

INFORMATION TRANSFER OF DITHERED A/D CONVERTERS

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Abstract: There are several methods to enhance the resolution or the linearity of an ADC by dithering. One can consider this as changing bandwidth against resolution. This paper analyses some dithering methods from the viewpoint of information transfer.

Keywords: Analogue-Digital converter, Dithering, Information transfer.

1 INTRODUCTION

There are several methods to improve the characteristic of an A/D converter [1]. Dithering makes this by adding a suitable signal (deterministic or stochastic) and calculating the final result as a mean value of several converted values. The averaging diminishes the effective bandwidth of the analogue-digital conversion. One can consider that dithering brings the possibility to change bandwidth against resolution.

Dithering has been discussed in several IMEKO Workshops. There was also a dedicated ONLINE-Workshop organised 1998 by the universities of the authors. Aumala and Holub have presented some results discussed in this workshop [2] in their paper [3].

This paper collects papers presented so far and discusses them from the viewpoint of information transfer.

2 SAMPLING AND QUANTISATION

Sampling and quantisation are the fundamental operations of the A/D converter. It is commonly known that the resulting data series describe the input signal waveform correctly only if the sampling does not cause any aliasing effect. This condition is formulated in the sampling theorem, which states that the sampling frequency has to be at least twice the highest frequency presented by the signal to be converted:

$$f_c = \frac{1}{t_c} \geq 2 f_{\max} \quad (1)$$

It is not so widely known that in the amplitude domain there is a similar condition. This is presented by Widrow, Kollár and Liu [4]. They show that the resolution of the A/D conversion should fulfil a condition regarding the form of the probability density function of the signal. The resolution has to be smaller than half of the biggest wavenumber of the characteristic function:

$$\Psi_x(u) = 0 \quad \text{for } |u| > \frac{p}{q} = \frac{\Psi}{2} \quad (2)$$

where $\Psi_x(u)$ is the characteristic function (Fourier transform of the probability density function), and q is the quantisation step value.

Dallet presents in his paper [5] a set of concepts used for characterising the A/D converters. The analysis includes statistical analysis as well as time domain and spectral analysis.

Bellan *et al.* [6] presents a theory for calculating the spectra of the quantisation error for deterministic waveforms. This corresponds to the case when the digital data series is used for sound reproduction or other similar purposes where the spectrum is important.

3 PURPOSE OF DITHERING

The purpose of dithering is to improve the effective resolution, or to increase the effective number of bits of the conversion. The aim can be accurate reproduction of very small changes in the incoming signal, or it may also be improvement of the overall measurement uncertainty. In the first case only some uncertainty components have to be taken into consideration, because the measurand moves only slightly, and stable error components have no effect on the conclusions.

Holub and Smid present in their paper [7] how dithering may be used also to linearise the characteristic of the A/D converter. In this case the dithering amplitude is rather large in order to cover areas of nonlinearities.

Dunay, Kollár, and Widrow discuss dithering for floating-point number representation in their paper [8]. Their basic idea is to implement dither that is a uniform or a triangular-shaped one *for the mantissa*, with the same exponent as of the numbers. A floating-point D/A converter is very suitable for producing this type of dither signal.

3.1 When to Use Dithering

Measurement uncertainty is a limiting factor in amplitude domain. If the quantisation step is the largest uncertainty component, it is probably useful to diminish the effective resolution by dithering. But if this is not the case, one should make an uncertainty evaluation and look particularly on the role of the rounding uncertainty.

The ideal quantiser characteristic is in Fig. 1. If the step size of the A/D converter is q , the rounding uncertainty (standard deviation) is

$$s(e_q) = \frac{q}{2\sqrt{3}} \quad (3)$$

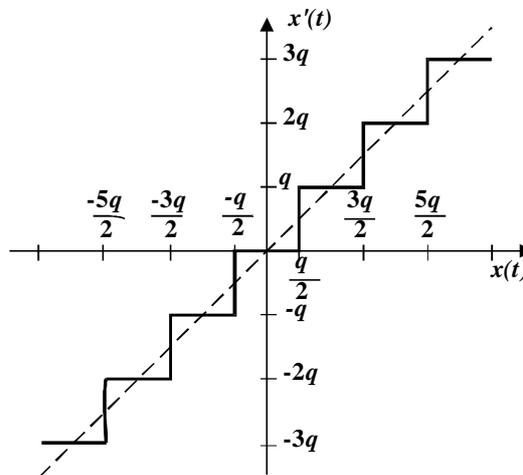


Figure 1. Ideal quantiser characteristic.

If this is one order of magnitude smaller than other uncertainty components, resolution improvements have generally no practical value, and dithering would be meaningless.

4 DITHERING PROCEDURES

Petri [9] presents topologies of dithering and analyses quantisation effects both for deterministic and for stochastic dithering signals. The topologies are shown in Fig. 2.

4.1 Deterministic Dithering

The goal of the deterministic dithering is to add to the signal consecutively such a set of dither values that the mean of the conversion results gives an unbiased estimate of the signal s and produce a minimal value of effective resolution. The common method is to add N values distributed uniformly over an interval $(-\frac{2k+1}{2}q, \frac{2k+1}{2}q)$, most often $(-q/2, q/2)$. Other distributions are also possible. It can be shown that every distribution obtained from a sum of the above mentioned signal of uniform distribution and any independent signal with symmetric distribution can be used.

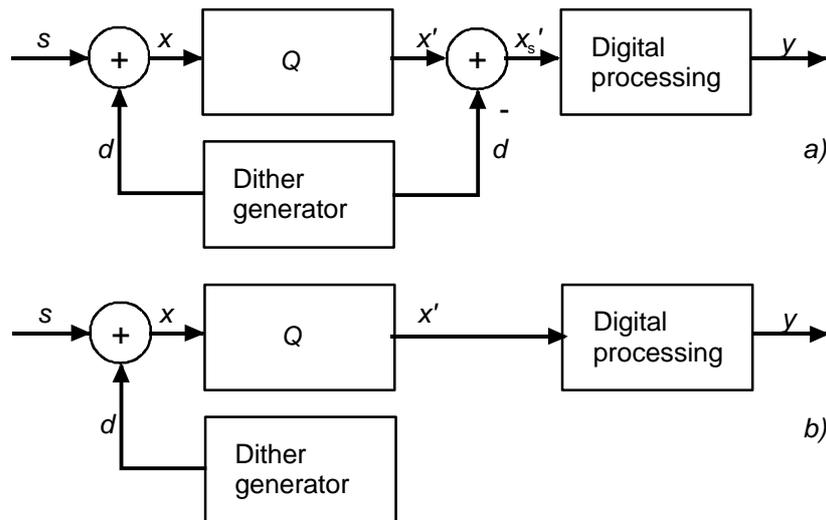


Figure 2. Subtractive (a) and non-subtractive (b) dithering topologies.

Two cases of sum signals are interesting. One is a sum of two independent uniformly distributed signals that produces a triangular distribution. The other one is a sum of a dither signal distributed uniformly or triangularly and a sinusoidal signal covering exactly a number of cycles. This indicates that network interference can be taken as a dithering component, and dithering may be combined with interference suppression in a very useful way [10].

An idea how the presented case distributions produce results is depicted in Fig. 3.

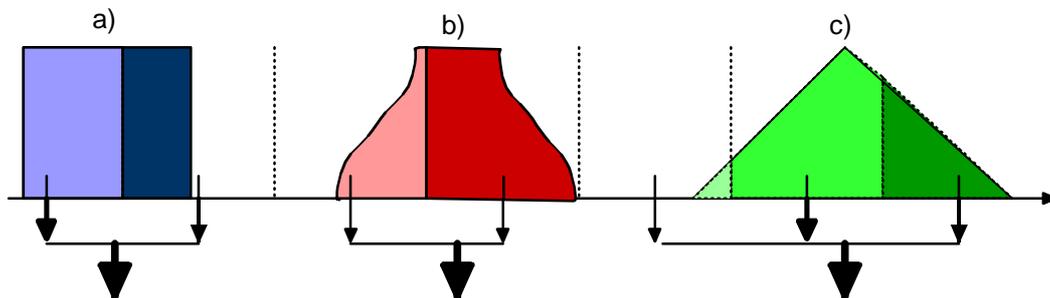


Figure 3. Case dither distributions. a) uniform distribution $(-q/2, q/2)$, b) uniform distribution plus interference, c) triangular distribution $(-q, q)$. For each case the arrows depict how the conversion result is formed.

4.2 Stochastic Dithering

The dithering signal can also be stochastic (e.g. noise or randomly sampled deterministic signal). If this stochastic signal is uniformly distributed, all the above mentioned conditions for unbiased results are valid. More often, the stochastic dither is normally distributed. In such case, the dithering signal should have RMS value higher than certain value to provide unbiased results. For details, see [3] or Chapter 5.1 of this text.

In all cases of stochastic dithering signal, more samples must be acquired and averaged to obtain the results with the same resolution as for the deterministic case - compare equations (5) and (7). Also, it is usually impossible to use subtractive topology for stochastic dither because the values of dither signal in moments of sampling are not known.

5 INFORMATION CHARACTERISTICS

The most practical importance of a measurement is its information content [11]. The classical information theory presented by Shannon analyses however only one aspect of the information: the syntactic information. Other aspects are the pragmatic information (meaningfulness, dependability,

etc.) and the semantic information (usefulness, specificity, etc.). Mari drafts in his paper [12] a theory where the semantic and pragmatic aspects can be taken as components of the information.

In this paper only the syntactic information is analysed, and it is discussed here as an equivalent to the measurement uncertainty. For dynamic measurements the transinformation flow is discussed.

5.1 Information entropy

Aumala [13] presents three basic cases of dithering, and gives measurement uncertainties for quasistationary signals. He also presents a virtual instrument [14] for studying dithering cases.

Baccigalupi [15] suggests according to an IEEE standard draft [16] that the effective number of bits can be defined as

$$n_{\text{eff}} = \log_2 \left(\frac{\text{full_scale_range}}{\text{rms_noise} \cdot \sqrt{12}} \right) \quad (4)$$

where the *rms_noise* is defined according to a best fit (sine wave) data set.

Information entropy may be identified with the effective number of bits. Its value is for the most common deterministic dithering structure [10]

$$n_{\text{eff}} = n + \log_2(N) \quad (5)$$

where *n* is the effective number of bits for the undithered A/D converter and *N* the number of dither values averaged at a time.

Dithering may be also stochastic. If other uncertainty components are not taken into consideration, the uncertainty is made up of the inherent uncertainty (caused by the distribution of the stochastic noise) and the number of averaged values. For stochastic dithering with normally distributed noise the inherent uncertainty [3] is for most cases negligible for $s_{\text{noise}} > 0,7q$, and the uncertainty is in these cases [13]

$$s_N = \frac{\sqrt{q^2/12 + s_{\text{noise}}^2}}{\sqrt{N}} \quad (6)$$

Thus the effective number of bits is

$$n_{\text{eff}} = n + \frac{1}{2} \log_2 \left(\frac{Nq^2}{q^2 + 12s_{\text{noise}}^2} \right) \quad (7)$$

Figure 4 shows the theoretical effective number of bits for a 10 bit A/D converter.

5.2 Information Bandwidth

The bandwidth of a transfer device or system (amplifier etc.) used for sinusoidal signals is usually given as the -3 dB bandwidth. This is well suited for A/D converter tests with sinusoidal signals. The characteristic operation of dithered converters is the averaging. If it is calculated as a moving average, the amplitude characteristic obeys the formula

$$\frac{Y(f)}{S(f)} = \frac{\sin(2\pi f N \Delta t)}{2\pi f N \Delta t} \quad (8)$$

where Δt is the sampling period. The value corresponding to -3 dB is

$$f_{-3 \text{ dB}} = 0,227 / (N \Delta t) \quad (8)$$

For many measurement purposes it is better to use the concept of response time. Because the response time is directly equivalent to the averaging time of the converter, the corresponding bandwidth is

$$f_r = 1 / (N \Delta t) \quad (9)$$

If the converter is used to suppress network interference, the averaging time is equal to the network cycle time: $T = N \Delta t$.

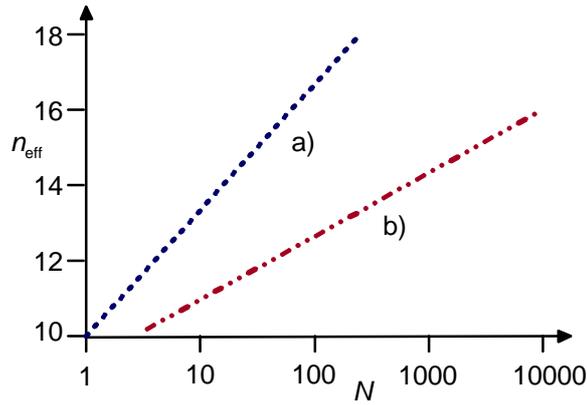


Figure 4. Dithering increases the effective number of bits n_{eff} of an A/D converter. 10 bit converter, a) standard dithering $(-q/2, q/2)$, b) dithering with normally distributed noise, $\sigma_{\text{noise}} = 0,35 q$. N is the dithering number.

5.3 Information Flow

The transinformation flow is a very important concept for information transfer. In measurement applications it can often be evaluated as the product of the effective number of bits and the bandwidth. In some cases of measurement of quasistationary signals the bandwidth is even not important.

Figure 5 presents the theoretical transinformation flow for the standard dithering case (uniform dithering $-q/2, q/2$) and for the stochastic dithering with normally distributed noise, $\sigma_{\text{noise}} = q$.

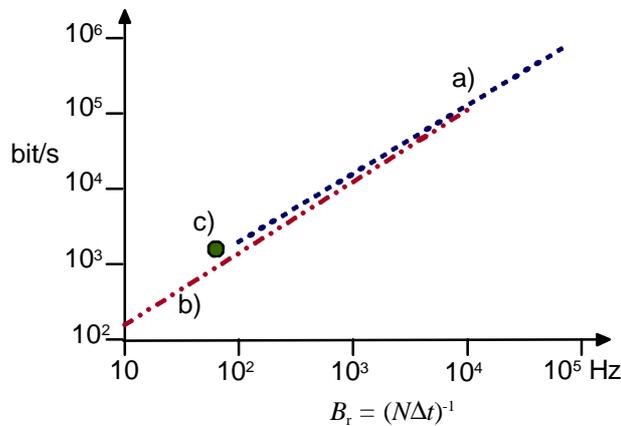


Figure 5. Transinformation flow for some dithering cases. A/D board resolution 10 bits, conversion rate 80 kS/s. a) standard dithering $(-q/2, q/2)$, b) dithering with normally distributed noise, $\sigma_{\text{noise}} = 0,35 q$, c) combination of standard dithering and 50 Hz interference suppression. B_r is the response time based bandwidth f_r .

One can see immediately that the information flow depends mainly of the bandwidth. If the information flow is important, dithering should be restricted to the minimum needed.

6 DISCUSSION

It is possible and widely applicable to change bandwidth against resolution of A/D conversion by dithering. Even disturbing (originally undesired) noise or interference can be used as useful dither signal (self-dithering). The information approach confirms that dithering is interesting and useful method but not a universal medicine – all improvements that are achieved in the area of resolution or even linearity are compensated by losses in effective bandwidth and response time. Definitely,

the opportunity to balance both groups of parameters adaptively according the concrete input signal or concrete application is the advantage of dither-based ADCs.

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