k_1}T_C}\right\}M\left\{e^{-j(r-n_1)\omega_0 x_{k_2}T_C}\right\} \quad \text{(B-2)}$$

By considering (A-3) and (A-5), the equation (B-2) turns into the following one:

$$\sum_{n_1=-M}^{+M}\sum_{n_2=-M}^{+M}\underline{A}_{n_1}\underline{A}_{n2}^{*}e^{j(n_1-r)\pi(N-1)\frac{T_C}{T_\beta}}\frac{\mathrm{sinc}\left((n_1-r)NT_C/T_\beta\right)}{\mathrm{sinc}\left((n_1-r)T_C/T_\beta\right)}e^{j(r-n_1)\pi(N-1)\frac{T_C}{T_p}}\frac{\mathrm{sinc}\left((r-n_2)NT_C/T_\beta\right)}{\mathrm{sinc}\left((r-n_2)T_C/T_\beta\right)}\cdot$$

$$\cdot\,\mathrm{sinc}\left((n_1-r)T_C/T_\beta\right)\mathrm{sinc}\left((r-n_2)T_C/T_\beta\right)+\sum_{n_1=-M}^{+M}\sigma_{n_1}^2 e^{j(n_1-r)\pi(N-1)\frac{T_C}{T_\beta}}\frac{\mathrm{sinc}\left((n_1-r)NT_C/T_\beta\right)}{\mathrm{sinc}\left((n_1-r)T_C/T_\beta\right)}\cdot$$

$$\cdot\, e^{j(r-n_1)\pi(N-1)\frac{T_C}{T_\beta}}\frac{\mathrm{sinc}\left((r-n_1)NT_C/T_\beta\right)}{\mathrm{sinc}\left((r-n_1)T_C/T_\beta\right)}\mathrm{sinc}\left((n_1-r)T_C/T_\beta\right)\mathrm{sinc}\left((r-n_1)T_C/T_\beta\right)=$$

$$=\sum_{n_1=-M}^{+M}\sum_{n_2=-M}^{+M}\underline{A}_{n_1}\underline{A}_{n_2}^{*}e^{j(n_1-r)\pi(N-1)\frac{T_C}{T_\beta}}e^{j(r-n_2)\pi(N-1)\frac{T_C}{T_\beta}}\mathrm{sinc}\left[(n_1-r)NT_C/T_\beta\right]\mathrm{sinc}\left[(r-n_1)NT_C/T_\beta\right]+ + \sum_{n_1=-M}^{+M}\sigma_{n_1}^2 e^{j(n_1-r)\pi(N-1)\frac{T_C}{T_\beta}}e^{j(r-n_2)\pi(N-1)\frac{T_C}{T_\beta}}\mathrm{sinc}\left[(n_1-r)NT_C/T_\beta\right]\mathrm{sinc}\left[(r-n_1)NT_C/T_\beta\right]. \quad \text{(B-3)}$$

By recalling (B-1) and taking into account (A-6), from (B-3) it follows:

$$Var\left\{\underline{\tilde{X}}_r\right\}=\sum_{n_1=-M}^{+M}\sigma_{n_1}^2 e^{j(n_1-r)\pi(N-1)\frac{T_C}{T_\beta}}e^{j(r-n_2)\pi(N-1)\frac{T_C}{T_\beta}}\mathrm{sinc}\left[(n_1-r)NT_C/T_\beta\right]\mathrm{sinc}\left[(r-n_1)NT_C/T_\beta\right]. \quad \text{(B-4)}$$

For $n_1=r$ it is $Var\left\{\underline{\tilde{X}}_r\right\}=\sigma_r^2$, whilst for $n_1\neq r$ the variance (B-4) tends to zero asymptotically. Finally we can write that:

$$\underrightarrow{Var}\left\{\underline{\tilde{X}}_r\right\}=\sigma_r^2 \quad \text{(B-5)}$$

which coincides with (15).

REFERENCES

[1] R. Sasdelli, G. Del Gobbo, G. Iuculano, Quality management for electricity as a processed material, in: V. Piuri and M. Savino (ed.), *Proceedings of the 16th IEEE Instrumentation and Measurement Technology Conference "Measurements for the New Millennium"* (Venice, May 24-26, 1999), Venezia, Italy, 1999, **1**, p. 214-219.
[2] ISO-International Organization for Standardization, *Standard 8402: "Quality management and quality assurance-Vocabulary"*, Geneva, Switzerland, 1994.
[3] A. Ferrero, R. Sasdelli, Revenue and harmonics: a discussion about new quality-oriented measurement methods, in: *Proceedings of the 9th International Conference on "Metering and Tariffs for Energy Supply", IEE Conference Publication 462*, (Birmingham, May, 25-28, 1999), Birmingham, UK, p. 46-50.
[4] D.Mirri, G.Iuculano, G.Pasini, F.Filicori, L.Peretto, A Broad-Band Power Spectrum Analyzer Based on Twin-Channel Delayed Sampling, *IEEE Trans. on Instr. and Meas.*, **47**, (5), (1998) 1346-1354.
[5] G. Iuculano, *Introduction to probability, statistics and stochastic processes in engineering and physical sciences* (in Italian), Pitagora, Bologna, Italy, 1996.
[6] A. Papoulis, *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, New York, USA, 1965.
[7] D. Mirri, G. Iuculano, A. Menchetti, F. Filicori, G. Pasini, M. Catelani, A broad-band harmonic vector voltmeter based on a random sampling strategy, in: *Proceedings of the IEEE Instrumentation and Measurement Technology Conference "Advanced Technologies in I & M"*, (Hamatsu, May 10-12, 1994), Hamatsu, Japan, 1994, **2**, p. 1012-1015.