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TIME-DOMAIN TESTING OF A/D CONVERTERS

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Abstract: *Time-domain testing of A/D converter in single tone mode amounts to the identification of fundamental and harmonic tones in noise. The variable projection method reduces this problem to frequency estimation by minimising a one-dimensional function. In this paper it is shown that the bias due to ignoring the harmonic distortion effect is negligible, and this approximation leads to fast frequency estimation algorithms. Three methods are examined (Newton, Gauss-Newton and Golden section search) and compared to the method which takes into account the effect of the distortion through simulations and experimental data. The convergence and implementation issues of the algorithms are also treated.*

Keywords: ADC dynamic testing, sine-wave fitting, variable projection method, global convergence.

1 INTRODUCTION

The characterisation of analogue-to-digital converters (ADCs) by time-domain analysis has the advantage of not requiring an accurate coherent sampling. The test problem amounts to the identification of fundamental and harmonic tones in noise. The use of the variable projection (VAPRO) method [1] splits the identification problem into two parts. Firstly, the fundamental frequency is estimated by minimising a one-dimensional function. Then, the model linear parameters (combination of the amplitudes and phases of the different tones) are estimated by solving a linear set of equations. Thus, the problem with which we are concerned is the frequency estimation and more precisely the speed and the convergence of the estimation algorithms.

In [2] an algorithm which combines the Gauss-Newton method with the singular value decomposition (SVD) was proposed for estimating the fundamental frequency. The problem is that this algorithm is slow when the number of samples is large, that is, when dealing with medium-high resolution ADCs. Also, the speed of the algorithm decreases with the number of harmonics taken into account. This is due to the fact that at each iteration the algorithm uses the SVD to calculate the pseudo-inverse of a matrix whose size is proportional to number of samples and tones. The advantage of the SVD is that no assumption is made on the rank of this matrix which may be singular. In [3] we have shown that the combination of the Newton method with the algorithm described in [2] results in a significant improvement of the algorithm speed, and we have determined the required initial frequency accuracy in order to ensure the global convergence.

The above problem arises because the effect of harmonic distortion on the frequency estimation is taken into account. As the distortion of ADCs is small and in general the dominant contribution is due to the noise, one can neglect the effect of the distortion in the frequency estimation. In this paper, it is shown that this reasonable approximation leads to fast algorithms without a significant accuracy loss. The convergence and implementation issues of the algorithms are treated, and this approach is compared to that taking into account harmonic distortion through simulations and experimental data.

2 PARAMETER ESTIMATION

The estimation of the ADC parameters requires the knowledge of the fundamental frequency and the amplitudes and phases of fundamental and harmonic tones. The VAPRO method reduces this problem to frequency estimation. Knowing the frequency, the amplitudes and phases are estimated by solving a linear system [3]. Thus, the main problem is the frequency estimation. If we ignore the effect of the harmonic distortion, the frequency is estimated by minimising the following nonlinear least squares (NLS) cost function

$$\begin{aligned} &F(w,\mathbf{x}) = (\mathbf{y}-\mathbf{s}(w,\mathbf{x}))^T(\mathbf{y}-\mathbf{s}(w,\mathbf{x})) \quad with \\ &\mathbf{y} = [y_0, y_1, \cdots, y_{M-1}]^T; \mathbf{s} = [s_0, s_1, \cdots, s_{M-1}]^T \\ &s_n = a\cos(nw) + b\sin(nw) + c; \mathbf{x} = [a, b, c]^T \end{aligned} \tag{1}$$

where $\mathbf{y}$ represents the data record; $\mathbf{s}$ is the sampled test sine wave; the superscript T stands for the transpose operator; M is the number of samples; $\mathbf{x}$ are the test sinewave linear parameters, and $w = 2pf_t / f_s$ is the normalised angular frequency, with f_t and f_s the test and sampling frequencies. For any fixed w, the least squares estimate of $\mathbf{x}$ is given by

$$\begin{aligned} &\hat{\mathbf{x}}(w) = \mathbf{H}^{-1}(w)\mathbf{v}(w) \quad \text{with } \mathbf{v}(w) = \sum_{n=0}^{M-1} y_n [\cos(nw), \sin(nw), 1]^T \\ &\mathbf{H}(w) = \begin{bmatrix} 0.5(1+S_{c,0}(2w)) & 0.5S_{s,0}(2w) & S_{c,0}(w) \\ * & 0.5(1-S_{c,0}(2w)) & S_{s,0}(w) \\ * & * & M \end{bmatrix} \end{aligned} \tag{2}$$

where

$$S_{c,m}(q) = \sum_{n=0}^{M-1} n^m \cos(nq), \quad S_{s,m}(q) = \sum_{n=0}^{M-1} n^m \sin(nq), \quad \text{with } m = 0, 1, 2 \tag{3}$$

The matrix $\mathbf{H}$ is symmetric positive definite and hence nonsingular (the elements represented by $*$ are obtained by symmetry). Substituting (2) in (1) yields

$$L(w) = F(w, \hat{\mathbf{x}}(w)) = \mathbf{y}^T\mathbf{y} - \mathbf{v}^T(w)\mathbf{H}^{-1}(w)\mathbf{v}(w) \tag{4}$$

The minimisation of this reduced cost function allows us to find an estimate of w. Useful relationships to avoid the calculation of the summations (3) are obtained from the fact that these functions represent the real and imaginary parts of

$$S_m(z) = \sum_{n=0}^{M-1} n^m z^n, \quad \text{with } m = 0, 1, 2; \; z(q) = \exp(jq) \tag{5}$$

which has the following analytical expressions

$$S_0(z) = \frac{z^M - 1}{z-1}, \; S_1(z) = \frac{Mz^M - zS_0(z)}{z-1}, \; S_2(z) = \frac{M^2 z^M - z(S_0(z) + 2S_1(z))}{z-1} \tag{6}$$

We have studied three methods for the minimisation of the cost function (4), namely Newton, Gauss-Newton and Golden section search with parabolic interpolation. For the Golden method we have used the Matlab software function *fminbnd*. The Golden method only requires the cost function and bounds for the global minimum, the choice of these bounds is discussed below.

2.1 The Newton and Gauss-Newton Methods

The application of the Gauss-Newton and Newton methods to the minimisation of $L(w)$ leads to the following iterative scheme [4]

$$w_{k+1} = w_k - \Delta w_k \quad with \quad \Delta w_k = \begin{cases} \dfrac{\mathbf{j}^T(w_k)\mathbf{r}(w_k)}{\mathbf{j}^T(w_k)\mathbf{j}(w_k)} & Gauss-Newton\ Method \\[2ex] \dfrac{L'(w_k)}{L''(w_k)} & Newton\ Method \end{cases} \tag{7}$$

where $\mathbf{r}(w)$ is the residual $\left(\mathbf{y} - \mathbf{E}(w)\mathbf{H}^{-1}(w)\mathbf{v}(w)\right)$, $\mathbf{E}(w)$ being the matrix $[\cos(w\mathbf{t}) \quad \sin(w\mathbf{t}) \quad \mathbf{1}]$ with $\mathbf{t} = [0 \quad 1 \quad \cdots \quad M-1]^T$ and $\mathbf{1} = [1 \quad 1 \quad \cdots \quad 1]^T$; $\mathbf{j}(w)$ is equal to $d\mathbf{r}(w)/dw$; $L'(w)$ and $L''(w)$ are the first and second derivatives of the cost function. The derivatives of $\mathbf{v}$ and $\mathbf{E}$ can be easily calculated, those of $\mathbf{H}^{-1}$ are given by

$$\left(\mathbf{H}^{-1}\right)' = -\mathbf{H}^{-1}\mathbf{H}'\mathbf{H}^{-1}, \quad \left(\mathbf{H}^{-1}\right)'' = -\left[\left(\mathbf{H}^{-1}\right)'\mathbf{H}'\mathbf{H}^{-1} + \mathbf{H}^{-1}\mathbf{H}''\mathbf{H}^{-1} + \mathbf{H}^{-1}\mathbf{H}'\left(\mathbf{H}^{-1}\right)'\right] \tag{8}$$

The derivatives of $\mathbf{H}$ can be calculated by using the following simple derivation rules

$$\frac{dS_{c,m}(lw)}{dw} = -lS_{s,m+1}(lw), \frac{dS_{s,m}(lw)}{dw} = lS_{c,m+1}(lw) \quad \text{with } m = 0, 1 \tag{9}$$

The process (7) is iterated until $\left|\Delta w_k / w_{k+1}\right|$ is less than the typical tolerance value $[machine\ precision]^{\frac{2}{3}}$.

2.2 Initial guess and global convergence

Figure 1 shows the variation of the cost function around the global minimum w^*. As it can be seen, the cost function has local minima where the iterative methods will converge in the case of bad frequency starting value w_0. Note that the width of each lobe is equal to $4p/M$ and hence it only depends upon the number of samples. As the signal to noise ratio is high, the global convergence depends on the accuracy of w_0. By following the method described in [3], we have found that the Gauss-Newton and Newton methods require an initial frequency accuracy of about $2p/M$ and $2p \times 0.3/M$. Thus, the Gauss-Newton method tolerates more initial frequency uncertainty that the Newton method. However, it is well known that the convergence of the Newton method is faster than the Gauss-Newton one [4]. As a result, we have now a criterion for choosing the suitable initial frequency estimation method that ensures the global convergence.

Using the tolerated initial frequency uncertainties thus obtained, both Newton and Gauss-Newton methods can be modified as explained in [3] in order to obtain algorithms which are robust with respect to the initial guess of the frequency. This modification requires the knowledge of the worst case value of the difference between the ADC number of bits and its effective number of bits. In [3] we have proposed to set this value equal to 4 bits. This choice is based on the experimental data given in [5] where it is shown that the variation of this value as a function of the sample rate is always less than 4 bits. The advantage of this modification is that the estimation of w_0 using signal processing techniques is no longer necessary, since it can be simply generated from the test generator frequencies. In any case, the frequency estimation techniques based on the interpolated fast Fourier transform largely satisfy the initial frequency accuracy required by the Newton and Gauss-Newton methods.

The Golden section search algorithm proposed in the Matlab software requires lower and upper bounds for the global minimum w^* instead of w_0. As the global minimum width equals $4p/M$, we propose to set these bounds equal to $w_0 \pm 2p \times 0.5/M$.

3 SIMULATION RESULTS

The aim of this section is to examine the speed and statistical performances of the frequency estimation algorithms described above and to compare them with the algorithm taking the harmonic distortion effect into account. For this purpose, the ADC output data were described by a pure sine wave (test signal) plus harmonics and additive gaussian noise. The contribution of the distortion and noise are controlled by the THD and SNR parameters. The harmonic distortion power is concentrated in the second harmonic. The frequency variance was calculated from 100 realisations (R = 100) from the following equation

$$\mathrm{var}(\hat{w}) = \frac{1}{R}\sum_{k=1}^{R}\left(\hat{w}_k - w^*\right)^2 \tag{10}$$

where $\hat{w}_k$ is the frequency estimate corresponding to the *k-th* realisation, w^* being the true normalised angular frequency. This definition of the frequency variance takes into account the bias. Figure 2 shows the variation of the frequency variance with the SNR for the algorithms described above compared to the CRB. Throughout this paper CRB correspond to the results obtained by the sine-wave fit algorithm taking into account the harmonic distortion effect, since this algorithm reaches the CRB [2].

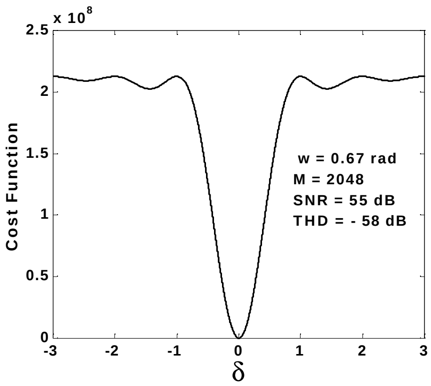

Figure 1. Variation of the cost function around the global minimum. $d = M\left(w - w^*\right)/2p$.

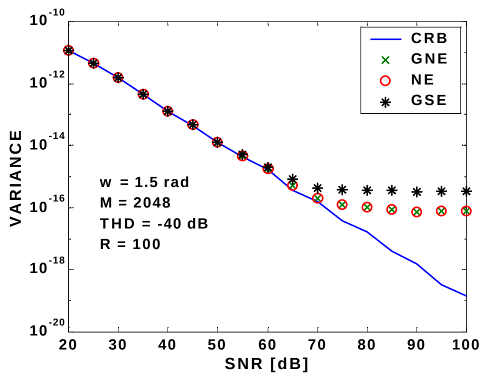

Figure 2. Log plot of the frequency variance variation versus the SNR.

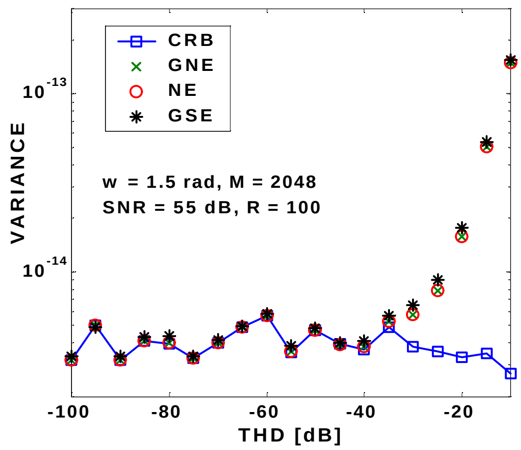

Figure 3. Log plot of the frequency variance variation versus the total harmonic distortion.

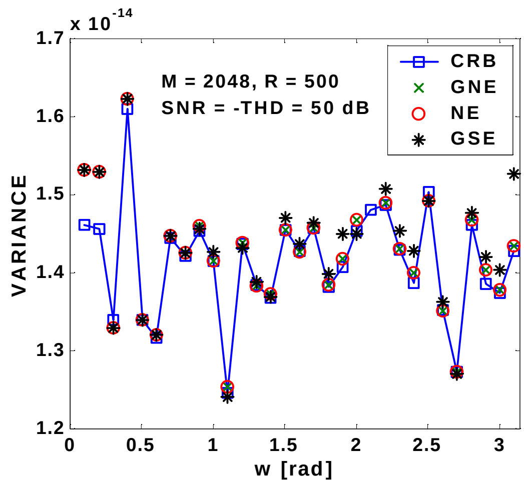

Figure 4. Variation of the frequency variance versus the frequency.

Figures 3 and 4 show the variation of the frequency variance with respect to SNR and THD. As it can be seen, the bias of the algorithms described above is negligible in the region where the noise contribution is dominant compared to that of distortion. The Gauss-Newton (GNE) and Newton (NE) give the same results, whereas the Golden search algorithm (GSE) has slightly lower accuracy. Figure 3 shows the variation of the variance with the frequency for the situation where the noise and distortion contributions are equal. One can note a small bias for small frequency values, elsewhere the bias is negligible. Also, one can see that the Golden algorithm is slightly less accurate than the other methods especially near the Nyquist limit.

We have also examined the speed of the different algorithms as a function of the number of samples. The obtained results are reported in Table 1. These results correspond to the average execution time of 50 realisations, the algorithms were developed using Matlab 5.3 on 32 MO RAM, 400 MHz PC. One can see that a significant time is saved by neglecting the effect of harmonic distortion, especially for large number of samples. On the other hand, the algorithms described above are fast notably the Newton method.

Table 1. The speed of the algorithms. Simulation parameters: $w = 0.7$ rad, $M(w_0 - w)/2p = 0.01$, SNR = 55 dB, THD = -58 dB, number of realisations = 50, number of harmonics = 4.

	Execution Time [seconds]			
Algorithm/Number of samples	*Newton*	*Gauss-Newton*	*Golden*	*Gauss-Newton with harmonic distortion*
2048	0.082	0.093	0.14	0.52
4096	0.12	0.21	0.18	1.17
8192	0.24	0.44	0.27	2.63
16384	0.54	1.08	0.6	6.5
32768	1.77	3.2	2.22	17.3

4 EXPERIMENTAL RESULTS

We have applied the algorithms described in this paper to various ADCs, and the obtained performance parameters are identical to those given by the algorithm taking into account the effect of harmonic distortion. Here we give the results obtained with the AD9432 device (12 bits, 105 MHz). The data spectrum and the test parameters are given in figure 5. The time-domain algorithms gave identical results which are in good agreement (see Table 2) with those obtained by the spectral-domain analysis described in [6].

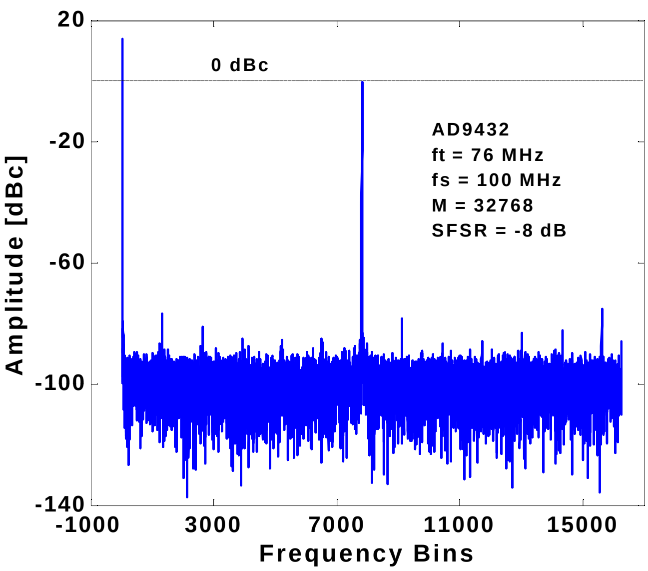

Figure 5. The spectrum of the AD9432 data weighted by the 7 term Blackman-Harris window in order to reduce the leakage effect.

Table 2. AD9432 performance parameters obtained by time-domain and spectral-domain analyses.

	SINAD [dB]	**SNR [dB]**	**THD [dB]**
Time-Domain Analysis	58.67	58.98	-70.3
Spectral-Domain Analysis	58.6	58.9	-70.2

5 CONCLUSION

Time-domain testing of A/D converters in single tone mode is reduced by the variable projection method to frequency estimation by minimising a one-dimensional function. In this paper, it has been shown that the bias due to ignoring the effect of harmonic distortion on the frequency estimation is negligible, and this approximation leads to fast frequency estimation algorithms. The performances of three methods have been examined through numerical simulations and experimental data, namely Newton, Gauss-Newton and Golden section search with parabolic interpolation. It has been found that the Gauss-Newton method tolerates more initial frequency uncertainty than the Newton method, but it is slightly slower than the other methods. The Golden search method is slightly less accurate than the other methods which have the same accuracy. Implementation issues have been also treated.

These algorithms, however, are to be only used when the thermal and quantisation contributions are dominant in the additive noise. If the aperture jitter is dominant a weighted nonlinear least squares procedure should be used.

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