WAVELET NETWORKS FOR ADC MODELLING

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Abstract: The use of the wavelet network for ADC modelling is proposed in the paper. In particular, an original technique is set up capable of identifying the domain of each mother wavelet involved and establishing a compact and complete training set. Moreover, to reduce the complexity of the architecture of the wavelet network and its learning stage, the initialisation is based on the evaluation of all voltage values related to the transitions of the ADC. The technique is applied to a simulated ADC as well as an actual ADC, and several tests are carried out both in the time and frequency domain. The obtained results highlight the advantages of the proposed model if compared to those granted by a neural network-based model.
Keywords: ADC Modelling, Wavelet Network.

1 INTRODUCTION

Analogue-to-Digital Converter (ADC) modelling is of great interest from a design as well as simulation and testing point of view. The greatest difficulties in setting up a suitable model of an ADC arise from the shape of its transfer function, which is typically piecewise linear. Different models have already been proposed in literature, each of which shows some limitations due mainly to: (i) its complexity, (ii) its difficulty of representing error sources, (iii) the particular architecture of the ADC, and (iv) the adopted technique for parameter identification [1-8].

A new approach based on Artificial Neural Networks (ANNs) has recently been proposed for ADC modelling [9]. This approach succeeds in overcoming some of the aforementioned limitations, and, thanks to ANN generalisation capability, it shows itself independent of the ADC architecture. Moreover, it can be applied in a wide frequency spectrum which avoids the necessity of re-identifying the model. However, due to finite value of the steepness of the sigmoidal transfer function of each neuron, the higher the ADC resolution, the more complex the ANN structure and the greater the number of iterations needed for the ANN learning stage.

To reduce the complexity of both the structure and learning stage without compromising all the advantages of ANN-based ADC models, a suitable technique is proposed and experimented in the paper. The technique makes use of the so-called Wavelet Networks (WNs) for classification [10,11], and it exploits an innovative procedure capable of optimising WN design.

WNs are widely used in the approximation of non-linear systems [12-14]; they consist of a computational scheme that combines the mathematical rigor of the wavelet theory with the adaptive learning properties of ANNs. Their architecture is similar to ANNs': with one hidden layer and is based on the inverse discrete wavelet transform. Differently from the ANN, the architecture and the network parameters are determined directly by means of both the initialisation procedure based on the wavelet theory and the training set. The following training phase is used to increase the WN output accuracy only.

After a brief description of the theory underlying the definition, design, and implementation of the WN, the fundamental stages of the proposed technique are described in detail with reference to a numerical example related to the ADC model presented in [15]. An actual ADC is then considered, and the performance of the obtained model is assessed by means of tests in both the time and frequency domain. A comparison is finally drawn between the achieved results and those furnished by an ANN-based model of the same ADC in order to highlight the reduced complexity of both the structure and learning strategy.

2 ADC MODELLING THROUGH WN

In the last years, WNs have been established as a general approximation tool for fitting non-linear model from input/output data [10,11]. A brief overview is given in the following.

The general structure of a WN showing universal approximation property has the following expression:

$$g(x) = \sum_{i=1}^{n} w_i \O[d_i(x - t_i)] + c^T x + b\,, \qquad w_i, d_i, r_i, t_i, c, b \in R \tag{1}$$

where Ø(.) is the mother wavelet, d_i the dilatation parameter, t_i the translation parameter, w_i the weight, n the number of wavelets. The additional term, c^Tx+b, is useful in the presence of functions defined on a finite domain and characterised by a non-zero mean value [11]. If the family of wavelets in the relation (1), is the orthonormal base of L^n (R), the WN is capable of approximating all functions defined in L^n(R). Indeed, if the mother wavelet is bounded in the definition domain, each wavelet both appropriately translated and dilated gives contribution to the over all approximation. The dimension of the definition domain establishes the scale factor of the wavelet. The initial values of the dilatation parameter, translation parameter and weights are established by a suitable procedure. Successively, the same parameters are adjusted by a learning algorithm based on a sample of input/output pairs $\{x, f(x)\}$, where f(x) is the function to be approximated; The Gauss-Newton algorithm is commonly used in these cases. Problems due to local minima are avoided because the initial values of the parameters are close to the final ones.

The proposal of the WN as a tool for ADC modelling claims for a structure characterised by both approximation and classification properties. The network structure, mother wavelet, and parameter values influence the WN's properties and, consequently, they have to be fixed appropriately. To this aim, a technique for designing a WN characterised by approximation and classification properties is presented in the following. In particular, the technique operates so as both the WN structure and the initialisation procedure are modified and adapted to the new goal of the WN.

First of all, a modification is addressed to the structure given in (1) by suppressing the term c^Tx+b. The resulting structure is illustrated in Fig.1. The combination of the translation t_i, dilation d_i, and wavelet, on the same line, constitutes the wavelon (wl_i).

The classification properties depend on the mother wavelet, which must have a step size form. The approximation properties depend on the final values of the parameters of each wavelon. Moreover, to make it competitive and useful compared to other modelling techniques, the WN for ADC modelling must achieve the following objectives in determining the mother wavelet and the parameters:

1. good classification property have to be shown;
2. the learning phase has to be fast.

The classifying capacity of this WN strongly depends on the distribution of the training data and on the scale levels utilised to construct the mother wavelet; a long and complex stage, like that characterising ANNs, for establishing the training parameters of the WN may be necessary. However, several experimental observations have allowed the understanding that fewer wavelets are capable of providing a greater number of degrees of freedom, whilst maintaining a good classification capability. On the contrary, too many wavelets make the WN static; more restrictions are imposed and, consequently, the classification capability is reduced.

To understand the relation between the number of wavelets and training curves, it is necessary both to highlight how the scale levels are generated and establish the number of wavelets needed to cover the input range. Each level covers a portion of the range of the input data with a resolution increasing upon the number of levels' increasing.

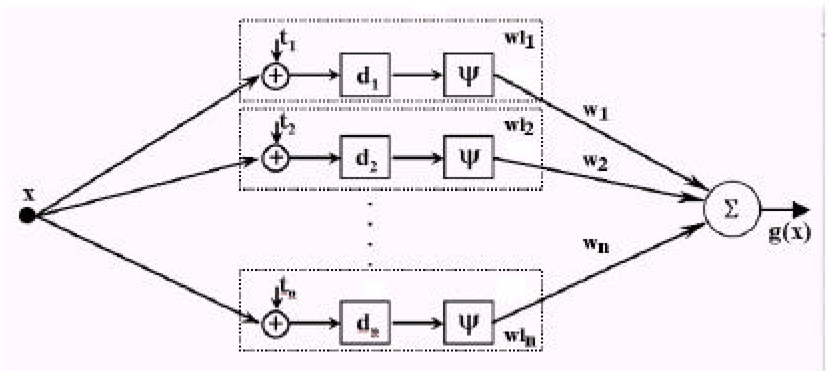


Figure 1. Structure of a wavelet network for classification; the wavelons $wl_1,\dots,wl_n$ are highlighted.

Taking into account the nominal way of operation of an ADC, the definition domain D of the WN is constituted by the union of n+1 voltage interval I_i, compact and disjoined from one another

$$D = \bigcup_{i=1}^{n+1} I_i = \{v : v \in I_i, \forall i = 1,\ldots, n+1\} \tag{2}$$

where n denotes the number of transitions occurring. Each interval, I_i, is defined by two successive voltage values, V_i and V_{i+1} for i=0,…,n (Fig.2). In particular, the voltage values V_i, (for i=1,…,n) stand for the nominal transition levels; V_0= 0 if the ADC is uni-polar or V_0= $-V_{fs}$ if not, and $V_{n+1}=V_{fs}$.

In Fig.2, the scale level, sl_i, of the i-th wavelet is related to the length of the segment the extremes of which are the two transaction values V_{i-1} and V_i; specifically, sl_i is given by V_{i-1}-V_i, for i=1,…,n+1. Furthermore, this segment also establishes the domain of the i-th wavelet. It can be noted how, for each level, the related wavelets cover the input range with a more and more increasing resolution, $r_i=2/sl_i$ for i=1…n+1. The segment associated to the scale level contains all the resolution and separation values of the lower levels.

Once the desired number of scale levels is fixed, it is possible to determine both the training set and the number of wavelets, which the WN has to contain. In particular, the training set consists of the separation and resolution values of the highest level; furthermore, the number of scale levels determines also the number of wavelets to be used.

The lowest number of scale levels (L_m) is

$$L_m = \lfloor \mathrm{Log}_2(n-1) \rfloor + 1. \tag{3}$$

The number of scale levels given by (3) has significance when the domain D is constituted by ordered sub-sets satisfying the cardinality property. For a real ADC, being its operation modes quite different from those characterising an ideal one, it is necessary to generate new scale levels in order to achieve the requested accuracy. As a consequence, the number of scale levels is $L>L_m$.

The total number of candidate wavelets for modelling a real ADC is:

$$NW = \sum_{i=1}^{L_m} 2^{i-1} + (L - L_m)(n-1). \tag{4}$$

A selection algorithm can be used in the training phase for reducing the number of candidate wavelets. This algorithm eliminates the wavelets contributing very little in the classification of the input signal, thus reducing architecture complexity and speeding up both the learning and production phase. The initial value of the weights can be obtained by solving a linear system.

The wavelet that can be used for ADC modelling is a on/off function with only two decisional levels in its domain. This is the Haar function in the form:

$$H_i(d_i(x-t_i)) = \begin{cases} -1 & 0 < d_i(x-t_i) \le 0.5 \\ 1 & 0.5 < d_i(x-t_i) \le 1 \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

The dilation parameter d_i is used to adapt the wavelet to the specific domain, the translation parameter t_i serves to adapt the wavelet to the transition level.

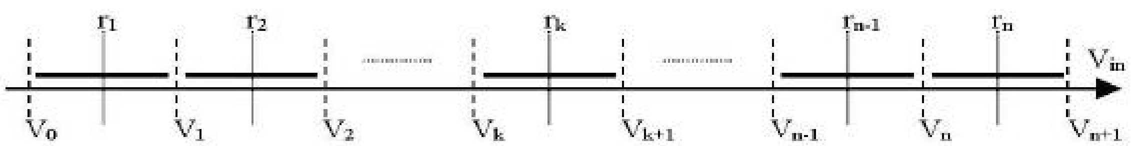


Figure 2. Domains of the wavelets constituting the WN for ideal ADC modelling with n transition levels.

3 APPLICATION EXAMPLE

A numerical example is reported in the following. The goal of this example is both to define all the steps to be taken and clarify all the rules that allow the optimisation of the structure of the WN.

An ideal 2-bit ADC with full-scale voltage, V_{FS}, equal to1V is considered. The number of transitions, n, is equal to 3. These transitions occur at the voltage values:

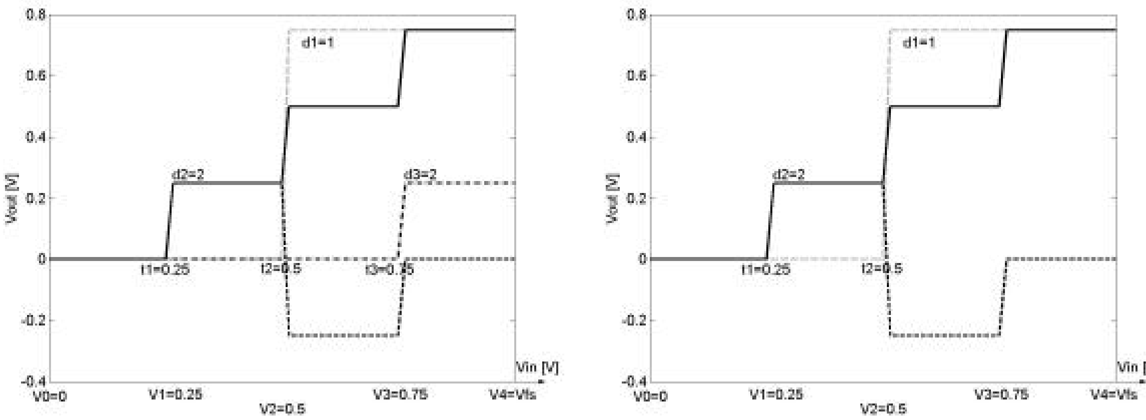
As a consequence, the domain, D, is the union of the disjoined and compact voltage intervals, I_i for i=1,…,4, given by:

$$I_1 = [0,...,0.25], \quad I_2 =]0.25,...,0.5] \quad I_3 =]0.5,...,0.75] \quad I_4 =]0.75,...,1]. \tag{6}$$

For each interval, the wavelet is constructed according to the relation (4). The related resolutions, r_i for i=1,…, 4, are equal to 2 (Fig.3a). Due to the specific symmetry of the characteristic in Fig.3b, only two wavelets have to be used, as stated by the relation (3). The training set consists of:

$$TS = \{(0,0),(0.125,0),(0.25,0),(0.375,0.25),(0.5,0.25),(0.625,0.5),(0.75,0.5),(0.875,0.75),(1,0.75)\}. \tag{7}$$

In the modelling of real ADC the scale levels, the voltage intervals I_i, i=1,…,4: and the training set will be modified according to the required accuracy.



a) b)

Figure 3. Candidate wavelets for modelling the ideal 2-bit ADC: a) three candidate wavelets placed in correspondence with the transition voltages; b) two candidate wavelets are selected due to the symmetry of the ADC characteristic; d_1, d_2, and d_3 represent the values of the dilatations parameters.

4 ACTUAL ADC MODELLING BY WN

To assess the performance of the proposed technique, the mathematical model shown in [15] has been used as the ADC under test. The model gives the possibility of constructing the learning, test and validation sets for the WN. Moreover, delay, distortion, gain and offset errors have been introduced in the model in order to force non-ideal behaviour. Finally, the same values given in [9] have been adopted for the parameters of the model in order to draw a reliable comparison between the performance of the WN-based model and that granted by ANN-based model.

Fig.4 shows the output both of the WN and the mathematical model for a 6-bit ADC; a sinusoidal signal with unitary amplitude has been used as input. The difference between the two outputs is below 1.25×10^{-13} LSB. The WN is characterised by 58 wavelons and 14 scale levels. Fig.5 shows the spectra of the output signal.

Fig.6 shows the output both of the WN and the mathematical model for a 10-bit ADC; a sinusoidal signal with unitary amplitude has been used as input. The difference between the two outputs is below 5×10^{-12} LSB. The WN is characterised by 887 wavelons and 21 scale levels. Fig.7 shows the spectra of the output signal.

Fig.8 compares the output both of the WN and an actual 6-bit ADC; a sinusoidal signal with unitary amplitude has been used as input. The difference between the two outputs is below 1×10^{-13} LSB. The WN is characterised by 62 wavelons and 15 scale levels. Fig.7 shows the spectra of the output signal.

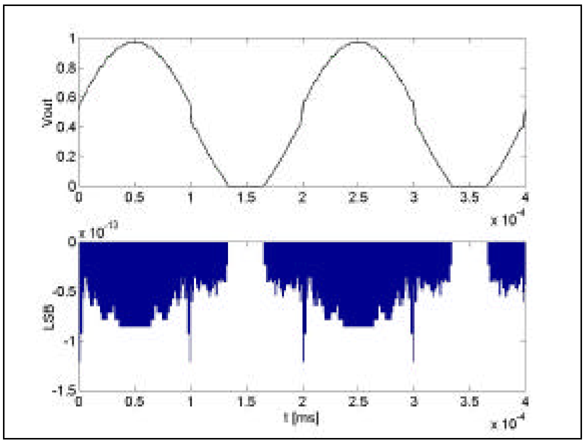


Figure 4. Output of both the WN and mathematical model for a 6-bit ADC with a sinusoidal signal as input (upper); difference between the two outputs (lower)

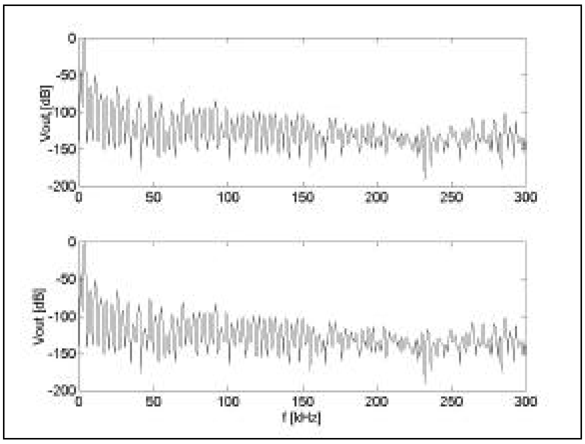


Figure 5. Spectra of the output both of the WN and mathematical model for the 6-bit ADC in the operating conditions of Fig. 4.

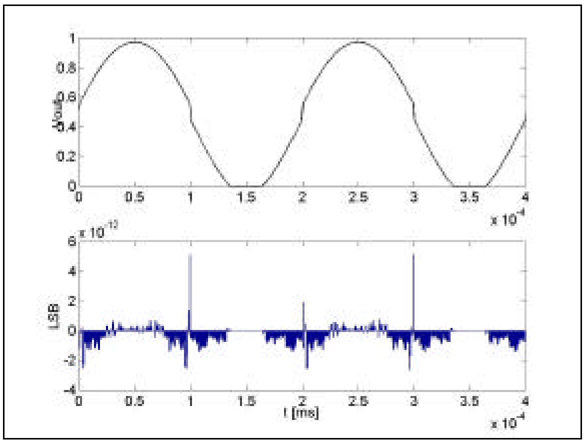


Figure 6. Output of both the WN and mathematical model for a 10-bit ADC with a sinusoidal signal as input (upper); difference between the two outputs (lower).

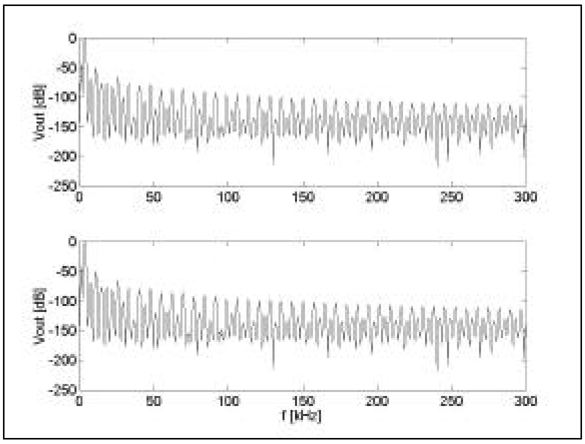


Figure 7. Spectra of the output both of the WN and mathematical model for the 10-bit ADC in the operating conditions of Fig. 6.

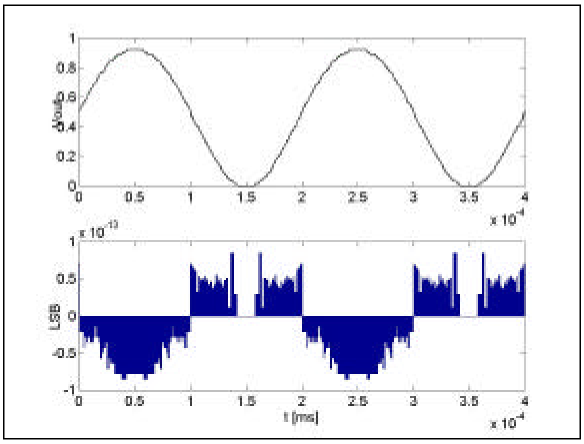


Figure 8. Output both of the WN model and an actual ADC with a sinusoidal signal as input (upper); difference between the two outputs (lower).

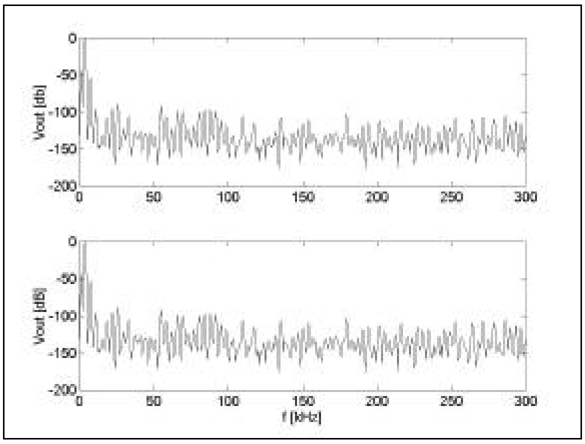


Figure 9. Spectra of the output both of the WN and mathematical model for the actual ADC in the operating conditions of Fig.8.

In all the previous models, the scale level is greater than the ADC resolution. This is due to the fact that the characteristic of the ADC, which includes delay, distortion, gain, and offset errors, is quite different from the ideal one; a greater number of wavelons is needed if compared to that required in the ideal case. The increasing number of wavelons allows to reduce the number of elements of the training set used by each wavelon and consequently increases the model accuracy.

The architectures of the ANN in same case studies [9] are more complex, and require a greater number of iterations in the learning phase; moreover, the difference between the output of the ANN model and that of the mathematical model, or the actual ADC, is always in the range 0.5 LSB÷1LSB.

5 CONCLUSIONS

The paper has proposed a new Wavelet Network-based technique for ADC modelling. The technique has been described in detailed and its generality, i.e. independence of a particular ADC architecture, has also been highlighted.

The validity of the proposed technique has been proved by means of a simulation study as well as an application to an actual ADC. The obtained results have established the advantage of the WN-based model with respect to an ANN-based model. In particular, the following conclusions can be drawn:

1. the approximation and generalization properties of the WN depend on the number of scale levels;
2. the training set must be organised in such a way that the different slopes in the ADC characteristic have to be taken into account;
3. the initialisation procedure and learning phase are faster with respect to ANN's;
4. the structure of the WN has a more reduced complexity than ANN's.

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