

CHARACTERISATION AND MODELLING OF THE ADC JITTER

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Abstract. The use of the Allan variance for the characterisation of the jitter error in analog-to-digital converters (ADCs) is proposed. In particular, the Allan variance is a sound basis for defining a figure of merit for jitter errors, diagnosing the jitter noise type, and including a jitter block into a previously proposed ADC model. Experimental results highlighting the effectiveness of the Allan variance in the characterisation of the ADC jitter error are discussed.

Keywords: Jitter, A/D Converters, Allan Variance

1 INTRODUCTION

In last years, research on analogue-to-digital converters (ADCs) received a big impulse by wider and wider applications in communications and related applications. At highest sampling frequencies, an error-free reconstruction of the digitised signals depends strongly on the capability of rejecting the whole of phenomena of time instability inside the ADC architecture [1]. Such phenomena give rise to an uncertainty on the sampling instant indicated mainly as timing jitter (or simply jitter), or as aperture uncertainty, or even as timing-phase noise [2]. In particular, the IEEE 1057-94 Standard on ADC metrological characterisation defines the jitter "the standard deviation of the sample instant in time" as a measure of the short-term stability taken during the longest single-record acquisition [2]. Since only the nominal sample times are known, the effect of the time jitter is to introduce random errors in the amplitude values which are assumed to correspond to the sample times. The magnitude of this apparent amplitude error is standardised as the timing error multiplied by the slope of the signal recorded at that instant.

The intrinsically random nature of the jitter makes usual its characterisation by traditional statistical techniques [2]-[3]. Several techniques have been proposed in order to evaluate the jitter effect on the ADC dynamic behaviour by separating all the error components both in the statistical, and in the time-domains [3]-[10]. However, the definition of a suitable figure of merit still needs for more attention. In particular, phenomena related to random variations of the frequency over the time are usually modelled by using power spectrum density (PSD) laws proportional to a power of the sampling frequency [4]. In some cases, such as the jitter noise, the variance and the standard deviation defined in classical statistics diverge [4]-[5]. Therefore, random processes like jitter are not correctly characterised by the classic variance intrinsically. In last years, such phenomena have been modelled successfully in terms of Allan variance, and specifically by its "modified" version [11]-[12].

In this paper, the usefulness of the Allan variance in the metrological characterisation of jitter errors in ADCs is shown. At the first, the characterisation problem is restated by defining a more appropriate figure of merit for jitter errors based on the Allan variance. Then, the diagnostic worth of the proposed figure of merit in determining the particular type of jitter noise is shown. Furthermore, the modification of an actual ADC model [13] including also jitter noise effects is proposed. Finally, experimental results highlighting the effectiveness of the Allan variance in the characterisation of ADC jitter error are reported.

2 THE ALLAN VARIANCE FOR ADC METROLOGICAL CHARACTERIZATION

The IEEE 1057-94 Standard defines the jitter "the standard deviation of the sample instant in time" as a measure of the short-term stability taken during the longest single-record acquisition of the ADC under test at a given sampling frequency [2]. Apart from more advanced techniques [3], a procedure was standardised to measure the jitter (section 4.9.2) [2]. At first, a sample of phase deviation F is estimated for each point of the record by: (i) applying a sine-fit algorithm to the samples; (ii) computing the corresponding residuals in each sampling instant; and (iii) calculating the time aperture errors in each sampling instant by multiplying the amplitude residuals for the related time derivatives. By repeating this procedure, the standard deviation of the jitter probability density function is estimated in each point of the longest ADC record. Then, an upper bound of the jitter is measured as the standard deviation of the statistical sample distribution of the standard deviations achieved inside the record. This involves the acquisition of several records in order to estimate adequately the sample distribution of the standard deviations inside the record. Moreover, this is valid for the operating sampling frequency, and the experiment should be repeated, if the jitter would be estimated at a different sampling frequency.

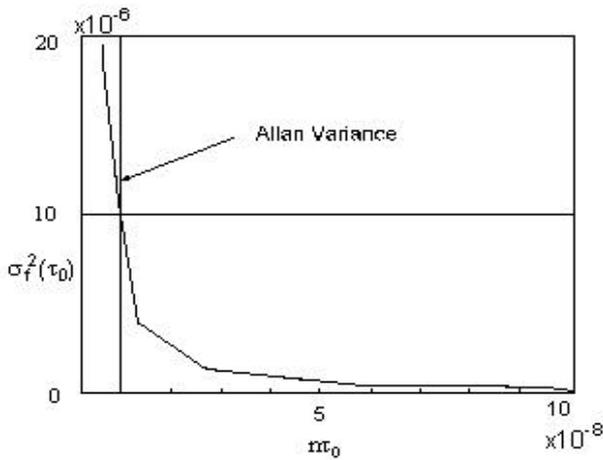


Figure 1. Allan variance trend with the sampling period.

A more suitable figure of merit can be introduced to assess the jitter error inside the record by exploiting the concept of variance of a time-varying random process.

Given the sequence F_i of the phase deviations in the points of the longest ADC record taken at a sampling frequency $1/t_0$, the first-order finite difference Df between two successive phase deviation samples, i.e. a discrete estimate of the instantaneous frequency, can be obtained as $Df = (F_i - F_{i-1})/t_0$.

The Allan variance of the random process of the instantaneous frequency is defined as [11]

$$\sigma_f^2(t_0) = 1/2 \langle \Delta f^2 \rangle, \quad (1)$$

where the symbol $\langle \rangle$ stands for the mean operator on infinite samples. In this way, the Allan variance expresses concisely the spread of the instantaneous frequency point by point along the record at the sampling frequency $1/t_0$.

The biggest advantage of the Allan variance as a figure merit for the ADC jitter error is the possibility of expressing the trend of the maximum jitter error as a function of the sampling frequency by only one record acquisition. Let a reference sine wave be acquired for the ADC longest record at the maximum ADC sampling frequency $1/t_0$. According to the above procedure, the Allan variance is computed by the equation (1). A change in the sampling step t can be performed easily by successively computing the (1) on successive couples of instantaneous frequency samples spaced by $t = nt_0$ in the original sample sequence. In this way, a new estimate of instantaneous frequency is obtained for each time interval nt_0 . This allows information on the maximum time jitter to be gathered also at sampling frequency different from the effective one by means of a unique experimental acquisition. The trend of the Allan variance with the sampling period n is shown in Figure 1.

Moreover, this allows also an easy definition of the maximum sampling frequency compatible with a given limit of the jitter error. Overcoming the limit of jitter specifications over sampling frequency is immediately argued by Fig. 1: given the jitter limit expressed in terms of maximum Allan variance, the corresponding achievable maximum sampling frequency is immediately determined by the law $S_f^2(n t_0)$.

Finally, the Allan variance is also convergent in some cases where the traditional variance diverges [11]-[12]. In particular, for the case of the jitter PSD such as w^{-1} , the classical standard deviation expressed by the integral of the Parseval theorem:

$$s = \frac{1}{P} \int_0^\infty S(v) dv \quad (2)$$

where $S(w)$ is the symmetrical two-side spectrum density, diverges in both the integration limits. Conversely, the Allan variance was shown to be convergent for the most common type of jitter power laws [11]-[12]. An exhaustive discussion about the link between the standard and the Allan variances is reported in reference [14].

3 THE ALLAN VARIANCE FOR ADC JITTER DIAGNOSIS

The Allan variance is useful also to detect and identify the jitter type for the diagnosis of the ADC errors.

In problems dealing with the PSD of random processes $S(w)$ proportional to a power of the pulsation such as w^α , the Allan variance was shown to be proportional to a power law of the sampling period: $\sigma_f(\tau) \propto \tau^\mu$, where μ is usually a constant value for constant values of α . In particular, assuming that $S(w) \propto w^\alpha$, α and μ are related as follows: $\mu = -\alpha - 1, \forall \alpha \in [-3, 1]$; or $\mu \equiv -2, \forall \alpha \geq 1$ [11]-[12]. In particular, the relation between α and μ is shown in Table 1 for the most common types of noise. In case of PSD with $\alpha = 2$, the apex b indicates the need of using the modified Allan variance to solve the ambiguity in the definition when $\mu = -2$. In fact, this ambiguity does not allow a flicker noise phase modulation (PM) with $\alpha = 1$ to be distinguished from a white noise PM with $\alpha = 2$. This problem is solved by using the modified Allan variance with $\mu = -\alpha - 1, \forall \alpha \in [1, 3]$. Consequently, a logarithmic diagram $\sigma_f(\tau)$ allows the jitter noise present in the ADC under test to be identified easily by estimating the slope of the diagram.

The corresponding diagnostic process can be divided into 4 phases:

Table 1. Allan variance for some types of noise.

a	Noise type	Allan var.
2	White-noise PM	$a_2\tau^{-3b}$
1	Flicker-noise PM	$a_1\tau^{-2}$
0	Flicker-noise PM	$a_0\tau^{-2}$
-1	Flicker-noise FM	$a_{-1}\tau^0$
-2	Random-walk FM	$a_{-2}\tau$

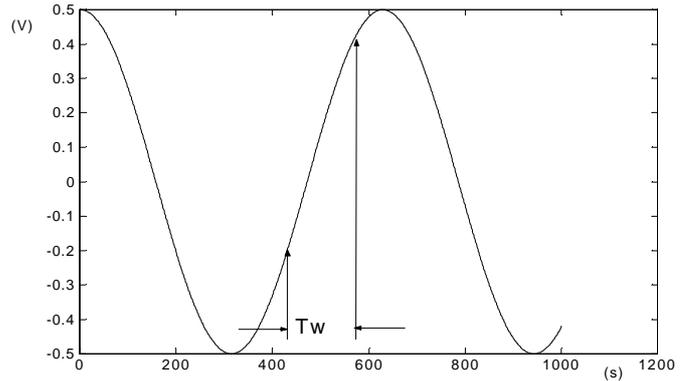


Figure 2. The Allan variance capability of detecting very-slow jitter on small time windows.

1. Acquisition of a suitably statistically significant number of periods and samples of a sine wave input signal; determination of the phase deviation sequence via the standard procedure; calculus of the instantaneous frequency sequence;
2. Estimation of the Allan variance for various values of $t=n\tau_0$;
3. Experimental evaluation of μ of the logarithmic law;
4. Characterisation of the jitter type by the knowledge of the w^α law.

An important application is related to the capability of detecting very-slow jitter on quite small time windows. As an example, the phenomenon of Fig.2 can not be detected by an analysis based on the classical variance carried out by acquiring data during the time window T_w . Conversely, the above analysis based on the Allan variance aimed at identifying the noise type allows the problem to be solved.

4 THE ALLAN VARIANCE FOR ADC JITTER MODELLING

The above analysis also allows an ADC error model taking into account jitter errors to be set up. With this aim, a previously proposal actual ADC model [13] was suitably modified such as depicted in Fig. 3. The new model differs from the former one for the input block taking into account an improved characterisation of the jitter noise by means of the Allan variance. The model is the cascade of three sections. The first section includes only an input block ω^α modelling the jitter effects (time jitter block). This model block is identified according to the procedure described in section 3. However, the jitter was measured through the zero crossing technique [15] in order to have a phase deviation estimate independent from other sources of noise. The previous version of the model [13] embodied here only a simple delay block. Conversely, the inclusion of the jitter error allows a more complete characterisation of the ADC error sources. The second section contains three parallel branches: (i) the upper branch models the saturation effects by the odd function “hyperbolic tangent” $a*\tanh$ (amplitude compression block); (ii) the middle branch models the effects of the nonlinear transfer function by the even function “hyperbolic cosine” $d*[1-\cosh(v_1/c)]$ (distortion block); and (iii) the lower branch models the gain error effects by the constant k (gain block). Finally, the third section takes into account the effects of all the noise sources producing a reduction of the signal-to-noise ratio evidenced in the frequency domain by a noise floor growth apart the jitter noise effects. This residual noise is expressed by subtracting the jitter contribution assessed in the time jitter block to the overall noise.

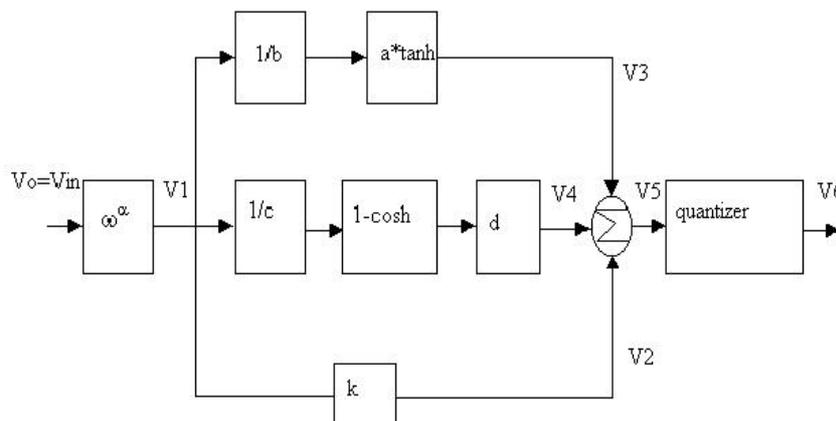


Figure 3. New ADC model including jitter effects.

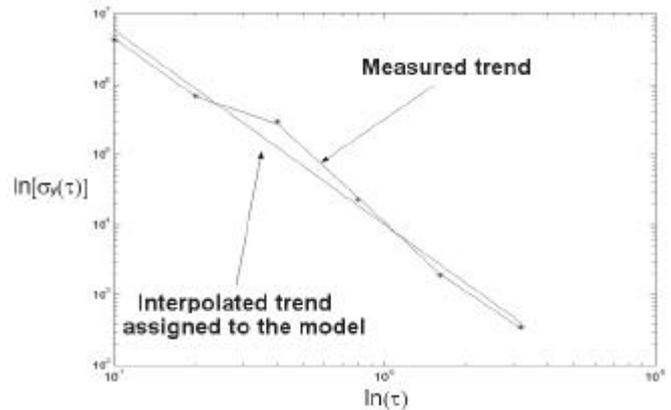
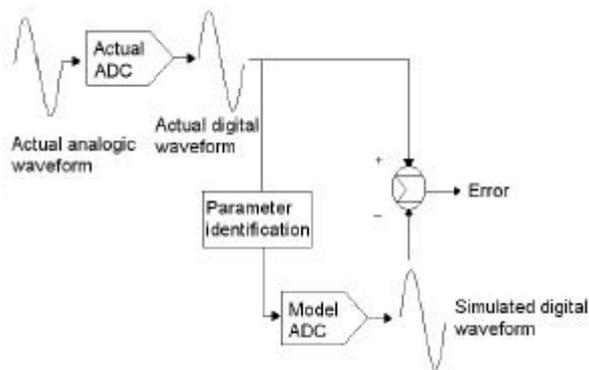


Figure 4. Validation procedure of the new ADC model.

Figure 5. Logarithmic graph of the Allan variance.

5 EXPERIMENTAL RESULTS

A series of experimental tests on actual ADCs has been carried out in order to validate the proposed approach. A calibrated signal generator Stanford Research DS360 was used to characterise a Tektronix Waveform Analyser VX4240. The validation scheme is shown in Fig. 4. The model parameters were identified by imposing the simulated ratio of the harmonic magnitudes equal to the experimental one through a sine wave acquisition [13]. The model error is evaluated by calculating the amplitude differences between the ADC actual and model outputs.

An example for a test sine wave of amplitude 10 V, and frequency 100 kHz acquired at a sampling frequency of 10 MS/s for a total of 10000 samples is considered. In Fig. 5 is reported the experimental behaviour of the Allan variance in versus τ in a logarithmic scale computed according to the procedure of section 3. The resulting value of α indicates a random noise process whose PSD is located between $\alpha=-1$ and $\alpha=0$.

The identified values of the other model parameters are: $a=0,001$; $b=0,10$; $c=0,60$; $d=0,0025$; $k=1,00$. The validation error of the new model is reported in Fig. 6. For the sake of the clarity, in Fig.7 a particular of the error trend is zoomed on a different time window. Experimental results of Figs.6 and 7 highlight the effectiveness of the proposed model in taking into account ADC jitter errors.

6 CONCLUSIONS

A new approach to the metrological characterisation, the diagnosis, and the modelling of the ADC jitter based on the Allan variance has been presented. The new figure of merit based on the Allan variance expresses concisely the spread of the instantaneous frequency over the record at different sampling frequencies. Moreover, it allows the diagnosis of the jitter error type, as well as the update of a previously proposed error model. The new ADC model is different from the former one for the use of a block taking into account the jitter noise. Preliminary experimental results showed the effectiveness of the proposed approach in characterising the ADC error jitter.

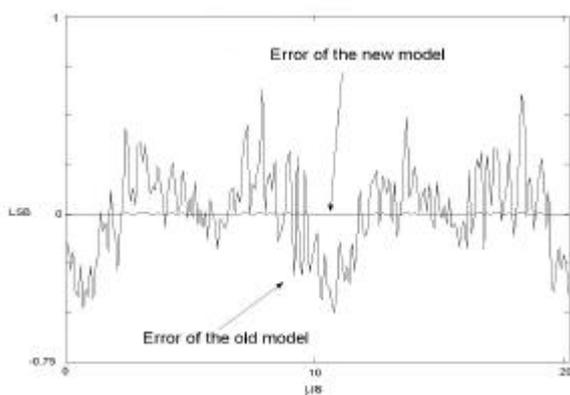


Figure 6. Errors of the new and old models.

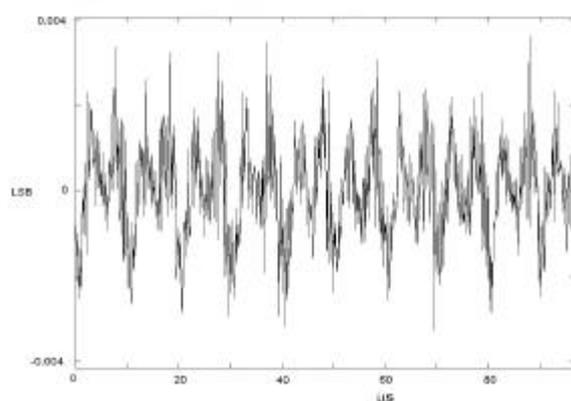


Figure 7. Particular of the error of the new model.

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