XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

FAULT DIAGNOSIS VIA DUPLICATE

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Abstract. The problem of fault detection and isolation in logical digital systems is studied. It is suggested using a duplicate as an observer. The procedures of designing the residuals and fault detection logic are developed.
Keywords: fault detection, observer, duplicate

1 INTRODUCTION

This paper deals with the problem of fault-tolerant systems which can be solved both by improving reliability of the functional units and by an efficient fault detection, isolation and accommodation methods. We accent on fault detection and isolation (FDI) that provides on-line checking of monitored system. On-line checking quarantees that a fault will be detected in time. This allows to prevent undesirable consequences of the fault or reconfigurate the system to recover a normal operation.

Well-known approaches to solve a fault detection problem use a duplication or triplicated voter. These approaches are good only for systems of invariable structure. However, if the considered system may be reconfigurated when the faults occure, such approaches are not effective.

On the other hand, there are well-developed FDI methods intended for dynamic systems described by difference or differential equations [1,2]. They allow to isolate the faulty unit of system but demand to use specially disigned observers.

We suggest combining the advantages of both approaches for providing the fault-tolerance property in the systems which may be reconfigurated. Namely, it is suggested using the duplicate as an observer. To solver the task either the faulty unit is in the main system or in the duplicate, one can use a few methods:

(1) testing both systems after detection of the fault by the FDI process;
(2) using two main systems and single duplicate one as observer (Figure 1);
(3) using the vote scheme which contains three identical systems.

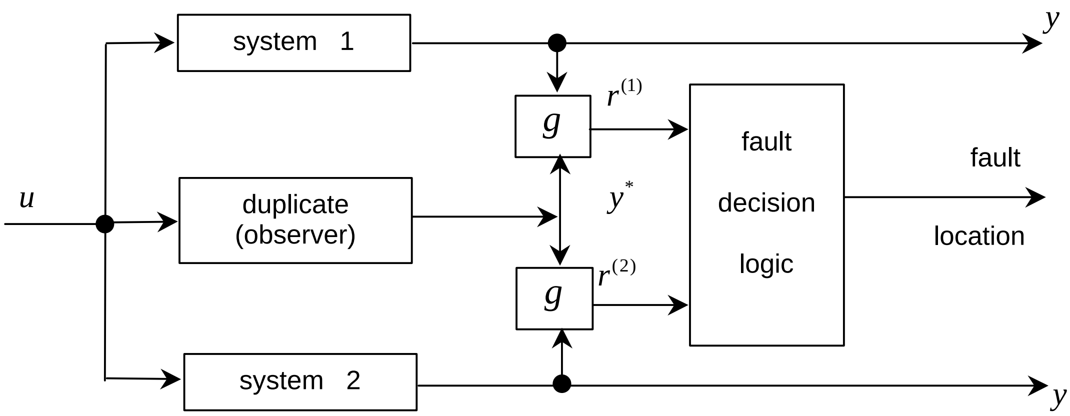

Figure 1. Scheme of fault detection and isolation.

So, the problem of the observer designing is droped (in particular, the problems of stability and modeling errors), and main problem becomes to provide the FDI process on the basis of the duplicate. Namely, the problem is to design such set of the residuals which are the most suitable for fault isolation. In our case, the vector of residuals is generated as follows:

$$r = g(y) - g(y_*)$$

where g is some vector function, y and y_* are the vectors of output of the system and duplicate respectively. So, the problem is to obtain the function g.

It is supposed that the considered system is described by the equations

$$x(t+1) = f(x(t), u(t), g), \quad y(t) = h(x(t)) \tag{1}$$

where x, u, y are the vectors of state, input and output, g is the vector of parameters, f and h are vector functions. If there are no faults, $g = g_0$; if the *i-th* faults d_i occurs, the i-th component of the vector g become an unknown function of time, $i = 1,2,..., q.$

We consider two approachs to solve the problem: the precise one based on special mathematical techniques and the approximate ones.

2 MATHEMATICAL TECHNIQUES

Vector functions with the domain X are elements of these techniques including some binary relations, operations and operator.

1. Partial ordering relation $\leq$: for some vector functions $a : X \to S$ and $b : X \to T$ denote $a \leq b$ if $ha = b$ for some function $h : S \to T$, i.e. $h(a(x)) = b(x)$ for all $x \in X$ where S and T are some sets. When $a \leq b$ and $b \leq a$, denote $a \cong b$. For example, if $a = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $b = x_1 x_2$, then $a \leq b$; if $a = \begin{bmatrix} x_1 + x_2 \\ x_2 \end{bmatrix}$ and $b = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, then $a \cong b$; if $a = \begin{bmatrix} x_1 \\ x_2 x_3 \end{bmatrix}$ and $b = \begin{bmatrix} x_1 x_2 \\ x_3 \end{bmatrix}$, then neither $a \leq b$ nor $b \leq a$ hold.

If the functions a and b are linear and $a(x) = Ax$, $b(x) = Bx$ for some matrices A and B, then $a \leq b$ (or $A \leq B$) when $NA = B$ for some matrix N. Clearly, $A \leq B$ if $rank(A) = rank\begin{bmatrix} A \\ B \end{bmatrix}$ and $a \cong b$ if $rank(A) = rank(B)$.

2. Operations $\times$ *and* $\circ$: $a \times b = \max(d \,\vdots\, d \leq a, d \leq b)$, $a \circ b = \min(d \,\vdots\, a \leq d, b \leq d)$. It is clear that $a \times b \leq a$ and $a \times b \leq b$ therefore $a \times b = \begin{bmatrix} a \\ b \end{bmatrix}$. By analogy, $a \leq a \circ b$ and $b \leq a \circ b$ thus every component of the function $a \circ b$ depends both on the components of the function a and the function b. One can use this rule to calculate the function $a \circ b$ by hand in the simple cases; for example, if $a = \begin{bmatrix} x_1 \\ x_2 x_3 \end{bmatrix}$ and $b = \begin{bmatrix} x_1 x_2 \\ x_3 \end{bmatrix}$, then $a \circ b = x_1 x_2 x_3$. In the general case, one has to solve special differential equations.

If $a(x) = Ax$ and $b(x) = Bx$, then $A \times B = \begin{bmatrix} A \\ B \end{bmatrix}$; the matrix N corresponding to the function $h = a \circ b$ can be obtained as follows. Let $[Q \,\vdots\, P]$ be the matrix of full rank such that $[Q \,\vdots\, P]\begin{bmatrix} A \\ B \end{bmatrix} = 0$, then $N = QA = -PB$. It can be shown also that N is the matrix of full rank such that each row of N linearly depends both on the rows of the matrix A and the rows of the matrix B therefore the matrix N corresponds to $span\{A\} \cap span\{B\}$.

3. Binary relation $\wedge : (a, b) \in \wedge$: for a diven function f, the function a and the differentiable function b from a pair, i.e. $(a, b) \in \wedge$ if $h(a(x), u) = b(f(x, u))$ for some function $h : S \times U \to T$ and for all $x \in X$, $u \in U$. In the linear case, $(A, B) \in \wedge$ if $rank(A) = rank\begin{bmatrix} A \\ BF \end{bmatrix}$. The relation $\wedge$ is of secondary importance, it is used only for the operator m definition.

4.Operator m: $m(a)$ is the function satisfying the conditions $(a, m(a)) \in \wedge$, $\forall (a,b) \in \wedge \Rightarrow m(a) \leq b$. Thus, the function $m(a)$ is the minimal one with which the function a forms a pair.

To calculate the operator *m* in the general case, one has to solve special differential equations. To obtain an approximate solution in the form $m(a)(x) = Qx$ for some matrix Q of full rank, one has to solve the algebraic equation $Qf(x,u) = h(a(x),u)$ for any vector function h. If the system (1) is described by the linear equation

$$x(t+1) = Fx(t) + Gu(t)$$

for some matrices F and G, then $m(A) = Q$ where $[Q \vdots N]\begin{bmatrix} F \\ A \end{bmatrix} = 0$ and $[Q \vdots N]$ is the matrix of full rank. If the matrix F is nonsingular, then $m(A) = AF^{-1}$.

if the function f is onto and $f(x,u,g_0) \circ (b(x) \times u) = a(f(x,u,g_0))$, then $m(b) = a$.

The main properties of the relation $\leq$, the operations and the operator are:

1. $a \leq b \quad \Leftrightarrow \quad a \times b \cong a \quad \Leftrightarrow \quad a \circ b \cong b$;
2. if $a \leq b$ and $b \leq a$, then $a \circ b \leq d$;
3. if $a \leq b$, then $m(a) \leq m(b)$.

3 EXACT SOLUTION.

There is the set of the functions $a^i,\ i = 1,2,...,q$, playing an important part in our approach [4]: a^i satisfies the inequalities $a^{i0} \leq a^i,\ m(a^i) \leq a^i$ where a^{i0} is the minimal function satisfying the condition

$$\partial / \partial g_i \, (a^{i0}(f(x,u,g))) = 0, \quad i = 1,2,...,q .$$

The demand of minimum means that if $\partial / \partial g_i \, (a(f(x,u,g)) = 0$ for some function a , then $a^{i0} \leq a$.

There is the procedure for the function a^i calculation: let $a^{i,j+1} = a^{ij} \circ m(a^{ij}),\ j = 0,1,...$ When $a^{i,j+1} \cong a^{ik}$ for some k, then $a^i = a^{ik}$. The function a^i determines such part of the system which is uneffected by the fault d_i.

Consider the case when the first component r_1 of the residual r has to be invariant to the fault d_1 and sensitive to others, i.e. $r_1 = 0$ if the fault d_1 occurs and $r_1 \neq 0$ for other faults. It can be shown that $a^1 \leq g_1 h$ in this case where g_1 is the first component of the function g . The function g_1 can be obtained from equation $a^1 \circ h = g_1 h$. If $a^1 \circ h = const$, then the function g_1 does not exist.

4 APPROXIMATE SOLUTION

If the considered system is not too comlex, one can use the described approach that gives a precise solution of the problem. Otherwise one can use another approach based on the model (1).

Define the relation $\mapsto$ on the set of the components of the vectors x and y: $x_i \mapsto x_j$ if x_i is in the right-hand side of the equation for x_j in the model (1); the relations $x_i \mapsto y_k$ and $g_j \mapsto x_i$ are defined by analogy

Define the matrix C as follows: C(k,j)=0 if there exists the shoetast chain of the components $x_{i1}, x_{i2}, ..., x_{is}$ such that $g_j \mapsto x_{i1} \mapsto x_{i2} \mapsto ... \mapsto x_{is} \mapsto y_k$, otherwise C(k,j)=1, $k = 1,2,...,l,\ \ j = 1,2,...q.$

The matrix C can be used instead of the functions $\{a^i\}$ as follows. If r_1 has to be invariant to the fault d_1 and to be sensitive to others, one can choose the first column of the matrix C as the function g_1.

The second approach is far simpler than the first one because the calculation of the functions $a^i, i = 1,2,...q$, is much more laborious than the procedures of obtaining and analysing the relation $\mapsto$. So, this approach may be used for more complex systems but it gives the approximate solution as a rule and demands an analytical description of the considered systems as the first one. If one has a functional description such as the list of the system units and their connections, this approach cannot be used. We suggest the third approach in this case.

Define the relation $\mapsto$ on the set of the units of the system: $e_i \mapsto e_j$ if at least one of the outputs of the unit e_i is connected with the unit e_j. By analogy, $e_i \mapsto y_j$ if at least one of the outputs of the unit e_i forms the *j-th* component of the vector y.

The matrix C^* is determined by analogy with C: C^*(k,j)=0 if there exists the chain of units $e_{i1}, e_{i2},..., e_{is}$ such that $e_{i1} \mapsto e_{i2} \mapsto ... \mapsto e_{is} \mapsto y_k$, and the unit e_{i1} contains the fault d_j; otherwise C^*(k,j)=1, $k = 1,2,..., l, \quad j = 1,2,...q.$. The matrix C^* has to be used instead of the matrix C for the function g obtaining.

More sophisticated analysis in the approximate approaches may be achieved if one takes into consideration the time of the signal propagation from the component g_j (or the unit e_{i1}) to the output

REFERENCES

[1] P. Frank, Fault Diagnosis in Dynamic Systems Using Analytical and Knowledge-based Redundancy - A Survey and Some New Results, *Automtica,* **26**, 1990, 459-474.
[2] J. Gertler, Residual generation in model-based fault diagnosis, *Control-Theory and Advanced Technology,* **9**, 1993, 259-285.
[3] A. Zhirabok, A.Shumsky, A new mathematical techniques for nonlinear system research, Procedings of 12th World Congress IFAC (Sydney, 18-23 July 1993), Sydney, Australia, **3**, 1993, p.485-488.
[4] A. Zhirabok, O.Preobrazhenskay, Robust fault detection and isolation in nonlinear systems, Proceedings of the IFAC Symposium “Safeprocess’94” (Espoo, 13-16 June 1994), Espoo, Finland, 1994, p.244-248.

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