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REPRODUCIBILITY OF THE JITTER MEASUREMENT

J. Halámek, M. Kasal, M. Villa*, P. Cofrancesco*
ISI Brno AS CR, Královopolská 147, 612 64 Brno, Czech Republic
*Unità INFM di Pavia,Via Bassi 6, 27100 Pavia, Italy

Abstract: The validity of the ADC noise model and jitter error are discussed and tested by measurement.
Keywords: ADC measurement, ADC noise, jitter.

1 INTRODUCTION

Jitter or aperture uncertainty is the standard deviation of the sample instant in time. The terms phase noise, timing jitter or timing phase noise are used interchangeably [1]. Aperture uncertainty arises from the phase noise of the sampling frequency f_s and from the ADC itself. Generation and distribution of f_s, along with its spurious signals, usually contribute significantly to the resulting aperture uncertainty. The corresponding ADC error signal depends upon the slew rate of the input signal and reduces the dynamic range and the frequency bandwidth which may be achieved. The standard measurement of the aperture uncertainty requires evaluating two levels of ADC output noise – one with the input signal and one without; the aperture uncertainty is obtained from the slew rate of input signal and the increase in output noise associated with it.

When the f_s itself is used as the input signal, the phase noise of f_s is not contributing and the ADC aperture uncertainty is determined [1]. The limitation of this method can be: a) Input signal slew rate – the influence of the ADC frequency response on it and how to measure it. b) Input signal purity, i.e.; its noise floor. c) The influence of ADC differential nonlinearity (DNL) – different noise contribution according to input signal amplitude. d) The influence of other ADC dynamic errors, such as limited slew rate of sample-and-hold circuit. The limitations b), c) are eliminated when measurements with different input frequencies are performed [2].

Our aim is to test the reproducibility of the aperture uncertainty measurement, to verify whether the error signal has a white noise spectrum or not, and to check the validity of the theoretical model in the area where the ADC frequency response cannot be neglected. We will consider the acquisition system as a whole, and will not discuss wheter te aperture uncertainty is given by generation and distribution of f_s , or by the ADC itself.

2 APERTURE UNCERTAINTY ERROR

Consider a sampling system which samples at time $t = n\ T_s + \text{Ä}t_n$, where $T_s = 1/f_s$ is the sampling period and $\text{Ä}t_n$ is a random variable. We will suppose, that $\text{Ä}t_n$ follows a Gaussian distribution with mean level 0 and standard deviation $\text{Ä}t$, which is aperture uncertainty. The input signal is s(t) and corresponding error signal is given by

$$err(n\ T_s)= s(n\ T_s + \text{Ä}t_n) – s(n\ T_s). \qquad (1)$$

When the input signal is a sine wave, $s(t) = A \cos(2\pi f_{in} t)$, where A is input amplitude and f_{in} is input frequency, the power of the error is given by

$$P_{err}= 1/N\ \Sigma(A \cos(2\pi f_{in} (n\ T_s + \text{Ä}t_n) - A \cos(2\pi f_{in}\ n\ T_s))^2. \qquad (2)$$

When f_{in} , $\text{Ä}t_n$, and f_s are not correlated, and the number of samples is large ($N \rightarrow\infty$), we can integrate over a full period of the input sine wave, and the error power is

$$P_{err}= 2 (A \sin(\pi f_{in} \text{Ä}t))^2 . \qquad (3)$$

In the region where

$$\pi f_{in} \text{Ä}t << 1, \text{ or } \text{Ä}t << 1/ f_{in} , \qquad (4)$$

we can replace $\sin(\pi f_{in} \text{Ä}t)$ with $\pi f_{in} \text{Ä}t$ and the well known result is:

$$P_{err} = 2 (A\ \pi f_{in} \text{Ä}t_n)^2. \qquad (5)$$

The signal to noise ratio, when only aperture uncertainty is present, is given by

$$SNR= -20 \log(2 \pi f_{in} \text{Ät}). \qquad (6)$$

This is theoretical limit, since other sources of error signals always exist. If we indicate with P_0 the power of this extra noise, the SNR is given by

$$SNR = 10 \log(A^2/2 / (2 (A \pi f_{in} \text{Ät})^2+ P_0)). \qquad (7)$$

2.1 Validity of the aperture uncertainty error model

First we have to consider condition (Eq. 4); with an aperture uncertainty to 50 ps and f_{in} = 500 MHz, the corresponding level of πf_{in} Ät is 0.078 and the approximation of replacing sin(x) with by x changes the result by ~0.1%. Therefore, Eq. 4 is satisfied in real conditions. We have also assumed that f_{in}, f_s and Ät_n are not correlated. However, a correlation between f_s and Ät_n exists when some coupling between analog and digital part is present. The study of this case involves an analysis of the existing coupling, which will not be discussed here. With a ratio f_s / f_{in} higher than four and coherent sampling, the power is approximately given by Eq. (5), but amplitude modulation of the error signal occurs because of coherency. When f_{in} / f_s = 0.5 k, where k is prime number, the error power depends upon the phase difference between f_{in} and f_s and is determined by the slew rate of the signal during sampling.

In the previous analysis we have assumed an ideal ADC while a real ADC has limited frequency bandwidth, and limited slew rate. The frequency bandwidth is mostly limited by the input amplifier of the sample and hold circuit, which is characterized by an analog frequency response – K(f). The corresponding SNR is

$$SNR = 10 \log((A K(f_{in}))^2/2 / (2 (A K(f_{in}) \pi f_{in} \text{Ät})^2+ P_0)). \qquad (8)$$

The slew rate can be limited in the input amplifier and/or in the sample and hold circuit. Its influence upon the dynamic range depends upon its origin and includes harmonic and spurious contributions. In any case, the effects of limited slew rate and aperture uncertainty may be considered separately.

It would be nice to be able to deal with an input signal with negligible noise, but this is difficult to achieve in inexpensive systems over a wide band of f_{in}. Therefore we will assume the presence of two basic noise sources, one constant - P_0 , given by system, and the second given by input signal – $P_{ns}(A)$, which depends upon its amplitude A. The resulting SNR is

$$SNR = 10 \log((A K(f_{in}))^2/2 / (2 (A K(f_{in}) \pi f_{in} \text{Ät})^2+ P_{ns}(A) + P_0)). \qquad (9)$$

3 MEASUREMENT

As the source of the input signal we use a 500 MHz synthesiser (by PTS), followed by a good attenuator (with 1 dB steps). From the attenuator the signal goes directly to the ADC's input. The ADC was AD9042, a 40 MHz, 12 bits converter (Analog Devices). The data from ADC go to a home made digital receiver based upon the Harris HSP50016 Digital Down Converter. By changing mixing frequency and spectral width of the digital receiver, we may discriminate between the main signal, harmonics, and noise in different pass bands. The output dynamic range SNR_{out} is increased owing to digital filtering

$$SNR_{out} = SNR_{in} + 10 \log (f_s/2SW_{dec}), \qquad (10)$$

where SNR_{in} is input SNR (in the pass-band $f_s/2$) and SW_{dec} is output pass-band. The output data are 16 bits. The output signal amplitude and output noise rms were measured as a function of the input frequency and amplitude. The sampling frequency was 5 MHz. The input frequency was in the range from 1.24 to 163.74 MHz, with the 2.5 MHz step. So the ratio of aliased frequency f_a and sampling frequency f_s was nearly constant, and signal distortions given by S/H circuit did not occur. We performed measurements at five levels of the input amplitude, which was changed in the 3 dB steps; the maximal amplitude was only -0.57 dB below the full scale (FS) signal of ADC. The full scale signal was measured according to the ADC's digital output at f_{in}= 2 MHz. With every setting of f_{in} and A the signal amplitude and noise rms in three different pass bands of the digital receiver were measured. The measured signal amplitudes give us the frequency response. The pass bands where the noise was measured were chosen so that no harmonics of the input signal were present in these bands.

Two pass bands showed a low noise, only the basic system noise was present, and the third showed an the increased level of noise given by EMI to test this influence.

The normalized frequency responses are presented in Fig. 1. Except for the maximal signal. this response does not depend upon the signal amplitude. (The frequency response at the amplitude FS-0.57 dB is slightly different from others, but we will not deal with the problem of the FS definition and ADC linearity). The measured amplitudes give us the real attenuation of the signal too. The adjusted attenuation was [8, 11, 14, 17, 20] dB, the real attenuation, computed from amplitudes, was [8, 10.96, 13.83, 16.78, 19.68] dB. This real attenuation was used when the dependence of the output noise upon the signal amplitude was analyzed.

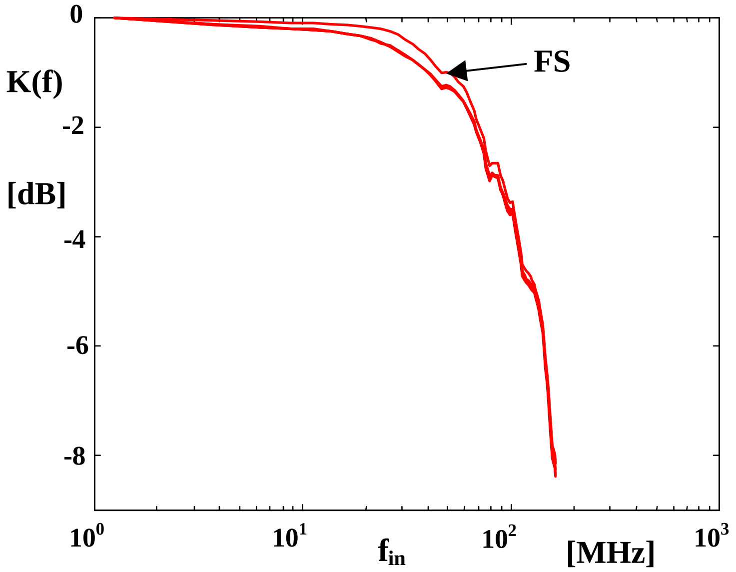


Fig.1. The normalized frequency responses for all input amplitudes.

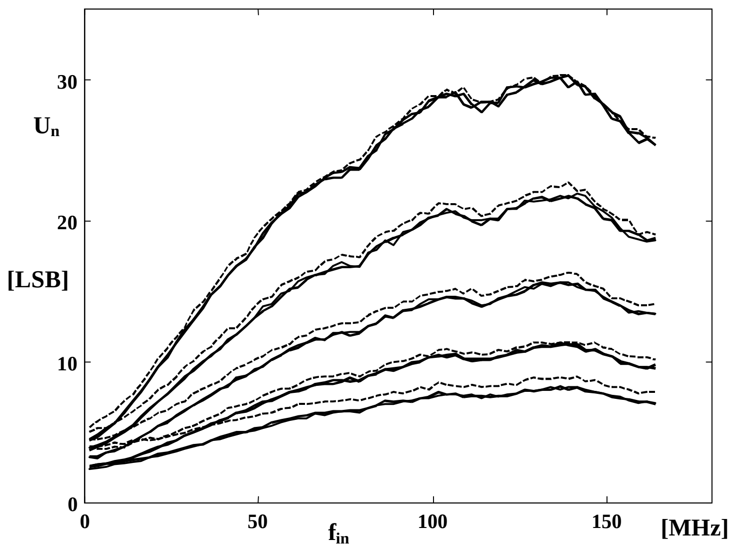


Fig.2. The output noise rms of the digital receiver for three pass bands and five input amplitudes. Dotted lines represent the passband with increased noise level.

Fig.2 shows the noise rms U_{nrms} measured at the output of the digital receiver for three pass bands and five input amplitudes. The dotted lines represent the pass band with the increased noise level. The noise contribution of the signal source can be seen at an attenuation lower than 17 dB and at low f_{in}. According to Eq. 9, the corresponding noise model should be

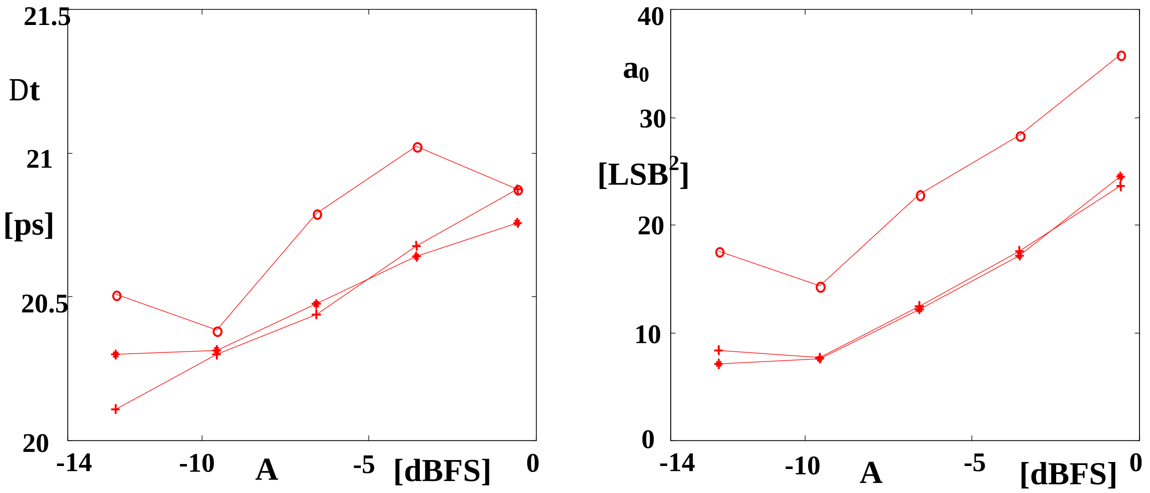
$$U_{nrms}(f_{in}) = SQRT(a_0 + a_2 (K(f_{in}) f_{in})^2), \quad (11)$$

where a_2 determines the aperture uncertainty

$$Ät = SQRT(a_2/2) / (A \pi f_{in}), \qquad (12)$$

and a_0 the basic acquisition system noise and noise contribution from the input signal source.

Fig.3a) shows the computed Ät as a function of A. Three different marks correspond to different pass bands, mark 'o' corresponds to the pass band with a higher basic noise. In our case, the main source of jitter is f_s generation and distribution. As a sampling frequency we use the logic signal that goes over more IC. Fig.3b) shows the basic noise power a_0 as a function of A. The dependence upon A is given by noise of the input signal, the dependence upon the pass band is due to EMI influence.



a) b)

Fig.3. The computed jitter a) and basic noise power b) as a function of input amplitude A. Different marks correspond to different pass bands.

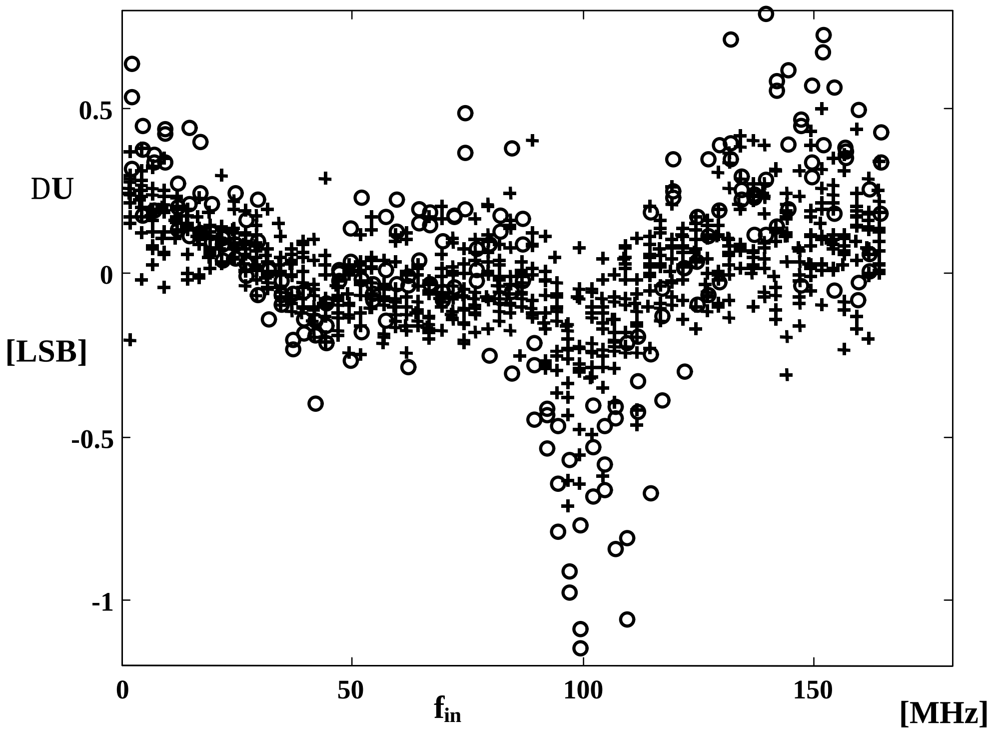


Fig.4. The differences between the noise model and measurement. The marks 'o' correspond to the maximal amplitude of the input signal.

Fig.4 shows the differences between the model and measurement. The marks 'o' correspond to the maximal amplitude of the input signal, marks '+' to other amplitudes. These differences demonstrate the validity of the theoretical model (11). When the model is valid, the differences should be random. While some dependence is observed at the maximal input amplitude, no other effect has been noted.

The noise contribution of the signal source P_{ns} and the basic ADC noise P_0 can be computed from a_0 as a function of input attenuation G,

$$a_0(G) = P_0 + G\,P_{ns}\,. \qquad (13)$$

The signal-to-noise ratio of the signal source (SNR_s) is given by P_{ns}. The best unit of SNR_s is dBFS/√Hz since it eliminates the influence of the frequency bandwidth is directly comparable with the ADC dynamic range. When computing, the aliasing of noise and ADC frequency response have to be assumed. The computed SNR_s were [137.3, 136.9, 136.3] dBFS/√Hz. We measured this parameter with a spectrum analyzer (Rhode&Schwarz) obtaining 136 dBFS/√Hz.

Another approach consists in analyzing first the dependence of the measured noise upon the signal amplitude and then studying how the constants change with frequency. However, the algorithm is more sensitive to irregularity at maximal A, as it can be demonstrated by comparing data sets which include the maximal amplitude with those which do not include it. While the computed Δt does not depend upon the data set, and agrees well with our previous result, the evaluation of the basic system noise P_0 is increased when data with the maximal A are included.

4 CONCLUSION

The reproducibility of jitter measurement is very good and the ADC noise model is accurate except for some irregularities at the maximal amplitude. However, these irregularities are due to ADC properties which are observed also in the ADC frequency response. We believe that more attention should be paid to the definition of the ADC full scale amplitude and the interval of signal amplitude, where the nonlinearity of ADC transfer curve is negligible.

The noise sources we have considered (jitter, system basic noise, input signal noise, EMI noise) are essentially independent, with little correlation. Only the basic noise in the pass band is influenced by EMI.

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AUTHORS: Josef HALÁMEK, Miroslav KASAL, Department NMR Spectroscopy, Institute of Scientific Instruments, AS CR, Královopolská 147, 612 64 Brno, Czech Republic, Phone (420-5) 41514311,
Fax (420-5) 41514401, E-mail: josef@isibrno.cz
Marco VILLA, Pacifico COFRANCESCO, Unità INFM di Pavia,Via Bassi 6, 27 100 Pavia, Italy, Phone (39-0382) 507220, Fax (39-0382) 507221, E-mail: mvilla@fisav.unipv.it