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NONLINEARITY OF CONVERSION A/C – CRITICAL REVIEW

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Abstract: Due to the complexity of processing, the total error of the analog to digital conversion being the difference between the input and output signal is set of several constituent errors. They include, among others, integral nonlinearity error and differential nonlinearity error. Different methods of defining the errors depending on their source of origin as well as certain ambiguity of the presentation manner regarding information on converter parameters between manufacturers makes a clear-cut assessment of the total error of analog - digital processing trajectory difficult. This paper is an attempt at describing mutual relationships between those errors and the effect of one error on the behaviour of the others.
Keywords: analog to digital conversion, nonlinearity, A/D converter

1 INTRODUCTION

Determination of deformation of the real characteristic of processing trajectory with analogue to digital conversion is one of the prerequisites for correct assessment of its measuring usefulness. In general, the real characteristic of such trajectory differs from the nominal characteristic in:

- position (offset error, gain error)
- shape (nonlinearity of the characteristic)

Defining, determination and correction of the zero shift error and gain error does not present a great problem. However, the situation is different when it comes to the determination of nonlinearity of the characteristic. In the literature and catalogue data of different manufacturers different parameters are used to determine that type of deformation. The lack of norms and standards regarding both the nomenclature and the manner of defining apparently identical parameters causes certain difficulties with the determination of the most important, total measurement error of a trajectory with analogue-digital conversion. Such situation also makes it difficult to compare products made by different manufacturers.

The most commonly used parameters which describe nonlinearity of trajectories with analogue to digital conversion include:

- differential nonlinearity DNL
- integral nonlinearity INL
- total nonlinearity TNL

There are certain interrelationships that occur between these parameters of one real characteristic. As a result of the said interrelationships, the occurrence of one parameter forces the occurrence of the other two parameters. It does not mean, however, that from the value of one parameter one may infer the values of the others.

2 ASSUMPTIONS

Prior to the analysis of nonlinearity of processing trajectory with analogue-digital conversion the exact position of real characteristic in relation to nominal characteristic needs to be determined. Therefore further herein we will follow the assumption that the trajectory under analysis has been correctly calibrated.

As it is generally known N-bit converter divides its analogue conversion range into 2^N equal quantum intervals of the width Q (except for the first and last interval). Each of these intervals has its digital representation in the form of an N-bit word (Fig.1 the dotted line). For numerous complex reasons, the width of the quantum intervals in the real converter is different than Q, which results in a deformation of nominal characteristic (Fig.1 the solid line).

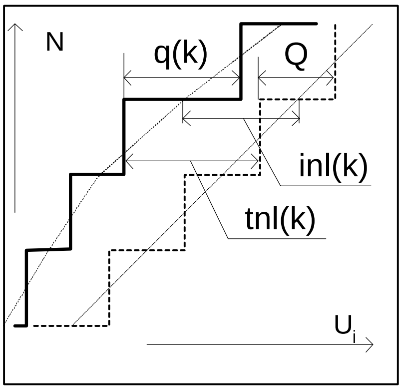


Fig. 1.The graphical definition of dnl(k), inl(k), tnl(k)

3 ASSESSMENT OF PARAMETERS DESCRIBING THE NONLINEARITY OF A TRAJECTORY WITH A/D CONVERSION

Differential nonlinearity is defined as:

$$DNL = \max_k |q(k) - Q| \tag{1}$$

The real width of the quantum interval q(k) that occurs in this definition is described in literature as: the difference between the adjacent values of input voltage U_i resulting in a change of the input word by the value of the least significant bit (1 LSB). This definition, however, is of no use in an event where differential nonlinearity is so big that the code losing effect has occurred. In such situation the difference between the two adjacent values of the input voltage will be determined for the change of the output code word by 2 LSB.

Therefore, the width of a quantum interval shall be understood as the difference between the two adjacent values of analogue input voltage at which a change of the input code word of the A/D converter occurs.

Another example for the ambiguities regarding the determination of differential nonlinearity is the fact of considering as equivalent of the expressions below:

a) not losing codes
b) differential nonlinearity smaller than Q.

If differential nonlinearity is smaller than Q then we may be sure that the converter does not lose codes. However, the fact that a converter does not lose codes does not mean that its differential nonlinearity is smaller than Q. An example of such characteristic is given in Fig. 2.

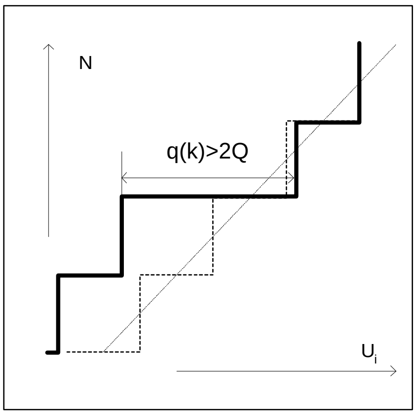


Fig.2 An example of a/d conversion characteristic, where despite the fact of not losing the code, differential nonlinearity DNL is bigger than Q.

Integral nonlinearity describes the shift of the centre of the real quantum interval in relation to the centre of the corresponding interval of the nominal characteristic (Fig. 1) and it is defined as:

$$INL = \max_k |inl(k)| \tag{2}$$

Integral nonlinearity is very often referred to just as nonlinearity which is inconsistent in regard of the nomenclature and may result in some confusion: it may suggest that this parameter determines the error related to the nonlinearity of a characteristic in a sufficient and unambiguous manner, which is not the case. Besides, integral nonlinearity is often confused with total nonlinearity (cf. Fig 1) which is defined as:

$$TNL = \max_k |tnl(k)| \quad (3)$$

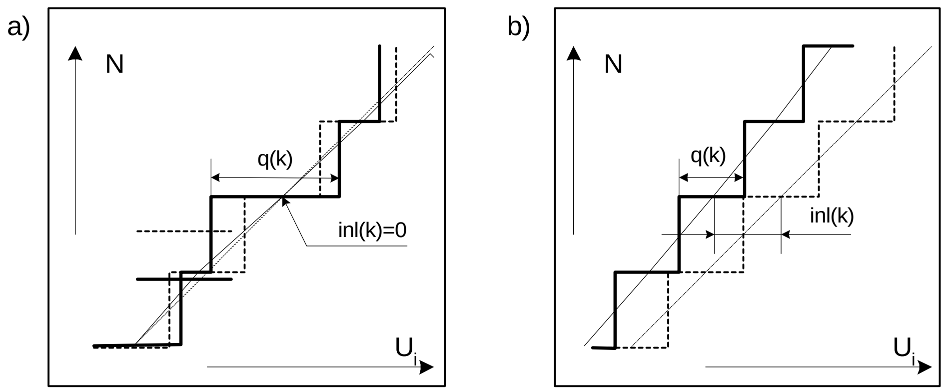
4 RELATIONSHIP BETWEEN DIFFERENTIAL AND INTEGRAL NONLINEARITY

Integral nonlinearity of the k-th quantum interval may be expressed as the sum of differential nonlinearities of the subsequent quantum intervals from 1 to k according to the following formula:

$$inl(k) = \sum_{n=1}^{k-1} dnl(n) + \frac{1}{2} dnl(k) \quad (4)$$

The above relationship may be confirmed by the determination of integral nonlinearity using the histogram method. The picture of integral nonlinearity is obtained by the integration of the signal of code density that provides direct information on the differential nonlinearity of the converter.

Equation (4) shows that if the differential nonlinearity dnl(n) has the same sign (i.e. is either positive or negative) within a certain interval $n \in \langle a,b \rangle$ (a, b $\in$ N) then integral nonlinearity inl(n) increases with the increase of n. For a sufficiently big interval $\langle a,b \rangle$ even very small differential nonlinearity results in the occurrence of substantial integral nonlinearity.



Q - nominal quantum
q(k) - k-th real quantum
inl(k) - integral nonlinearity of the k-th quantum
nominal characteristic
real characteristic

Fig.3 A/D converter's characteristics for: a) dnl(k)=|q(k)-Q|>>0 and inl(k)≈0, b) dnl(k)=|q(k)-Q|≈0 and inl(k)>>0

The situation is different when the sign of differential nonlinearity dnl(n) is of random nature. In such case, integral nonlinearity may be very small despite the fact that the differential nonlinearity of particular quantum intervals is substantial. Fig. 3 shows two extreme cases of the relationship between differential and integral nonlinearity.

5 RELATIONSHIPS BETWEEN DIFFERENTIAL, INTEGRAL AND TOTAL NONLINEARITY

Total nonlinearity may be expressed as the resultant of the differential and integral nonlinearity. (Cf. Fig. 1):

$$TNL = \max_k \left| inl(k) + \frac{1}{2} dnl(k) \right| \le INL + \frac{1}{2} DNL \quad (5)$$

The following conclusions may be drawn from Formula (5):

- in the worst case, total nonlinearity equals the sum of the integral nonlinearity and a half of the differential nonlinearity
- total nonlinearity takes into account mutual distribution of nonlinearities inl(k) and dnl(k) along the range of conversion
- only total nonlinearity smaller than 0,5 Q guarantees that differential nonlinearity DNL is smaller than 1Q and that code losing does not occur in the conversion characteristic.

6 CONCLUSION

In the paper, the basic parameters describing deformation of the characteristic of trajectory with analogue-digital conversion have been presented. These parameters include differential nonlinearity DNL, integral nonlinearity INL and total nonlinearity TNL.

The occurrence of one of the above parameters provides evidence for the occurrence of the other two.

The fact that integral nonlinearity of a k-th quantum interval inl(k) may be presented as the sum of differential nonlinearities of subsequent quantum intervals dnl(k) starting from 1 to k does not, however, allow to postulate any relationship between the given catalogue parameters of an a/d trajectory, differential nonlinearity DNL and integral nonlinearity INL. The fact that the system has small differential nonlinearity DNL does not mean that it also has small integral nonlinearity INL and vice versa.

Total nonlinearity TNL is a parameter describing the resultant of differential and integral nonlinearity, however only at TNL<0.5 Q we may be sure that no code losing occurs in the process of a-d conversion.

Neither of these parameters provides full information on nonlinearity of the trajectory, but only describes some of its aspects. Therefore, in order to correctly assess the nonlinearity of a trajectory with a/d conversion it is necessary to know two parameters.

- One of them is the differential nonlinearity DNL which describes the size of deformation of the quantum interval thus providing information on ‘local’ deformation of processing characteristic (monotony, code losing)
- the other, most often INL, specifies how much the real characteristic ‘diverged’ from the nominal characteristic as a result of summation of differential nonlinearity *dnl(k)* of subsequent quantum intervals.

In an event when instead of integral nonlinearity INL, the error of total nonlinearity TNL is known, the nonlinearity of the converter is described more precisely.

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