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REDUCTION OF SYSTEMATIC ADC ERRORS BY OVERSAMPLING

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Abstract: Oversampling, successive noise shaping, and final low-pass filtering is a well-known approach to enhance dynamic range of analog-to-digital converters. However, the potential of this approach can be limited by systematic errors overriding the quantization error. The paper deals with the reduction of this limitation: an interpolating algorithm, based on a moving-average FIR filter with Bayesian coefficients is proposed, and its configuration inside some correction schemes are presented. Effectiveness, drawbacks, and the corresponding evaluation results of these schemes are discussed.
Keywords: Oversampling, Noise Shaping, Systematic Error, ADC Correction.

1 INTRODUCTION

Oversampling is a basic technique for attenuating the error impact in analog-to-digital converters (ADCs) [1]-[2]: the oversampling frequency f_{OS} overrides the Nyquist frequency $f_S=2f_{max}$ by the oversampling ratio $FA=f_{OS}/f_S$ and allows the quantization noise spectra to be expanded by proportionally reducing its power density. Noise power is further reduced by noise-shaping modulation such as Sigma-Delta (Σ-Δ):unwanted spectral components are shifted beyond the frequency band of interest [2]. However, currently performance of noise-shaping Σ-Δ structures with multilevel quantizers are often limited by the systematic errors of the quantizer.

Systematic errors are reduced usually by interpolation-based methods. Generally, the interpolation process smoothes the distortion in the output signal caused by the differential nonlinearity (DNL) in the ADC transfer characteristic. Moreover, it suppresses also the random errors generated by aleatory error sources affecting the conversion process. However, the main drawback of interpolation algorithms is the generation of new spectral components by harmonic distortion of the processed signal. Thus, though the quantisation noise is reduced by noise modulation, the new spectral components introduced by the interpolation overcome the noise and reduce the final spurious-free dynamic range.

Most common interpolation algorithms are based on techniques of (i) polynomial (or spline), or (ii) higher-order regression. The former are known by their main property of repeating the values in the determining nodes, with the drawback of possible oscillations of the output values among the interpolation nodes. The latter are complicated from the computational point of view.

In this paper, the problem of reducing systematic errors via low-distortion interpolation in ADC architectures based on oversampling and noise shaping is faced. In particular, the use of an algorithm based on a moving averaging FIR filter, where every sample is weighted by Bayesian principle, is proposed. Then, the basic idea of systematic error correction by interpolation simultaneous to noise shaping is discussed by referring to Σ-Δ structures with multilevel quantizers. The enhancement of the ADC dynamic range obtained by some error correction schemes, where the output flux is pre-processed by oversampling, successively smoothed by interpolation, and noise shaped at low frequencies, is assessed.

2 THE BASIC IDEA

In actual ADCs, systematic errors in the ADC transfer characteristic affect the shape of the ideal quantization noise [3]-[4]. The final equivalent quantization noise $e_{real}(t)$ could be modelled as the sum of the ideal quantization noise $e_q(t)$ plus an additive signal n(t) correlated to the input signal:

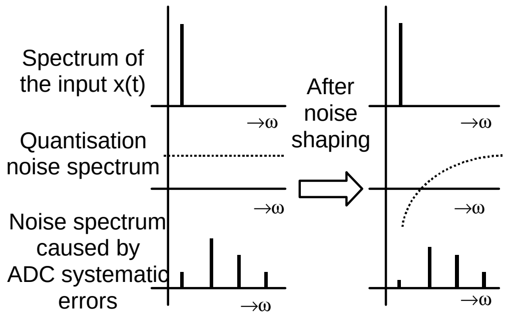

Fig. 1. Spectral components at the output of an actual noise-shaping ADC.

$$e_{real}(t) = e_q(t) + n(t)\ . \qquad (1)$$

Owing to the correlation, the signal n(t) is periodical with frequency components which are integer multiple of the input signal frequency f_x. Thus, the spectrum of the noise $e_{real}(t)$ is the superposition of the distribution of the ideal quantization noise and the line-wise spectrum of the periodical signal n(t). Oversampling, noise shaping, and final low-pass filtering could reduce the ideal quantization noise. Conversely, the systematic error n(t) can be suppressed by smoothing the output digital signal through an appropriate interpolation algorithm.

Models of Σ-Δ structures are well analysed in many sources [2], [6]. The ideal quantization noise from the input $e_q(t)$ is uniform in the Nyquist frequency band *(0,f_{OS}/2).* The power density $\varepsilon^2_{out}(\omega)$ is shifted according to the well-known formula:

$$e^2_{out}(\omega) = \frac{Q^2/12}{f_{OS}/2}\left|\left(1-e^{(jwT_{OS})}\right)^2\right| = \frac{Q^2/6}{f_{OS}/2}\left|\left(1-\cos\omega T_{OS}\right)\right| \qquad (2)$$

where $T_{OS}=1/f_{OS}$ is the oversampling period, and Q is the ADC quantization step.

The power spectrum η(ω) of the periodical signal n(t) could be obtained by the transformation from the z space:

$$h_{out}(z) = h(z)(1-z^{-1})^2 \Rightarrow \quad h^2_{out}(w) = h^2(w)\frac{4}{f_{OS}}\left(1-\cos wT_{OS}\right). \qquad (3)$$

These spectral components in the output spectrum are practically unchanged by the noise shaping procedure (Fig.1). They can be reduced by appropriate interpolation techniques applied to the output data.

Furthermore, after noise shaping, the noise level of the digitised signal at the zero frequency can not decrease below to the level corresponding to the quantization error. This limits the benefits of this structure implemented as post-processing procedure for improving the digital data of an ADC.

In line of principle, a first solution to achieve the advantages of both the approaches could be: (i) a preliminary interpolation of the data at the oversampled output data flux of an ADC, and (ii) a successive digital implementation of the noise shaping (Fig.2). The interpolation algorithm reduces not only the periodic noise caused by systematic errors, but also suppresses the noise generated inside the A/D conversion (i.e. by electromagnetic interference and hazards). Moreover, the interpolating procedure approximates the quantized data by their best fits expressed in floating-point numerical representation. Thus, the data in input to the noise shaper are "analog–like" and with a noise density reduced by the oversampling. In addition, the noise shaping block suppresses the low-frequency components of the quantisation noise on detriment of the high-frequency ones.

However, the main drawback of interpolation algorithms is the generation of new spectral components by harmonic distortion of the processed signal. In the following, an optimal interpolating algorithm reducing the generation of these new frequency components is proposed. The algorithm is based on a moving-average FIR filter where each sample is suitably weighted. The weighting coefficients take into account the probabilistic error characteristic of the ADC.

Let's suppose the oversampling frequency f_{OS} at the A/D conversion process so high that each code level has a high probability of occurrence in the output digital flux. This condition is accomplished

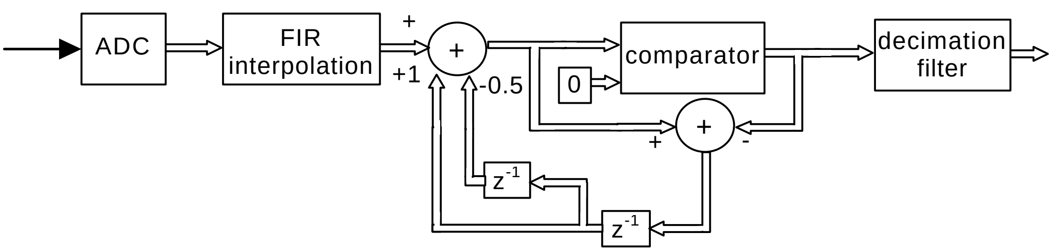

Fig. 2. Principle of digital correction based on a preliminary interpolation of samples and a successive 2nd-order noise-shaping.

when
accomplished when

$$\left.\frac{x((i+1)T_{OS})-x(iT_{OS})}{T_{OS}}\right|_{\max} << \frac{1}{Q}; \qquad T_{OS}=\frac{1}{f_s{*}FA}=\frac{1}{f_{OS}} \tag{4}$$

where x is the analog input signal. In actual ADCs, phenomena generating various types of stochastic noise occur. For this reason, the transition levels are defined in statistical terms, usually as the input producing two neighbouring codes k, and k+1 with the 50% of probability. This leads to the probabilistic definition of the characteristic of a N-bit ADC by 2^N probability density functions $p(k/x[i])$ of any digital output k given for the analogue input $x[i]$ converted in the time instant iT_S [5]. The probability density function $p(k/x[i])$ can be estimated from known values of the transition levels $T(k)$ or determined experimentally, e.g. by a histogram test. By using a Bayesian relation, the optimal reconstruction of the input voltage for the output code $k[i]$ is

$$\bar{x}[i]=\sum_{x=0}^{2^N-1} x\,.\,p\left(x/k[i]\right) \doteq \sum_{x=0}^{2^N-1} x\,\frac{p\left(k[i]/x\right)}{\sum_{x=0}^{2^N-1} p(x).\,p\left(k[i]/x\right)} \tag{5}$$

where $p(x)$ is the probability density function of the input signal. The best estimation of the representing value for the oversampled data flux is obtained by the averaged value $\bar{x}[i]$, calculated by a moving window of length $L=2L_S-1$. The optimal reconstructed sample $x_e[i]$ which represents the windowed output of a FIR filter with length L is

$$x_e[i]=\frac{1}{2L_S-1}\sum_{j=-(L_S-1)}^{L_S-1}\bar{x}[i]=\frac{1}{2L_S-1}\sum_{j=-(L_S-1)}^{L_S-1}\sum_{x=0}^{2^N-1} x\,\frac{p\left(k[i]/x\right)}{\sum_{x=0}^{2^N-1} p(x).\,p\left(k[i]/x\right)} \tag{6}$$

The denominator of the (6) is assessed in the condition that $p(x)$ is a constant equal to $1/2^N$. In this case, the weights in the formula (6) are fixed.

This moving-average FIR filter suppresses the errors caused by the differential nonlinearity effectively. Moreover, its low-pass filtering characteristic reduces the impact of the random error sources.

The performance of the filter in the structure of Fig.2 is assessed by comparing a simulated actual ADC affected by systematic errors, and an ideal ADC for an harmonic input signal. The improvement in the dynamic range is assessed as the gain in the effective number of bits:

$$G\_ENOB=\frac{1}{2}\log_2\frac{e_r^2}{e_{out}^2} \tag{7}$$

where the reference quantisation noise power e_r^2 is the real quantisation noise of the simulated ADC affected by systematic errors, and the final quantisation noise power e^2_{out} is the residual noise after digital low-pass filtering with a cut-off frequency of $f_s/2$, for the same input signal and sampling frequency f_s. As an example, a G_ENOB equal to 2.84 was achieved by the structure of Fig.2 including the proposed FIR filter, for a 7-bit ADC, an input frequency to the filter $f_{input}=4\cdot f_s=64$, FA=128, and an input signal $U_x=0.9U_{max}$.

The proposed interpolation algorithm has been compared also with some other interpolation algorithms (cubic spline, second-order Newton interpolation). The results showed worse performance owing to a higher distortion of the output signal. In particular, Fig 3.a shows the spectral components in the structure of Fig.2 after the noise-shaping block when a cubic-spline interpolation is employed.

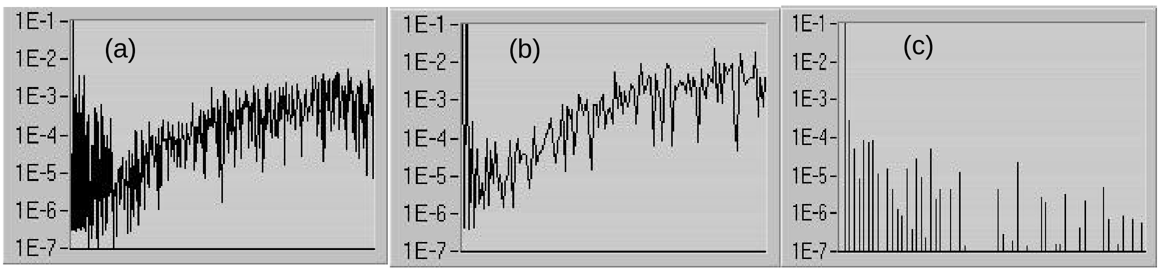

Fig.3. Noise spectrum at the output of (a) the noise shaping block after a traditional spline interpolation, (b) the noise shaping block after the proposed FIR interpolation, and (c) the proposed FIR filter directly, for a 7-bit ADC, $f_{input}=4\cdot f_s=64$, FA=128, $U_x=0.9U_{max}$, G_ENOB=2.84.

Better results are achieved if the proposed FIR interpolation is used (Fig 3.b). The effect of the proposed interpolation is highlighted in Fig.3c, where the spectrum is represented directly at the output of the proposed interpolation block: by comparing with the spectrum of Fig. 3a, a significant reduction of the low-frequency harmonic in the band of interest can be argued easily. However, an improper effect of spreading of the basic spectral components due to the low resolution of the digital input data can be still argued.

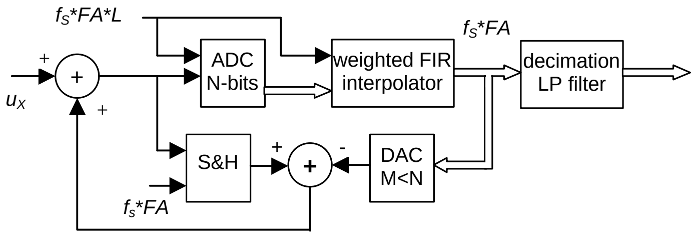

Fig. 4. Σ-Δ structure with analog noise-shaping and systematic error reduction by the proposed Bayesian FIR filter.

3 S-D STRUCTURES REDUCING SYSTEMATIC ERRORS BY THE PROPOSED INTERPOLATOR

Apart the above principle scheme of digital correction in Fig.2, real noise shaping structures where data are processed with high resolution are based on an analogue feedback.

An analogue-feedback structure, where the periodical noise $\eta(\omega)$ caused by the differential nonlinearity of the ADC is reduced by the proposed FIR filter, is shown in Fig.4. The DC shift is suppressed by the negative feedback. The weighted average value is calculated from $L=2*L_S-1$ samples around the sampling instant i. The delayed feedback value is added to the sample just after the end of the window ($i+L_S$). The calculated value is obtained as the difference between the delayed input and the output value of the quantizer of the Σ-Δ structure. The delay of the ADC is provided by an analog S&H circuit. The result of the subtraction is then added to the input voltage of the 1^{st} order Σ-Δ filter as a whole. In this way, the sampling rate at the DAC input is reduced by a factor L_S. A reserve in oversampling frequency for the implemented noise shaping procedure is provided by selecting a frequency at the filter output still overriding the Nyquist frequency of the input signal and accomplishing the condition $f_{out} >> f_S$.

The integral nonlinearity of the DAC must be smaller than the one of the ADC. Such a property is achieved by DACs with pseudocasual connection of k parallel resistances from 2^N of possible branches [6].

The drawback of this scheme is related to the need for two different oversampling frequencies: (i) f_S*FA*L_S, used in the A/D conversion, and (ii) f_S*FA from the output of the weighted FIR filter, used in the noise shaping procedure (Fig 4).

The higher-order noise shaping requires an additive Sample & Hold circuit which allows the analog samples to be shifted according to the corresponding filter structure.

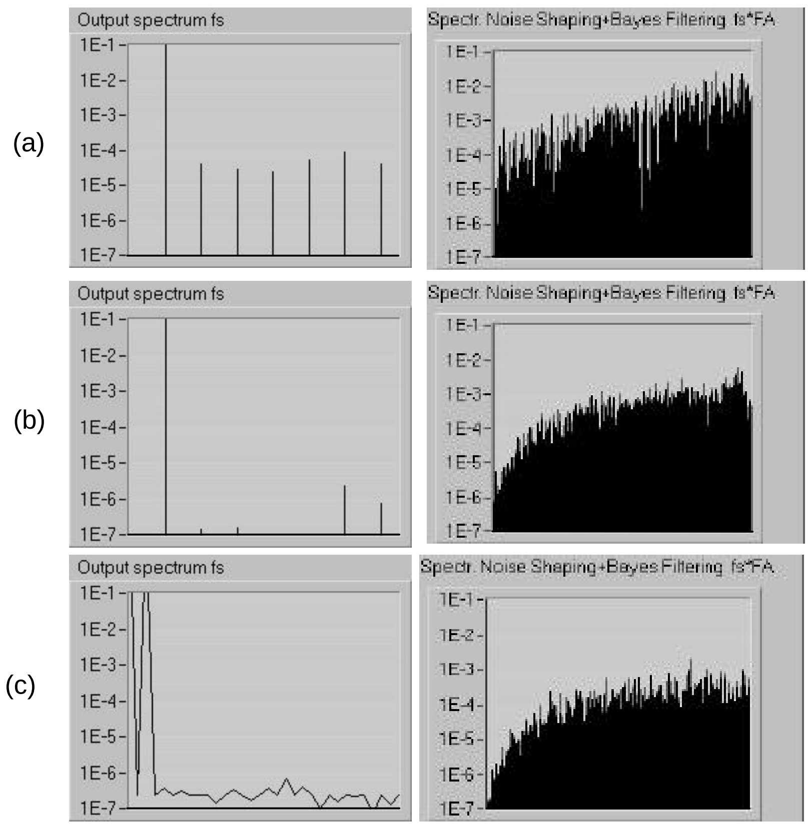

Fig.5. Spectrum (a) after noise shaping for a modelled actual 5-bit ADC without reduction of systematic errors (G_ENOB=6.1), (b) with suppression of systematic errors by the proposed FIR filter with Bayesian coefficients (G_ENOB=7.4), (c) after noise shaping with an ideal 5-bit ADC (G_ENOB=7.6).

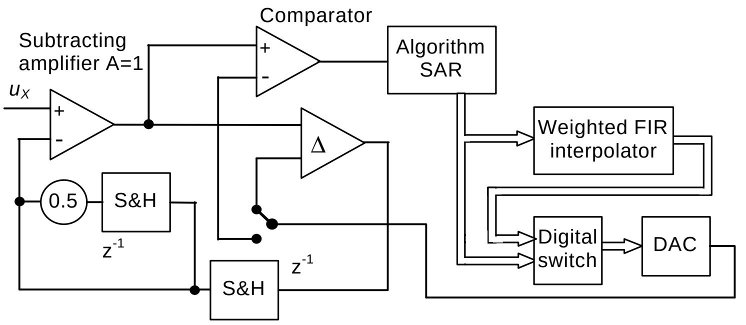

Fig. 6. Structure of a SAR ADC with last conversion step dedicated to noise shaping

This noise shaping structure elaborating the input analog signal does not spread the basic frequency line such as highlighted in Fig.3c for the previous one. The proposed FIR filter for the suppression of systematic errors improves the suppression of low-frequency components of the quantisation noise.

This is highlighted in the example of Fig.5. In particular, the output of the noise-shaping circuit is shown (i) in Fig.5a, for the case without any correction of the ADC systematic errors (G_ENOB=6.1), (ii) in Fig.5b, for the case of correction by the proposed FIR filter (G_ENOB=7.4), and (iii) in Fig.5c, in the case of an ideal ADC (G_ENOB=7.6). The differences are visible mostly at low frequencies. In particular, in the case of the not corrected ADC, the quantisation noise is overlapped by the spectral multiple of the basic frequencies.

The drawback of two sampling frequencies can be overcome by the structure shown in Fig.6. Here, the corrected value from the output weighted FIR filter is estimated by taking into account the optimal reconstruction $\overline{x}[i]$ of the input voltage from the output code k[i] by using the Bayesian relation (5). The estimated value in the time instant i is calculated from the actual reconstructed value $\overline{x}[i]$ and from the values averaged in the previous time instants by the recurrent algorithm. The coefficient $q \in (-1,1)$ strengthens or weakens the influence of the preceding samples.

$$x_e[i] = \frac{\overline{x}[i](1-q) + x_e[i-1](1+q)}{2} \qquad (8)$$

Then, such a value is added to the input signal. The structure of a SAR ADC corresponding to this process is shown in Fig.6. The A/D conversion is performed in N+1 cycles, where the first N ones are dedicated to the N-bit successive approximation process, and the last one is devoted to the noise shaping. Both the processes are controlled from a DSP. During the N cycles, the DSP controls the DAC in order to convert the input voltage for the SAR algorithm. After the conversion, the DSP calculates the best estimation of the actual sample by the (8). Then the DSP controls the conversion and noise shaping procedures, as well as the switches for both the working phases.

The performance of the ADC with such a post-conversion procedure has been assessed by computer simulations. The enhancement of the effective number of bits was evaluated for various values of the coefficients q utilized in the filtering algorithm (8). Fig. 7 shows the relation between the value q and the achieved improvement of the effective number of bits G_ENOB. The curve with the maximum value shows the results of the simulation for the case of an ideal ADC with no systematic errors (DNL(k)=0). The gain of such a converter has been enhanced by enlarging the length of the utilized averaging filter until a maximum value of q = - 0.8*.* A successive decrease in q causes the gain of ENOB to be smaller.

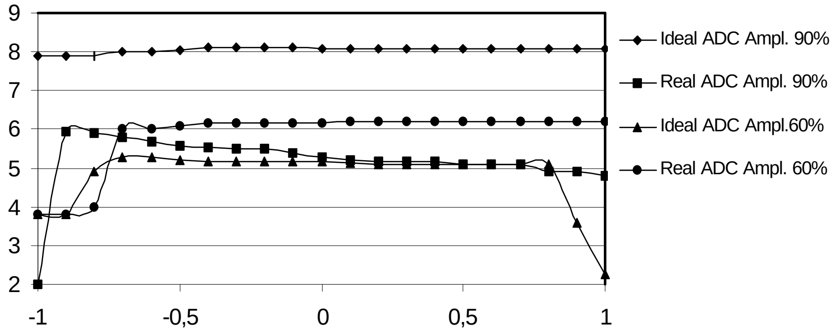

Fig. 7. Gain of ENOB vs. quotient *q* in weighted Bayesian filter (8) for various amplitudes of the input signal.

4 CONCLUSIONS

The oversampling was shown to be useful also for reducing systematic errors by interpolating the measured data of an actual ADC affected by systematic errors. In general, the harmonic distortion caused by the interpolator limits the effect of the successive digital noise shaping block. A suitable FIR filter with Bayesian coefficients suppressing the distorted spectral components overriding the reduced quantisation noise was proposed. The fixed digital value below quantisation level for DC signals can be reduced by dithering of the input signal.

In noise shaping structures with a multilevel ADC, the proposed FIR filter allows the effects of systematic errors to be suppressed by reducing the harmonics multiple of the input signal caused by the ADC nonlinearity.

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