

# DYNAMIC ERROR CORRECTION OF INTEGRATING ANALOG-TO-DIGITAL CONVERTERS BY USING VOLTERRA FILTRATION

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*Abstract: In analog-to-digital converters (ADCs), dynamic and memory nonlinear effects can contribute simultaneously to the distortion of the digitised signal. These effects can be modelled and compensated effectively via Volterra filters. The paper deals with a Volterra filter for ADC error correction based on an inverse model. An easy-to-implement correction technique, based on a efficient mathematical model of Volterra filter designed to reduce burden in model definition, in filter identification, and in experimental calibration is proposed. Preliminary simulation and experimental results for integrating ADCs highlight the effectiveness of the proposed modelling and correction approach.*

*Keywords: ADC, INL, Volterra filter, modelling, correction.*

## 1 INTRODUCTION

The ADC error is commonly characterised in terms of integral (INL) and differential (DNL) nonlinearity. The simplest approach describes the ADC static behaviour by considering the INL as a function of the output code  $k$  [1]-[2]. In dynamic conditions, the INL is considered as a two-dimensional function of the output code  $k$  and the time slope  $s$  of the input signal ("phase plane" approach) [3]-[6]. In particular, an a-priori approach to error modelling in the phase plane was proposed by exploiting specific features of the error for most common ADC architectures in order to reduce experimental burden in model identification [5]-[6]. However, in some cases, memory effects are predominating, and a "memory" modelling approach, e.g. considering the ADC error as a function of the current  $k$  and the previous  $k-1$  samples, is preferred [3]. In most critical ADC architectures, the combination of both the approaches is currently considered as highly promising, though it still requires for more efficient mathematical tools [2].

Volterra filtration [8]-[9] showed to be an advantageous approach to model a weakly nonlinear system, where dynamic and memory effects are present simultaneously. Moreover, the set of Volterra kernels can often have a physical interpretation, or can be related directly to the system's constituent elements [10]-[11]. Mirri et al. proposed a significant architecture-independent mathematical approach to model nonideal devices based on S/H (sample and hold) and ADC, under the hypothesis of only short-term memory effects [8]. The convergence properties of the Volterra series are increased by using the dynamic deviation  $e(t,\tau)=x(t-\tau)-x(t)$  of the input signal  $x(t)$ , instead of the input signal itself. Tsimbinos proposed an adaptive Volterra filter for the identification and the subsequent S/H correction by an inverse-model [9]: during identification, adaptive filters continuously correct their kernels by means of a convenient algorithm. This turns to be very useful for unknown and very time-varying error models.

However, in some ADC architectures, operations are carried out on a finite time and the constraints of only short-term memory effects is a drawback. Moreover, in experimental calibration, the adaptive process requires for a stationary signal. Conversely, a more complex signal is required by the twofold exigencies of mapping exhaustively the phase and the memory planes.

In this paper, an easy-to-implement error correction technique based on an inverse model using an efficient Volterra filter is proposed for integrating ADCs. The kernel is specifically designed according to the peculiarities of the IADC error. The burden for the inverse model computation is reduced by a direct calculation of the inverse Volterra kernels. In particular, in section 2, fundamentals of the use of Volterra filters in the identification of models for nonlinear dynamic systems with memory are stated. In section 3, the proposed correction technique is described. In section 4, preliminary results of simulation and experimental validation are discussed.

## 2 VOLTERRA FILTRATION FOR NONLINEAR SYSTEM IDENTIFICATION

The mathematical basis of the Volterra filtration is the Volterra functional. The infinite series of Volterra functionals can be expressed in the following form [8], [10]:

$$\hat{y}(t) = h_0 + \sum_{i=1}^{\infty} \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} h_i(\hat{t}_1, \hat{t}_2, \dots, \hat{t}_i) \prod_{j=1}^i x(t-\hat{t}_j) \prod_{j=1}^i d\hat{t}_j \right\}, \quad (1)$$

where  $x(t)$  is the input signal,  $\hat{y}(t)$  is the output signal, and  $h_i(\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_i)$  is the Volterra kernel of  $i$ -th order.

The discrete equivalent of the general formula (1) is [10]:

$$\hat{y}(n) = h_0 + \sum_{i=1}^{\infty} \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \dots \sum_{m_i=0}^{\infty} h_{im_1m_2\dots m_i} \prod_{j=1}^i x(n-m_j), \quad (2)$$

where  $x(n)$  is the discrete input signal,  $\hat{y}(n)$  is the filter output sequence, and  $h_{im_1m_2\dots m_i}$  is the element of  $i$ -th order Volterra kernel.

The (2) is the mathematical model of a Volterra filter of infinite order. In particular, the truncated model with a finite order  $M$ , and a finite memory of samples  $N+1$  is considered [10]:

$$\hat{y}(n) = h_0 + \sum_{i=1}^M \sum_{m_1=0}^N \sum_{m_2=0}^N \dots \sum_{m_i=0}^N h_{im_1m_2\dots m_i} \prod_{j=1}^i x(n-m_j). \quad (3)$$

By organising the Volterra kernel elements  $h_{im_1m_2\dots m_i}$ , and the input sample products

$\prod_{j=1}^i x(n-m_j)$  into convenient block vectors  $\mathbf{H}$  and  $\mathbf{X}$ , respectively, the (3) can be rewritten as [10]:

$$\hat{y}(n) = \mathbf{H}^T(n)\mathbf{X}(n) = \mathbf{X}^T(n)\mathbf{H}(n). \quad (4)$$

The design of an optimal Volterra filter requires the search for a set of Volterra kernels minimizing the error function  $e(n) = y(n) - \hat{y}(n)$  between the desired signal  $y(n)$ , and the filter output  $\hat{y}(n)$ , according to a given optimisation criterion. The least-mean-squares error optimisation criterion can be considered:

$$E[e^2(n)] = E\left[\left(y(n) - \hat{y}(n)\right)^2\right] \rightarrow \min.$$

The solution leads to the equation [10]

$$\mathbf{R}_{XX} \mathbf{H}^* = \mathbf{R}_{XY}, \quad (5)$$

providing that

$$\det \mathbf{R}_{XX} \neq 0, \quad \mathbf{H}^* = \mathbf{R}_{XX}^{-1} \mathbf{R}_{XY} \quad (6)$$

where:

$\mathbf{H}^*$  is the vector of the optimum Volterra kernels (optimal filter coefficients),

$$\mathbf{R}_{XY} = E[y(n)\mathbf{X}(n)] \quad (7)$$

is the higher-order mutual correlation vector of the input and the desired signals, and

$$\mathbf{R}_{XX} = E[\mathbf{X}(n)\mathbf{X}^T(n)] \quad (8)$$

is the higher-order autocorrelation matrix of the input signal.

This shows that the filter design is based on the knowledge of the statistical properties of the input and the desired signals.

## 3 THE PROPOSED CORRECTION TECHNIQUE

According to the Tsimbinos approach [9], an adaptive Volterra filter is utilised for the identification of the nonlinear system model (Fig. 1). Once adapted, the filter provides the Volterra kernels modelling the specific nonlinear system under test. From the measured kernels, the inverse Volterra kernels are analytically derived in order to determine an inverse model of the system. This allows the nonlinear system error to be corrected according to the inverse-model principle of Fig. 2.

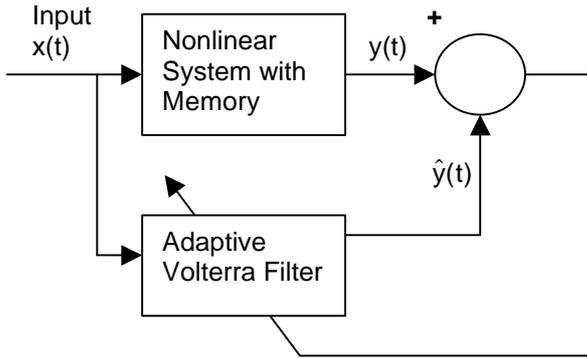


Figure 1. Volterra kernel identification by an adaptive method.

With respect to this approach, the computational burden due to Volterra inverse calculation is reduced by obtaining directly the inverse Volterra kernels. In principle, this is possible by modifying the configuration of Figure 1, by connecting the nonlinear block just in front of the adaptive Volterra filter. However, the principle of nonlinear system identification by means of adaptive Volterra filters requires a stationary input signal [9]. This causes the twofold problems of trying to find a proper calibration signal that not only (i) maps the phase plane  $k,s$  and the “memory” plane (e.g. the state-previous

state  $k,k-1$  plane) simultaneously, but also (ii) is stationary during the adaptation process in order to have invariant conditions during the adaptation.

For these reasons, a different approach was followed by exploiting the mathematical knowledge of the error in a given ADC architecture. This can be considered as the natural evolution of the a-priori approach to ADC error modelling [2], [5]-[6] toward a correction application. The Volterra filter is designed by taking into account the specific error features of integrating ADCs by a suitable mathematical model. Then, the set of the optimal Volterra filter coefficients are obtained according to the method shown in Fig.3. In particular, the inverse Volterra kernels are calculated directly from a sequence of known input values of the calibration signal, and the corresponding sequence of distorted output values of the IADC.

In typical IADCs, the integral nonlinearity is a two-dimensional error functions of the output code  $k$  and the Input slope  $s$  [3]-[5]. The  $INL_Q(k,s)$  presents a smooth surface and can be modelled with a low-order polynomial [5]:

$$INL_Q(k,s) = B_0 + B_1k + B_2k^2 + B_3s + B_4ks + B_5s^2 + \dots \quad (9)$$

Since  $INL_Q(k,s) \approx \Delta k = k - k_{id}$  [5], where  $k_{id}$  is the  $k$ -th ideal code, thus  $k_{id} \approx k - INL_Q(k,s)$ .

On this basis, the mathematical model of the Volterra filter is expressed in the following form:

$$\hat{y}(n) = k_{id}(n) = -B_0 + (-B_1 + 1)k(n) - B_2k^2(n) - B_3s(n) - B_4k(n)s(n) - B_5s^2(n) + \dots \quad (10)$$

The slope is estimated by the central difference equation [3]:

$$s(n) = [k(n+1) - k(n-1)] / 2T_s, \quad (11)$$

where  $T_s$  is the sampling period.

By using the substitution  $n=m-1$ , the second-order Volterra series corresponding to the inverse nonlinear model  $G_n$  of Fig. 2 can be immediately derived:

$$\begin{aligned} \hat{y}(m-1) = & g'_{0} + g'_{1_0} k(m) + g'_{1_1} k(m-1) + g'_{1_2} k(m-2) + g'_{2_{00}} k^2(m) + g'_{2_{01}} k(m)k(m-1) + \\ & + g'_{2_{02}} k(m)k(m-2) + g'_{2_{11}} k^2(m-1) + g'_{2_{12}} k(m-1)k(m-2) + g'_{2_{22}} k^2(m-2) \end{aligned} \quad (12)$$

By substituting (11) into (12), and by assuming from the properties of model (9) that:

$$g'_{1_0} = -g'_{1_2}; \quad g'_{2_{01}} = -g'_{2_{12}}; \quad g'_{2_{00}} = -g'_{2_{02}} / 2 = g'_{2_{22}}, \quad (13)$$

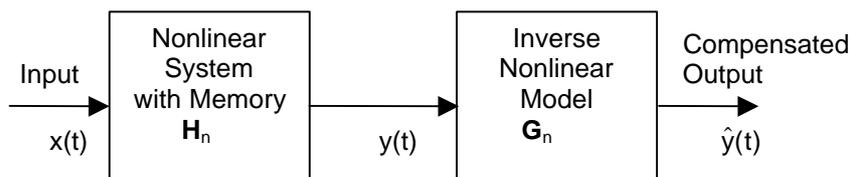
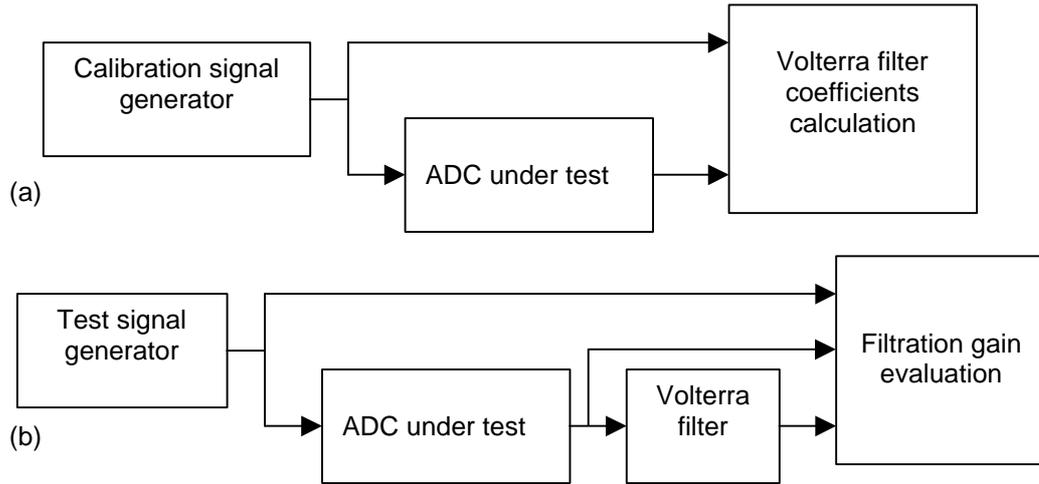


Figure 2. Principle of error correction by means of an inverse model.



**Figure 3** The proposed method: (a) calculation of the Volterra filter coefficients; (b) verification of the filter effectiveness.

the following equation is obtained:

$$\begin{aligned} \hat{y}(m-1) = & g'_0 + g'_{1_1} k(m-1) + 2T_s g'_{1_0} s(m-1) + g'_{2_{11}} k^2(m-1) \\ & + 2T_s g'_{2_{01}} k(m-1)s(m-1) + 4T_s^2 g'_{2_{00}} s^2(m-1) \end{aligned} \quad (14)$$

After further modification, it is obtained:

$$\hat{y}(n) = g_0 + g_1 k(n) + g_2 s(n) + g_3 k^2(n) + g_4 k(n)s(n) + g_5 s^2(n). \quad (15)$$

By comparing the (10) and the (15), the (15) provides the sought formula for the Volterra filter necessary for the correction of an IADC according to the inverse-model principle of Fig.2. The (15) shows also that the model (9) can be interpreted as a Volterra filter with memory order 3, once the slope is estimated via the (11). Moreover, the number of coefficients is reduced by the 40% (from 10 to 6) in comparison to traditional Volterra models [9].

The vector of the optimal filter coefficients is obtained analogously as in (5):

$$\mathbf{G}^* = [g_0, g_1, g_2, g_3, g_4, g_5]^T = \mathbf{R}_{XX}^{-1} \mathbf{R}_{XY}, \quad (16)$$

where  $\mathbf{R}_{XX}$ , and  $\mathbf{R}_{XY}$  are estimated from an experimental sequence of  $L$  known ideal input and actual output IADC values:

$$\mathbf{R}_{XY} \approx \frac{1}{L} \sum_{n=1}^L (x(n) \mathbf{X}(n)), \quad (17)$$

$$\mathbf{R}_{XX} \approx \frac{1}{L} \sum_{n=1}^L (\mathbf{X}(n) \mathbf{X}^T(n)), \quad (18)$$

where

$$\mathbf{X}(n) = [1, k(n), s(n), k^2(n), k(n)s(n), s^2(n)]^T. \quad (19)$$

#### 4 SIMULATION AND PRELIMINARY EXPERIMENTAL RESULTS

The proposed correction method was validated preliminarily on a model of nonideal IADC identified experimentally. The model generates the output codes  $k(n)$  for the input values  $x(n)$  represented in floating point with a quantization step  $Q$ , by fulfilling the inequality:

$$Q(k(n) - 0,5) + Q \cdot INL_Q(k(n), s(n)) \leq x(n) < Q(k(n) + 0,5) + Q \cdot INL_Q(k(n) + 1, s(n)). \quad (20)$$

Inside (20), the model (9) of  $INL_Q(k, s)$  was identified experimentally on an actual 12-bit dual-slope ADC Harris Semiconductors ICL7109CPL, with a full-scale of  $V_{fs} = \pm 2.048$  V, and a maximum conversion time of  $T_M = 33.3$  ms. A histogram test based on small triangular waves with a suitable superimposed dc level was used according to the technique described in [5]. A generator Hewlett Packard 33120A suitably calibrated, and a dc calibrator Fluke 5442A, were used. The small-wave

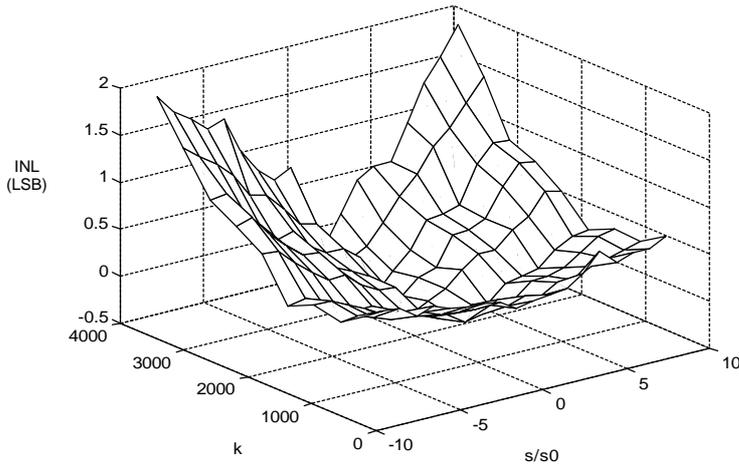


Figure 4. Experimental INL<sub>Q</sub>(k,s) used for the actual IADC model.

$g_0$	-0.081112
$g_1$	1.000180
$g_2$	-0.000491
$g_3$	0.000000
$g_4$	-0.000001
$g_5$	0.000010

Tab.1 Coefficients of the Volterra filter.

correction model was set up by computing the Volterra filter coefficients. In this phase, an appropriate selection of the calibration signal of Fig.3a is essential: a suitable algorithm was used in order to generate the combinations of code and slope corresponding to the selected calibration signal covering the phase plane as much evenly as possible. A calibration signal composed by a peak-to-peak sawtooth and an additive random signal of small amplitude was applied.

Then, the correction effects were assessed according to the scheme of Fig.3b by using multisinusoidal test signals. A test signal example constituted by sinusoidal components of 0.075 Hz, 0.008 Hz, and 0.003 Hz is highlighted in Fig. 5. The signal was converted by the modelled actual ADC and the distorted output was corrected through the inverse model based on the Volterra filter (Tab.1). The corresponding results achieved by the correction are highlighted in Fig. 6. In this example, the IADC performance improvement in terms of signal-to-noise ratio achieved by the proposed filtering technique was 4.8 dB.

## 5 CONCLUSIONS

A method using Volterra filtration for inverse-model correction of the ADC error has been proposed. The Volterra filter models memory and dynamic nonlinear error effects simultaneously. The method can be considered as the evolution of the a-priori approach to ADC error modelling [2],[5]-[6] toward a correction application. The a-priori approach makes easy the analytical

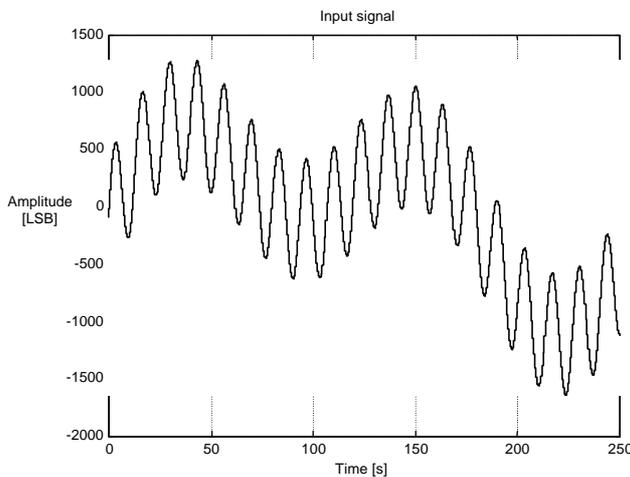


Figure 5. Test signal for filtration effects simulation.

histogram technique allowed the linearity limitations of the generator HP33120A to be overcome. In the case study, the selection of the second order for the polynomial (9) guaranteed a model accuracy of 0.196LSB in the range  $s/s_0 = [-10.0, 10.0]$ , with  $s_0 = Q_{id}/T_M$ , where  $Q_{id}$  is the ideal quantization step. The model parameters  $B_j$ ,  $T$ , and  $Q$  were identified according to classical least-square algorithm in the above range of  $s/s_0$ . The resulting experimental values of INL are shown in Fig.4.

Once determined the experimental model for the actual ADC, the inverse

derivation of filter expression by taking into account specific characteristics of the ADC error. This allows: (i) in model definition, a more compact expression to be used, (ii) in model determination, complicate adaptive scheme to be avoided (in each adaptation step a nonlinear equation system had to be solved), (iii) in filter identification, the Volterra filter coefficients to be computed very easily, and (iv) in experimental calibration, a simpler signal to be used.

In preliminary investigations on integrating ADCs, computer simulation using an error model identified by real measurements was carried out. A satisfying performance improvement in terms of signal-to-noise ratio was achieved.

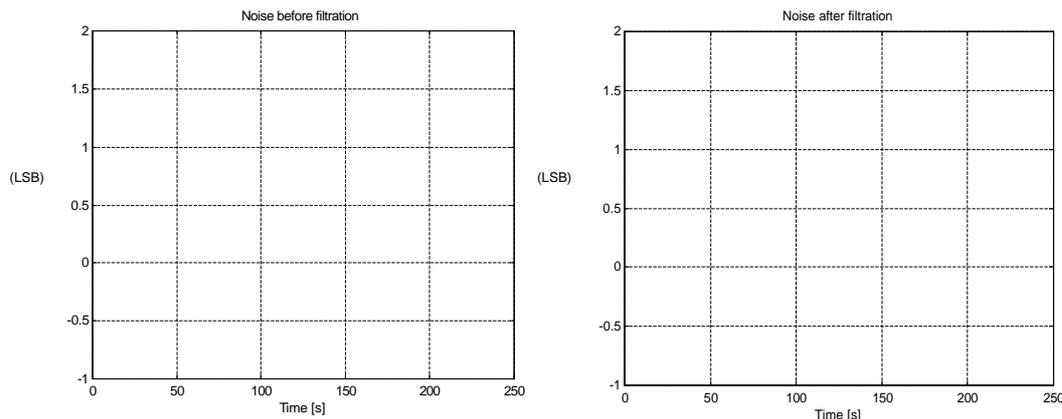


Figure 6. IADC output error: (a) before filtration, (b) after filtration

### ACKNOWLEDGMENT

The work was supported on behalf of National Research Council of Italy (CNR), Targeted Project "Special Materials for Advanced Technologies II" (No. 980014 PF34).

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