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MEASUREMENT OF AC VOLTAGES WITH DIGITAL VOLTMETERS

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Abstract: A principle of using high-resolution digital voltmeters (DVs) Hewlett-Packard 3458A[1)] for the measurement of AC voltages in DCV range is described. The method of approximate synchronous sampling is used, having only one sample per cycle of a working frequency close to 16 Hz, when the mean value of one-third cycle is measured. The beginning of integration is shifted for the same interval for every subsequent period of the measured voltage. From the samples, the RMS value of the "derived" sine is determined using subsequent numeric synchronisation. The phase angle of the first sample of the "derived" sine is calculated as regression line coefficient. To ensure accurate phase difference measurement between two voltages, a synchronised triggering is used. A method for the determination of the difference between the sampling frequency of two DVs and the main frequency is presented. For the voltage ratio and phase difference measurements, the uncertainties of 0,1 ppm and 0,1 mrad respectively were achieved.

Keywords: Digital voltmeter, RMS value, phase angle

1 INTRODUCTION

High-resolution digital voltmeters, like HP 3458A, have wide application in the field of measurements at a low level of uncertainty, especially when connected by the IEEE-488 bus with a computer in which the measured data are stored, processed and displayed. The best performance of the instrument is DC voltage (DCV) measurement by an integrating procedure. The integration time can be chosen from 500 ns to 1 s in steps of 100 ns and the interval between two samples to 6000 s in the same steps. This enables the measurement of AC voltage at a low frequency of 15,873 Hz in the DCV range by integration during the one-third cycle of the main signal [1].

AC voltage at the mentioned frequency is used in the procedure for calibrating the resistance standard of 100 MΩ with the capacitance standard of 100 pF by means of two HP 3458A at the Primary Electromagnetic Laboratory ETF in Croatia [2, 3]. For this purpose, the Fluke 5200A AC Calibrator[1)] is used as a very stable source of voltage with low harmonic components.

The method of RMS value calculation from the sampled data, in the case when the sampling rate and frequency of AC source are slightly different, is described. Since this yields a non-integer number of samples per period of the "derived" sine, an approximation with a 5^{th} order polynomial is used around the samples close to zero value. From the sampled data, the phase angle of the first sample and the phase angle between two adjacent samples are calculated as regression line coefficient. To ensure accurate phase difference measurement a synchronised triggering is used, where one voltmeter serves as a "master" and the other one as a "slave" voltmeter. Since the absolute frequencies of the main clocks in two voltmeters are slightly different, a method for the determination of their difference to the main frequency is presented. From these measurements follows the correction for the integration time and the interval between two samples for two DVs. This approach gives a possibility for the determination of the absolute RMS values and phase angles of the main signals, which yields to the measurement of other electric quantities at a lower frequency range.

2 DETERMINATION OF RMS VALUE AND PHASE ANGLE

Modern digital multimeters have the best performance when they measure DC voltage by an integrating procedure. Since the integration time and the interval between the samples can be chosen, this enables the measurement of AC voltage at a low frequency when the voltmeter is set to DCV range. The method is based on the integration during the one-third cycle of the main signal (T_m), while

[1]Brand names are used for the purpose of identification. Such use does not imply endorsement by authors nor assume that the equipment is the best available.

the beginning of each next measurement is shifted by $t_0=T/p$ (p is an integer number), and is shown in Figure 1. The samples are denoted as u_0, u_1 and u_2, they represent the mean value of the measured voltage during the integration, they also follow the sine function and form the so-called “derived” sine. In the first step the determination of RMS value of the “derived” sine will be described according to the method explained in [2], since it is important for the phase angle determination that depends on the effective value. Thus, in the next step, the attention will be concentrated on the phase measurement.

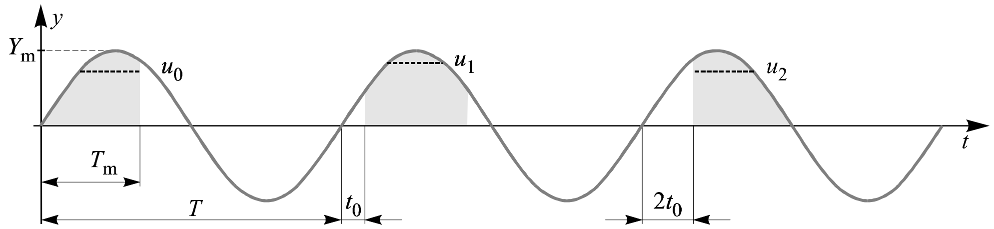

Figure 1. One data sample per cycle of the measured voltage

2.1 Calculation of RMS value of “derived” sine

For the measured voltage $y(t) = Y_m \sin(wt + j_1)$ with period T, the measured value u_i of the i-th sample according to Figure 1 will be:

$$u_i = U_m \sin\left[\frac{p}{T}(2t_0 i + T_m) + j_1\right], \tag{1}$$

where T_m is the integration time. Two terms determine the amplitude of the “derived” sine, U_m: the amplitude of the measured voltage and the ratio between the integration time T_m and period T, according to the following equation:

$$U_m = kY_m = \frac{\sin(pT_m/T)}{pT_m/T} Y_m . \tag{2}$$

Except the main harmonic, the measured voltage is contamined by the influence of DC component and higher harmonics as well, which can be described with the following function:

$$y(t)=Y_0 + Y_{m,1}\sin(wt+j_1) + Y_{m,2}\sin(2wt+j_2) + ... + Y_{m,n}\sin(nwt+j_n). \tag{3}$$

If we assume that the DC component is negligible or can be calculated and eliminated, the share of the n-th harmonic in the measured value u_i of the i-th sample is equal to:

$$u_{n,i} = \frac{Y_{m,n}\sin(npT_m/T)}{npT_m/T} \sin\left[\frac{np}{T}(2t_0 i + T_m) + j_n\right]. \tag{4}$$

From the previous equation follows that by adjusting the ratio T/T_m to be equal to n, the n-th harmonic will be eliminated, as well as the $2n$-th, $3n$-th, $4n$-th, etc. harmonics. For the chosen ratio $T_m/T=1/3$ the harmonics dividable by 3 do not affect the RMS value of the “derived” sine, U, which is calculated by:

$$U = U_1 \sqrt{1 + \left(\frac{Y_{m,2}}{2Y_{m,1}}\right)^2 + \left(\frac{Y_{m,4}}{4Y_{m,1}}\right)^2 + \left(\frac{Y_{m,5}}{5Y_{m,1}}\right)^2 + \left(\frac{Y_{m,7}}{7Y_{m,1}}\right)^2 + \left(\frac{Y_{m,8}}{8Y_{m,1}}\right)^2 + ...} \tag{5}$$

The influence of other harmonics is entirely insignificant if they are lower than 0,01 % of the basic harmonic, which is not a strict requirement. Thus, the effective value of the “derived” sine can be determined as the effective value of the main harmonic, described by equation (1).

Without synchronisation between the DVs and the AC source, there is a small difference between the sampling rate and the main frequency, which yields a non-integer number of samples per period of the “derived” sine. Thus, the RMS value of the samples is calculated by the expression:

$$U^2 = \frac{1}{k_{eff}} \sum_{i=0}^{k_{eff}-1} u_i^2, \qquad k_{eff} = k + \Delta k . \tag{6}$$

Here k indicates the integer number of the “derived” sine partition, while Δk is the correction for numeric synchronisation. This correction is calculated from the angles at the beginning and at the end of the sample array that is taken into account. It is done in following manner: as the first step it is necessary to find the first sample higher than zero (let us assumed its index is k1). In the second step, index k2 is attached to the last sample in the calculation, for which the condition $u_{k2}>u_{k1}$ is fulfilled. Since the samples of the "derived" sine follow the function $u_i = U_m \sin j_i$, it is very simple to calculate the associated angles j_{k1}, j_{k2} and j_{k2-1}. Thus, the correction is equal to:

$$\Delta k = \frac{j_{k1} - j_{k2-1}}{j_{k2} - j_{k2-1}} . \tag{7}$$

At this step, the amplitude U_m is determined from the 2nd order polynomial, calculated for three samples around the sample with the highest value.

The sum of squares in (6) needs to be calculated for $u_{k-1+\Delta k}$, which means for a non-integer number. For this purpose, an approximation with the 5th order polynomial is used, and the following sums are calculated:

$$s_0 = \sum_{i=k1}^{k2-4} u_i^2 , \quad s_1 = \sum_{i=k1}^{k2-3} u_i^2 , \quad s_2 = \sum_{i=k1}^{k2-2} u_i^2 , \quad s_3 = \sum_{i=k1}^{k2-1} u_i^2 , \quad s_4 = \sum_{i=k1}^{k2} u_i^2 \quad \text{and} \quad s_5 = \sum_{i=k1}^{k2+1} u_i^2 . \tag{8}$$

Since k-1=k2-k1-2, the needed sum is determined as follows,

$$\sum_{i=k1}^{k2-k1-2+\Delta k} u_i^2 = a + b\Delta k + c\Delta k^2 + d\Delta k^3 + e\Delta k^4 + f\Delta k^5 , \tag{9}$$

where the coefficients a to f are calculated from the sums in equation (8) according to the relations in [4]. Finally, the RMS value of the "derived" sine is determined from the following equation:

$$U^2 = \frac{1}{k2 - k1 - 1 + \Delta k} \sum_{i=k1}^{k2-k1-2+\Delta k} u_i^2 . \tag{10}$$

The numerical analysis performed on mathematical models shows that the limit of this approach is the chosen resolution in calculation, which has no influence at the level of interest (i.e. 1×10^{-7}).

2.2 Calculation of phase angle

The determination of phase angles follows from rearranging equation (1), since the phase angle of the first sample y_0 and the phase angle between two adjacent samples Δy can be denoted as:

$$y_0 = \pi T_m / T + j_1 , \quad \text{and} \quad \Delta y = 2\pi t_0 / T . \tag{11}$$

Obviously, the phase angle is a linear function of the sample index, which can be expressed as:

$$y_0 + \Delta y \, i = \arcsin(u_i / U_m) , \tag{12}$$

where u_i is i-th sample and U_m is calculated from the RMS value of the "derived" sine. Both unknowns, y_0 and Δy, can be determined according to the least squares theory as regression line coefficients. Because the errors of the measured voltages have much more impact on the peak of the "derived" sine than on its passing through zero, the weights which are equal to the square of the reciprocal value of the function derivation $\arcsin(u_i/U_m)$ are added, i.e. $p_i = 1-(u_i/U_m)^2$ [1]. If we denote that

$$\begin{aligned} \Delta y &= a \\ y_0 &= b , \\ \arcsin(u_i / U_m) &= f \end{aligned} \tag{13}$$

then the system of equations that have to be solved is (with the Gauss's signs for sums):

$$\begin{aligned} [pi]a + [p]b &= [pf] \\ [pi^2]a + [pi]b &= [pif] \end{aligned} . \tag{14}$$

A certain number of starting samples is excluded from the mathematical processing, and y_0 is determined from the phase angle of the first sample in calculation, decreased by $m\times\Delta y$ (m is the number of "discarded" samples).

2.3 Standard measurement procedure

The determination of RMS value and phase angle, described in two previous sections, is not limited to only one period of the "derived" sine. Even more, it is desirable and reasonable to obtain many periods (which means more samples) to produce a lower standard deviation and to allow that some periods can be omitted (e.g. the first and the last two periods) in the calculation process. In this way, some errors that can arise due to the starting of measurement sequence are simply avoided.

Having this approach in mind, the measurements were done with nominal values T_m=21 ms and t_0=0,9 ms (in the next section it will be shown that in the real situation these values should be slightly modified), which gave the interval between two samples equal to t_0+T=63,9 ms and partition per period p=70. For one measurement n=2381 samples were taken during the time of approx. 2,5 min (all together 34 periods of the "derived" sine) and that was repeated ten times in the usual procedure. The domain of the arcsine function is limited to the interval [-1, 1], while the sum in the system of equation (14) needed to be calculated outside those limits. Using a special routine in the phase calculation, that problem was solved. The DVs are connected and controlled by a computer, and so the measurement procedure is completely automated.

To ensure the simultaneous start of measurement, which is necessary to determine the phase difference between the first samples (i.e. the phase difference between two measured voltages), the DVs were initially set and connected for synchronised triggering [5]. In this method the trigger impulse activates the "master" voltmeter, which generates the trigger impulse for the "slave" voltmeter at the beginning of its integration. Thus, the unwanted delay is inserted in the triggering chain. Fortunately, it depends on the behaviour of digital circuits and for the use of this method it is not important to measure directly this delay, because it is incorporated in the determination of some other corrective terms, as explained in [6]. The only factor that affects the measurements is its time drift, but experimental results show that this delay is very stable for this purpose.

3 DETERMINATION OF VOLTMETER PARAMETERS

In section 2.1 the attention was paid to the influence of a small *difference between the sampling rate and the main frequency* on the calculation of the RMS value of the "derived" sine. It was shown that this situation, which in real measurement conditions cannot be avoided, leads to a non-integer number of samples per period of the "derived" sine. The main reasons that give rise to that difference are as follows: finite resolution of AC source frequency tuning, finite resolution of the timing parameters of voltmeters, and inaccuracy of the main oscillators in DVs. All parameters of DV, which determine the time interval, directly depend on the stability and accuracy of its time base (10 MHz). To determine the necessary correction for the integration time (aperture) and the interval between the samples (timer), a measurement method, according to Figure 2, has been analysed.

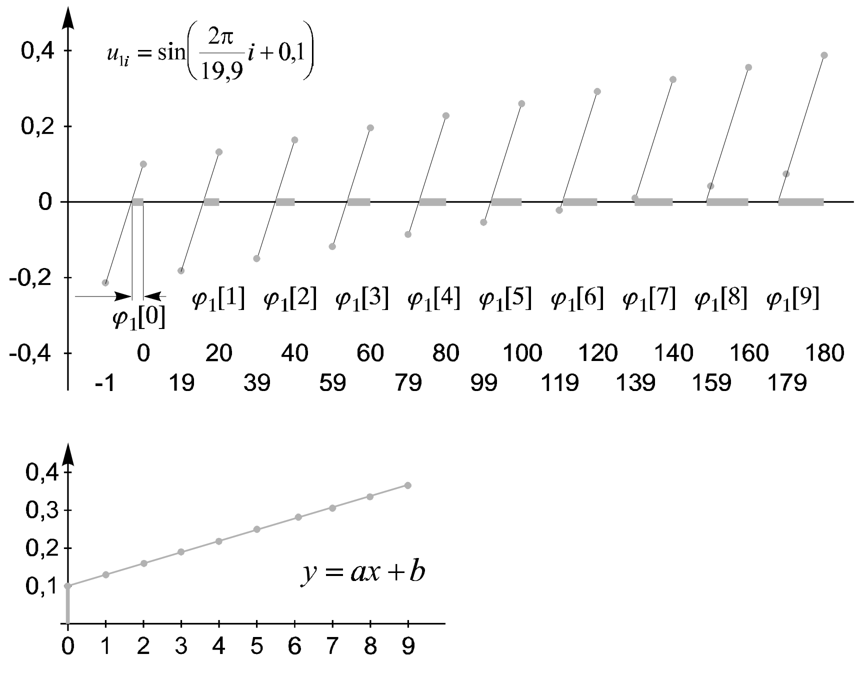

Figure 2. The influence of the difference between the sampling rate and the main frequency

In Figure 2 an example of the "derived" sine with a non-integer number of samples is presented. Here the partition per period is equal to p=19,9 instead of integer number 20 (this is chosen only for clarity), while in the standard procedure (section 2.3) nominal value is p=70. In the upper part only two subsequent samples around the zero value at the beginning of ten periods of "derived" sine are drawn, i.e. with indexes -1 and 0, after that with 19 and 20, etc. If the phase angle of the first sample greater than zero is calculated according to the relation j_1=arcsin(u_i), the values $j_1[0]$, $j_1[1]$), ..., $j_1[9]$ will be obtained. These values are drawn in the lower part in the same figure, and it is obvious that the phase angle linearly grows with the number of period. Thus, the slope coefficient of the regression line, calculated for these values, shows the difference between the sampling rate and the main frequency.

For measurements with two DVs, the regression lines for both of them have to be determined. For this purpose the standard measurement procedure is used (section 2.3) and the samples for all together 34 periods of the "derived" sine have been taken. In the initial step of this measurement, the frequency of AC source is measured by means of frequency counter and is denoted as f_{init}. In the second step, the timer (t_{mr}) and aperture (a_{pr}) parameters for two DVs are set as follows:

$$\begin{aligned} t_{mr1} &= 1/f_{init} + 1/(70\times f_{init}) + t_{kor1} \\ t_{mr2} &= 1/f_{init} + 1/(70\times f_{init}) + t_{kor2} \\ a_{pr1} &= (1+t_{kor1}/t_{mr1})/(3\times f_{init}) \\ a_{pr2} &= a_{pr1}+a_{kor}. \end{aligned} \tag{15}$$

The corrections for the timer parameters of two DVs are denoted as t_{kor1} and t_{kor2}, respectively. Since the integration time depends directly on the accuracy of time base, the correction is needed for the aperture too. This was taken into account by means of the term t_{kor1}/t_{mr1} for the first voltmeter (a_{pr1}), and with the correction a_{kor} for the second one; in an ideal case all corrections should be equal to zero.

Two examples of the measurement results obtained with the described method are presented in Figure 3. For the calculation of regression line coefficients, all together 30 values of phase angles are analysed, and both measurements are performed with the same difference t_{kor1}-t_{kor2}= -3,7 µs and with a_{kor}=1,2 µs. Two upper lines are determined for parameters t_{kor1}=-4,5 µs and t_{kor2}=-0,8 µs, while in the second case their values are -4,8 µs and -1,1 µs, respectively. The lower slopes of the regression lines mean better adjustment of the timer and aperture parameters for the two DVs. The limiting factor in the determination of needed corrections is finite resolution of their change, equal to 100 ns.

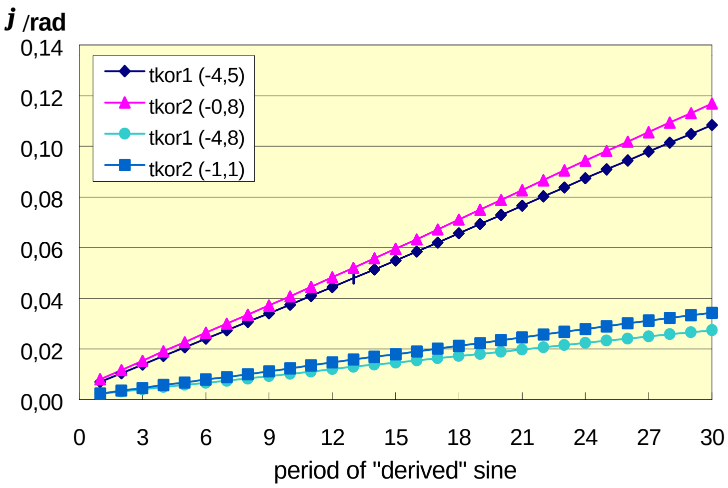

Figure 3. Adjustment of corrections for timer and aperture parameters of two DVs

The adjustment of the corrections was repeated during the period of seven months, and the obtained values are listed in Table I. Although they refer to the same type of voltmeters, it is obvious that the long-term stability of their time base is quite different.

Table I Time dependence of the corrections for two DVs

	t_{kor1}/µs	t_{kor2}/µs	a_{kor}/µs
17.02.1999.	-4,8	-1,1	1,2
07.06.1999.	-5,3	-1,2	1,4
14.09.1999.	-5,7	-1,2	1,5

The methods described in sections 2 and 3 have been used for the measurement of the phase difference correction (Δj_{10}) when both DVs are set on 10 V range and measure the same voltage with Y=8 V, as the first step in their use for complex voltage measurements [6]. The results of 40 repeated measurements, performed according to the described *standard measurement procedure*, are shown in Figure 4. In this figure the mean values of the phase difference are presented, where the error bars are the standard deviations of the mean from ten repeated measurements, lower than 0,02 µrad. The mean values were stable within 0,1 µrad during the twenty hours of measurements, which showed the applicability of the used method for precise measurements of the phase difference. The unbroken line shows the ambient temperature in the vicinity of the DVs. The AC source was frequency stabilised and the mean frequency for each presented measurement lied between (15,8727763÷15,8727781) Hz. In this case, the standard deviations of the ratio of RMS values were lower than 0,1 ppm.

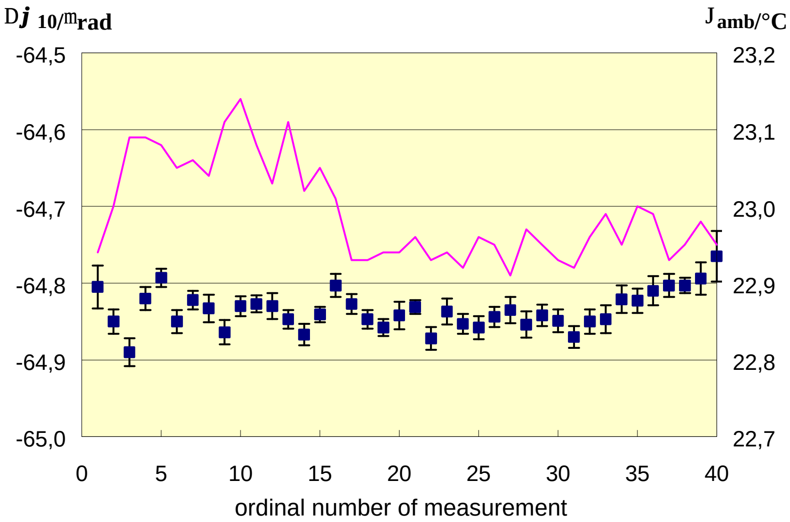

Figure 4. Results of Δj_{10} measurements with parameters t_{kor1}=-5,3 µs, t_{kor2}=-1,2 µs and a_{kor}=1,4 µs.

4 CONCLUSION

Methods for the determination of the RMS values and phase angles of AC voltage (with low harmonic components, i.e. “clear” sine) by means of high-resolution DVs is described. Special attention was paid to the phase angle and phase difference measurement. With the frequency stabilisation of AC source, and with both DVs measuring the same voltage, a standard deviation of phase difference was typically less than 0,1 µrad, and a standard deviation of voltage ratios was less than 0,1 ppm. The presented methods are used for the measurement of impedance at low frequencies with each DV measuring at a different range. Further investigations of these methods are in progress.

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