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NON-LINEAR DYNAMICS OF THE RESPIRATORY OSCILLATOR

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Abstract: Numerical procedures for detection of deterministic chaos in non-linear time series have been prepared. The procedures include reconstruction of the phase space, estimation of the maximal Lyapunov exponent and calculation of capacity and correlation dimensions. Prepared algorithms have been successfully tested with simulated data of the Hénon map and the logistic equation, and then used for investigation of the respiratory rate variability (RRV). The dimensions of the RRV signal in the phase space reconstructed are rational numbers and the Lyapunov exponent is positive, suggesting that the respiratory oscillator exhibits chaotic dynamics. An estimate of the RRV attractor dimension can be a potential indicator of abnormalities in respiratory system activity.
Keywords: non-linear dynamics, respiratory rate, invariant measures.

1 INTRODUCTION

A central neural oscillator, driving respiratory muscles, regulates breathing. It has been shown that the rhythm of the respiratory oscillator can be altered by a variety of stimuli [1]. An analogy between biological oscillators and a pendulum may be used to explain the variations in rhythm of physiological systems [1, 2]. Striking the pendulum just as it passes the bottom of its arc (a stable singular point) can stop it if the strength of the impulse exactly opposes the momentum of the pendulum. On the contrary, if the momentum is such that the pendulum would stop at the top of the arc (an unstable singular point) even the weakest and/or random impulse can change the direction of its movement. In such circumstances it is no longer possible to predict the pendulum behaviour. Other perturbations, having different strength or time of impact, cause shift in the pendulum movement. The more the disturbances the more irregularities of the rhythm are observed [3]. This kind of non-linear dynamic system behaviour is called the deterministic chaos [2].

Dynamical systems possess attractors in their phase space. Attractors of the chaotic dynamics with non-integer dimension, called the strange attractors, are a special case of them [4]. Though the systems are usually multidimensional, it is possible to reconstruct the phase space from a time series of only one variable [5-6]. There are some invariant measures characterising the strange attractors, such as: Lyapunov exponents; capacity, information or correlation dimensions [3, 7-9]. Time adjustment in such a measure can indicate pathological changes in a physiological system [3, 10].

Pathological disturbances in the breathing rhythm are observed in children and adults with neurological and cardiopulmonary diseases [1]. Irregularities in the pattern of breathing occur usually during sleep and their most common form is apnea, which is seen as an unexpected loss of inspiratory effort. Apnea of central neural origin is a cause of sudden infant death syndrome (SIDS). Direct studies on apnea in SIDS are very complicated and, moreover, it is difficult to distinguish between normal and abnormal respiratory rate variability (RRV) [1]. The aim of this work is to prepare computational algorithms for deterministic chaos investigation and to test if the RRV signal has the non-linear chaotic properties. This kind of study may lead to apnea detection in awake patients during spontaneous breathing by, e.g., measurement of a RRV attractor dimension [3, 10].

2 THEORY AND METHODS

2.1 Numerical procedures

All procedures for deterministic chaos investigation in time series have been written in Matlab - a professional tool for technical calculations and imaging [11]. Their correctness has been proved on the ground of simulated data of the Hénon map and the logistic equation. A user-friendly interface, enabling „artificial" data simulation, real data loading and execution of algorithms mentioned, has also been prepared.

2.1.1 Reconstruction of the phase space

One of steps in the dynamic system analysis is reconstruction of the phase space from the time series [5-6]. It converts the one-dimensional time series $x(t)$ of n values into a set of N D_e-dimensional points $\mathbf{x}(t)$:

$$\mathbf{x}(t) = \left[x(t)\ \ x(t+t)\ \ \ldots\ \ x\left(t+\left(D_e-1\right)t\right)\right]. \tag{1}$$

Parameter D_e is called the embedding dimension.

The choice of a value for the delay t depends on the properties of the time series. Various criteria of this value selection are based on the time scale at which the time series values start to be uncorrelated. In the present investigations the 1/e criterion of the unbiased estimate of the time series autocorrelation function has been used (i.e. t is equal to the number of the first lag, at which the autocorrelation function has a value lower than 1/e of the zero lag one).

2.1.2 Maximal Lyapunov exponent

Lyapunov exponents, if positive, are the most noticeable evidence for chaos in the non-linear time series. They are a measure of exponential divergence of neighbouring system trajectories in the phase space. Whereas it is fairly easy to determine them if one knows the equations of motion of a system, this is difficult if the only information is a univariate time series. In this instance two groups of algorithms are used: directly exploring the definition of the Lyapunov exponent or applying a tangent space method to reconstruct the system dynamics from the data. In practice, however, only the maximal exponent $l_{\max}$ can be correctly estimated [7]:

$$l_{\max} = \lim_{t\to\infty}\lim_{l\to 0}\frac{1}{t}\log\left(\frac{\left\|\mathbf{x}(t)-\mathbf{x}_l(t)\right\|}{l}\right), \tag{2}$$

where $l = \left\|\mathbf{x}(0)-\mathbf{x}_l(0)\right\|$.

To calculate the maximal Lyapunov exponent the algorithm of Kantz [7] has been used. It allows finding the distance between neighbouring trajectories on the basis of their scalar components, averaging these distances between all the trajectories and finally, after taking the logarithm, averaging the local effective Lyapunov exponents over the full length of the time series.

2.1.3 Capacity dimension

The capacity dimension D_0 is determined from the minimal number $M(l)$ of regions of size l that are needed to cover all the points of the attractor in the phase space. It is defined as

$$D_0 = \lim_{l\to 0}\frac{\log M(l)}{\log(1/l)}. \tag{3}$$

When the regions are non-overlapping boxes of rectilinear grid this dimension is called the box-counting one [3].

An efficient box-counting algorithm [8] has been used to calculate the capacity dimension. The co-ordinates of each point in the reconstructed phase space are rescaled to the interval [0, 2^k-1] and expressed in the binary form (for k=16). Then they are intercalated into a long bit string and sorted in lexicographical order. The benefit of the procedure is a single sorting. After sorting, the most significant bits determine boxes of different sizes, which contain the given point.

The capacity dimension is determined from the slope of fitting $\log_2 M(l)$ against $\log_2(1/l)$ in the region in which this relationship is linear. Moreover, D_0 determined from different embedding dimensions D_e must be the same. The scaling region of l and D_e over which this occurs is found interactively.

2.1.4 Correlation dimension

The correlation dimension D_2 is estimated by determining the relative number $C(l)$ of pairs of points in the phase space that are separated by a distance smaller than l. It takes into account not only the number of regions containing the attractor points but also the amount of points in the given region. The correlation dimension is defined as [4]

$$D_2 = \lim_{l \to 0} \frac{\log C(l)}{\log l}. \tag{4}$$

The correlation integral $C(l)$ is related to the probability that the distance between a pair of points is smaller than l and it is determined according to the following formula [3]

$$C(l) = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \Theta\left(l - \left\| \mathbf{x}_i - \mathbf{x}_j \right\|\right), \tag{5}$$

where Θ is the Heaviside function.

The correlation dimension, as previously D_0, is calculated from the slope of linear fitting $\log_2 C(l)$ against $\log_2 l$ in the scaling region.

2.2 Investigations of RRV

2.2.1 Respiratory signal acquisition

A respiratory signal of a healthy volunteer was recorded during a period of about 3 hours (2^{18} samples) (Fig. 1A). A thermistor sensor was placed into the outlet of a nose and its resistance changes, after conversion to voltage, were acquired via the DAS-1601 (Keythley) data acquisition board with 12 bit resolution. A virtual measurement panel was created on a PC using the TestPoint (CEC) software. The sampling rate was set to 25 Hz to guarantee good resolution of the RRV signal. The volunteer kept his mouth closed during the whole period of examination.

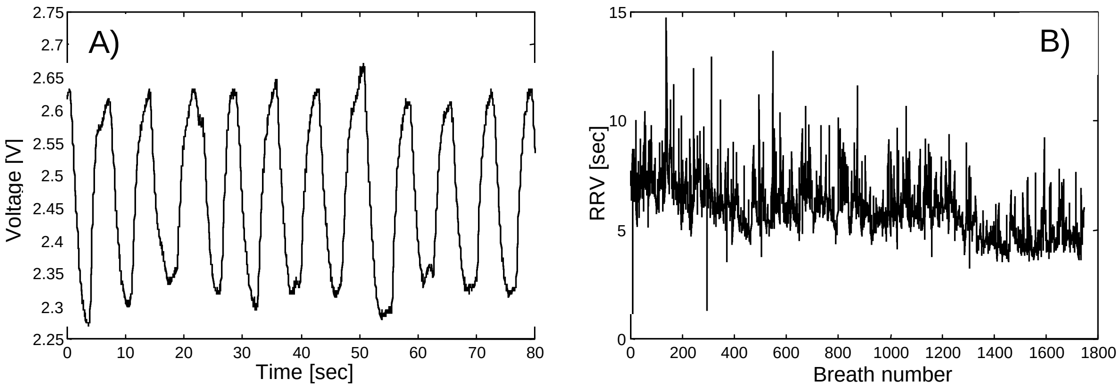

Figure 1. The first 2000 samples of the respiratory signal measured (A) and the constructed respiratory rate variability (RRV) time series (B).

2.2.2 Construction of the RRV time series

The recorded respiratory signal was processed in Matlab. First the linear trend was removed. Then the signal was filtered by a FIR passband filter with zero-phase shift. The cut-off frequencies were chosen as 0.05 and 1.0 Hz with the filter order equal to 100. Respiratory cycle values were determined by finding time instants in which the processed signal was changing its sign (from negative to positive). A vector of these succeeding values constitutes the respiratory rate variability (RRV) time series (Fig. 1B).

3 RESULTS

Analysis of the RRV time series autocorrelation function shows that this signal becomes uncorrelated after the second breath (t=2). The 3D phase portrait reconstructed is rather complex (Fig. 2A) without an apparent structure typical for low dimensional attractors. It may be caused by higher dimensionality of the RRV series, which disturbs the attractor shape after its projection into the 3D-space [12].

It was difficult to find the maximal Lyapunov exponent because there was not a clear scaling region in the resulting graph (Fig. 2B). For the first steps of system evolution the logarithm of the averaged trajectory divergences „jumps" and then increases in linear manner (chosen as the scaling region,

l_{max}=0.041±0.002). The last part of the plot is, as usually, almost constant. The too short for the averaging procedures RRV vector is the probable reason of the initial effect.

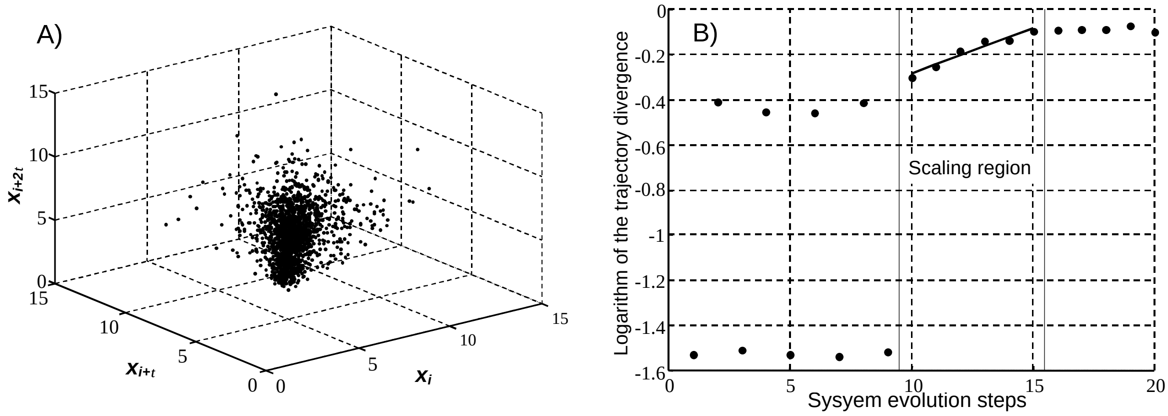

Figure 2. 3D-space phase portrait of the RRV time series (A) and plot of logarithm of the averaged trajectory divergences against the system evolution steps (B)

The scaling regions can be seen in figures created for the capacity and correlation dimension estimation (Figs 3A and 4A). Despite this, the dimensions calculated (see Figs 3B and 4B) have quite different values: $D_0 \approx 1.8$ and $D_2 \approx 4.8$. It is well known that for short time series the correlation dimension is a better evaluation of the attractor dimension than the capacity one [3]. The present investigations, were N = 1744, might be the case.

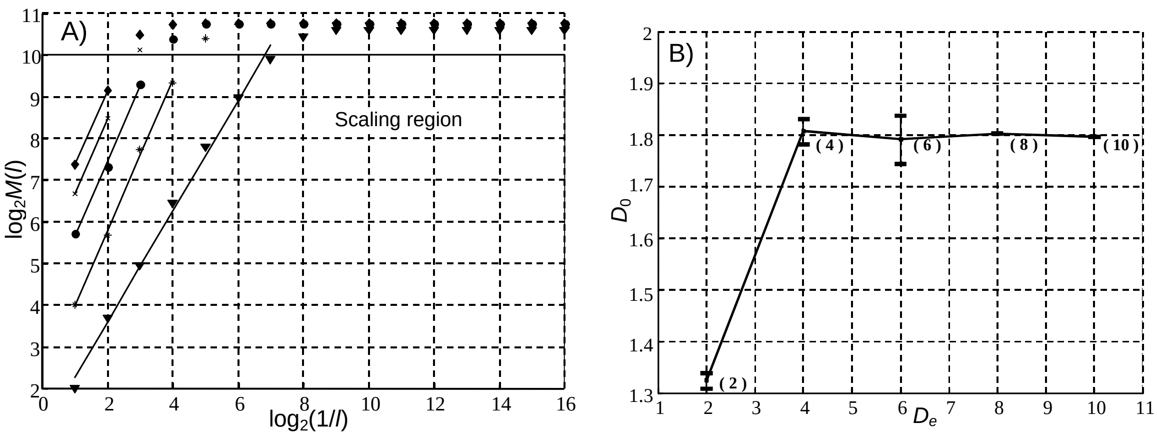

Figure 3. Plot of $\log_2 M(l)$ against $\log_2(1/l)$ for D_e = 2, 4, 6, 8 and 10 with the scaling region (A) and dependence of the estimated capacity dimension D_0 on the embedding dimension D_e (B).

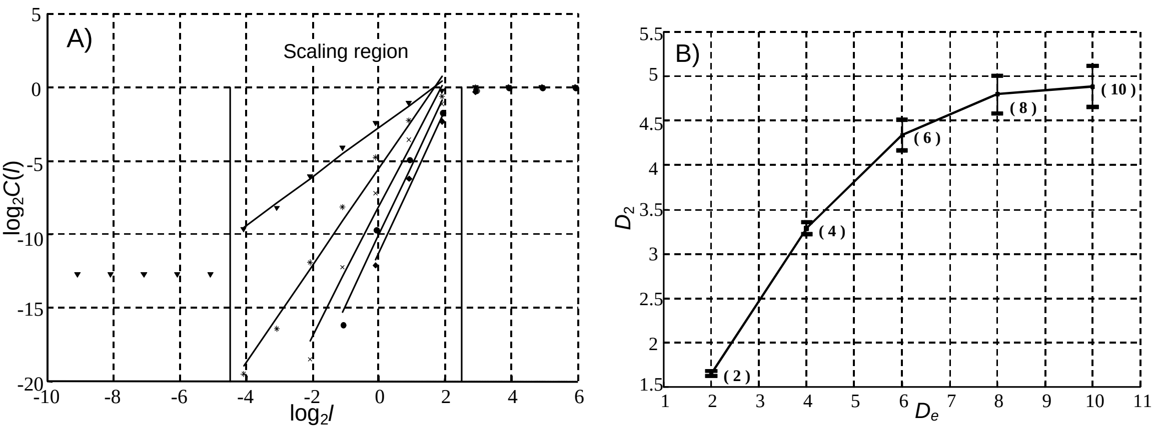

Figure 4. Plot of $\log_2 C(l)$ against $\log_2 l$ for D_e = 2, 4, 6, 8 and 10 with the scaling region (A) and dependence of the estimated correlation dimension D_2 on the embedding dimension D_e (B).

4 CONCLUSIONS

The algorithms for the phase space reconstruction, attractor dimensions and maximal Lyapunov exponent estimation have been used for investigation of deterministic chaos in the respiratory rhythm variability data. The time series analysed has a finite length and may be non-stationary. These are the possible reasons of some difficulties in consistent determination of the mentioned, well-defined invariant measures [3]. The 3D RRV data phase portrait has rather perturbed shape and does not exhibit any noticeable structure. On the other hand, projection of a high-dimensional attractor into a lower-dimensional space can yield such an effect. The attractor dimension measures used, the capacity and correlation dimensions, have different, rational values. The most likely reason is that the data vector was too short for appropriate estimation of the capacity dimension. In this instance the correlation dimension ($D_2 \approx 4.8$) is a better evaluation of the attractor complexity and thus it explains problems with phase portrait interpretation. The too short RRV time series is probably also responsible for difficulties with finding the scaling region in the plot for maximal Lyapunov exponent calculation. Within the chosen range the exponent is positive, suggesting chaos in the signal under consideration.

Though the results of the research carried out are not unambiguous, they show that the respiratory oscillator probably possesses the chaotic dynamics. To verify this some more investigations have to be performed, allowing analysis of longer RRV time series. Another interesting question is whether the invariant measures of attractor properties vary during pathological changes of the respiratory system, particularly in the case of the central origin apnea. Such a situation could make it possible to discriminate some respiratory system diseases in awake patients during spontaneous breathing.

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