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DETERMINATION OF THE DEFORMATION DEPENDENT STIFFNESS OF FIBER REINFORCED MEMBRANES

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Abstract: Some manufacturing processes of fiber reinforced soft membranes do not provide knowledge neither about the density nor about the orientation of the fairly random distributed fibers. As a consequence, the result of these processes is often an anisotropic membrane with an unknown, in general inhomogeneous, stiffness distribution.
In case of soft membranes both the distribution of the stiffness and the degree of anisotropy are in addition a function of the state of deformation. This leads in general to non-linear constitutive laws. For soft membranes which can only bear membran stresses parallel to its center plane it is evident that only correspondingly small (from the theoretical point of view infinitesimal small) deformation steps can be described using the generalized Hooke's law. This incremental procedure of the experiments is equivalent to the application of Henky's definition of strain by which the actual given strain results using a simple integration procedure. For the determination of the in general 6 independent coefficients of the generalized Hooke's law the results of 6 independent deformation steps are needed.
From experience with such kind of membranes, especially with biomechanical experiments with skin, it is known that all quantities have a considerable wide range of scattering. Therefore, a sufficient high number of measurements are necessary allowing the application of the methods of least mean square fits. This method has to be applied two times. First, to determine the stresses in each procedure with respect to an applied coordinate system, and second, for the coefficients of Hooke's law.
Key words : mathematical modelling, anisoptropy, inhomogeneous structures, biomechanics

1 INTRODUCTION

As a generalization of the methods developed by [1, 2] a two-dimensional analysis is used for the determination of a model of a flat anisotropic and structural inhomogeneous tissue. With the help of a special device developed by the authors [3, 4] the discrete forces in 6 directions necessary to apply plane strain deformations can be measured. With this equipment it was possible to determine the degree of anisotropy for each single deformation step. With these results it was possible to build up the real anisotropic behaviour of the structural inhomogeneous specimen to a homogeneous anisotropic mathematical model and to establish are more realistic definition of stresses for soft tissues.
In the present paper the above mentioned method is described and results from biomechanical experiments on different parts of human skin are presented.

2 THEORETICAL BACKGROUND

1. Generalized Hooke's law

Because of the general nonlinear and rheological behaviour of soft tissues the stress responds after relaxation can be described only for very small (theoretically infinitesimal small) deformation increments using the generalized Hooke's law, (1) shows this law for the case of a plane strain loading.

$$
\begin{aligned}
de_{xx} &= a_{11} ds^{*}_{xx} + a_{12} ds^{*}_{yy} + a_{16} ds^{*}_{xy} \\
de_{yy} &= a_{21} ds^{*}_{xx} + a_{22} ds^{*}_{yy} + a_{26} ds^{*}_{xy} \\
de_{xy} &= a_{61} ds^{*}_{xx} + a_{62} ds^{*}_{yy} + a_{66} ds^{*}_{xy}
\end{aligned} \qquad (1)
$$

The asterisk on the stresses denotes the use of the modified stress quantity according to [5] (s. (2) and fig. 1):

$$ds^{*}_{ij} = l(j) ds_{ij} \qquad l(j) = {dA}/{dA*} \qquad (2)$$

where $l(j)$ is the orientation dependent density distribution of the load bearing fibers.

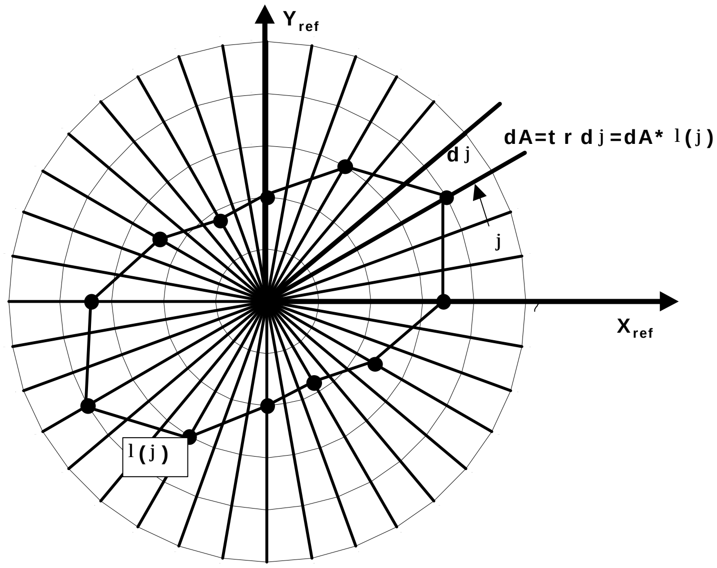

Figure 1: Functional cross-section area A*.

2. Determination of the coefficients a_{ij} from experimentally measured forces

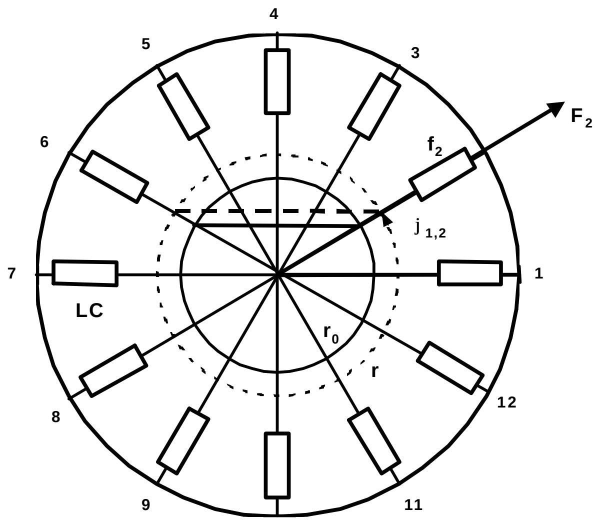

Figure 2: Scetch of the loading device. The situation is shown for the main-direction 2 (LC: load cells).

$$F_i = \sum_j^6 f_j \cdot \cos(\varphi_{ij}) \qquad (3)$$

Using (3) we have six equations to determine the forces $\mathbf{f_j}$ (fig. 2) from the measured forces $\mathbf{F_i}$. These forces as functions of φ can also be regarded as a representation of a density distribution λ(φ) given by (2).

To determine the three stress components $d\sigma_{ij}^*$ from six measured forces $F_i = F(\varphi_i)$ in six directions (fig. 2) we have an overdetermined measurement and may use a first least square procedure (4):

$$f_1 = \sum_{i=1}^{6} \left[\Delta\sigma_{xx}^* \cos^2\varphi_i + 2\Delta\sigma_{xy}^* \sin\varphi_i \cos\varphi_i + \Delta\sigma_{yy}^* \sin^2\varphi_i - \Delta F(\varphi_i)\right]^2 = \min \quad (4)$$

To determine the coefficients $\mathbf{a_{ij}}$ of the generalized Hooke's law we used a second least square procedure (5):

$$\begin{aligned} f_2 = \sum_{i=1}^{6} &\left[(a_{11}\Delta\sigma_{xx}^{i*} + a_{12}\Delta\sigma_{yy}^{i*} + a_{16}\Delta\sigma_{xy}^{i*} - \Delta\varepsilon_{xx}^{i})^2\right] + \\ &+ \left[(a_{21}\Delta\sigma_{xx}^{i*} + a_{22}\Delta\sigma_{yy}^{i*} + a_{26}\Delta\sigma_{xy}^{i*} - \Delta\varepsilon_{yy}^{i})^2\right] + \\ &+ \left[(a_{61}\Delta\sigma_{xx}^{i*} + a_{62}\Delta\sigma_{yy}^{i*} + a_{66}\Delta\sigma_{xy}^{i*} - \Delta\varepsilon_{xy}^{i})^2\right] = \min \end{aligned} \qquad (5)$$

3 EXPERIMENTAL PROCEDURE

Using a special developed loading and measurement device [5] we apply deformation steps in the following way. The starting point is a circular shaped tissue with a reference radius r_0 of 15 mm. In case of biomechanical tissues (e.g. skin) taken from the body shortly postmortem the reference shape is restored in a first experimental step [3, 4]. In a second procedure we applied to the first reference configuration (r_0=15 mm) "elliptical" states of incremental strains with fixed ratios a/r=1.0050 and b/r=1.0025 in each direction of the testing machine. Fig. 3 shows in principle the situation for a large nonlinear deformation.

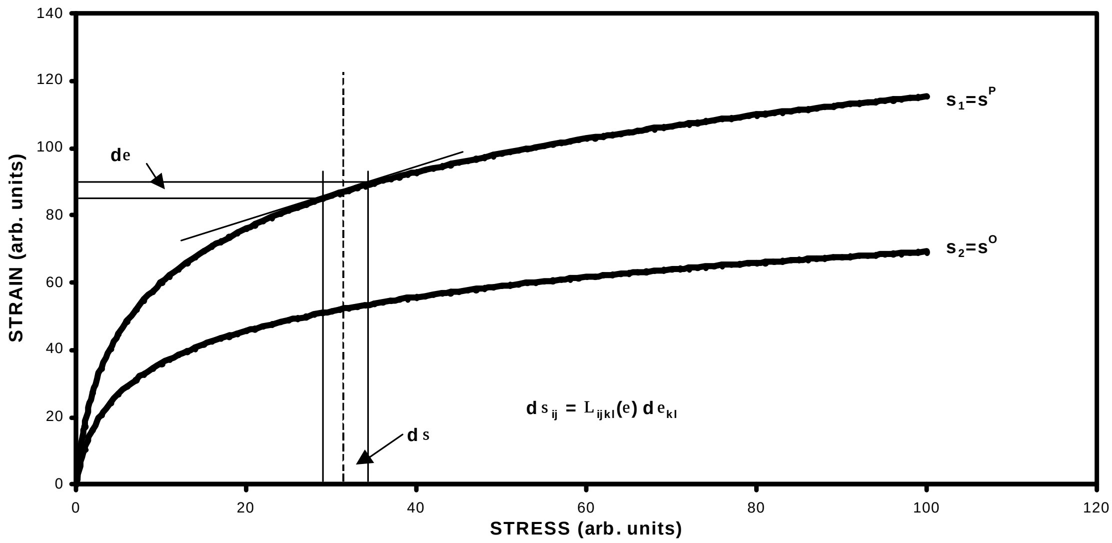

Figure 3: Main stresses (s_1: parallel, s_2: orthogonal to Langer lines) as a function of radial strains.

Keeping in mind that we usually deal with a material showing a rheological behaviour, the linearized Hooke's law can only be applied after the relaxation process of each deformation step is practically completed (figure 4).

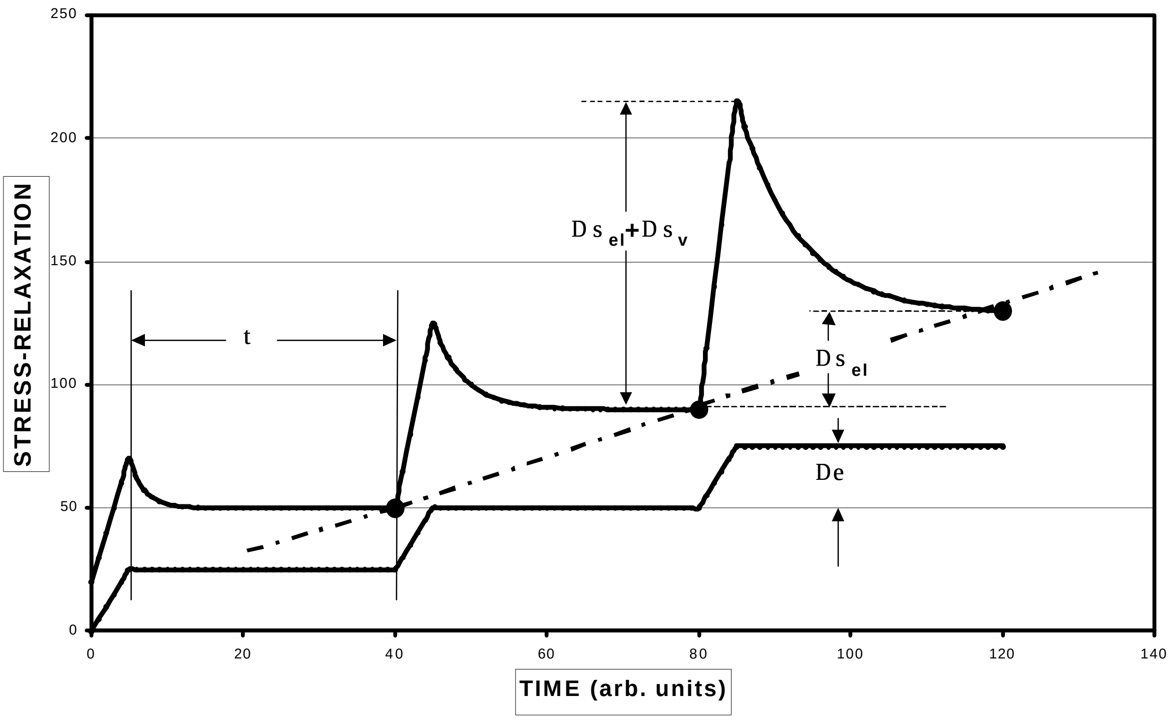

Figure 4: Strain controlled loading procedure (τ time for stress relaxation of each step)

With the result of this procedure the coefficients a_{ij} (1) can be determined for the first loading step using the least square procedure (5). In order to determine these coefficients also for higher levels of strain this procedure was repeated for the following circular reference configurations (r/r_0=1.025, 1.050, 1.075, and 1.100).

4 RESULTS AND CONCLUSIONS

Fig. 5.1 and 5.2 show the results $\lambda(\varphi)$ of the modelling in the described way for two specimens of human skin taken from the abdominal region and the forearm (palmar 10 cm distal from elbow) as determined on the basis of radial strain (r/t_0=1.10) gained by the first step of the procedure.

Figure 5.1: density distribution $\lambda(\varphi)$ of the fibers as a function of the direction in skin from the abdominal region for the radial strain level r/r_0=1.10.

Figure 5.2: density distribution $\lambda(\varphi)$ of fibers as a function of the direction in skin from the forearm region for the radial strain level r/r_0=1.10.

A comparison between these two results shows that in case of skin from the abdominal region (Fig. 5.1.) we have a fiber distribution with a preferred orientation whereas in case of the specimen from the forearm (Fig. 5.2.) we have a more or less uniform distribution of fiber directions (quasi isotropic situation).

In the second step described in point 2 elliptical states of strain were superimposed with the fixed ratios of a/r=1.0050 and b/r=1.0025 in the following directions α=0°, 30°,.., 150° to the chosen reference direction given by the surgeon. For each step the main direction for the state of stress was determined from the measurement using the fiber distribution resulting from the first step using (4). Fig. 6.1. shows for each deformation directions (–––) the corresponding main axis (—) of the resulting state of stress for the abdominal region. A comparison of these results with density distribution shown in fig. 5.1. shows that for the material main directions the deviation between main strain and main stress is nearly neglectible. Fig. 6.2. shows the same result for the palmar side of the forearm.

In terms of Young's moduli skin from the abdominal region shows a pronounced orthotropic behaviour whereas skin from the forearm display nearly isotropic behaviour.

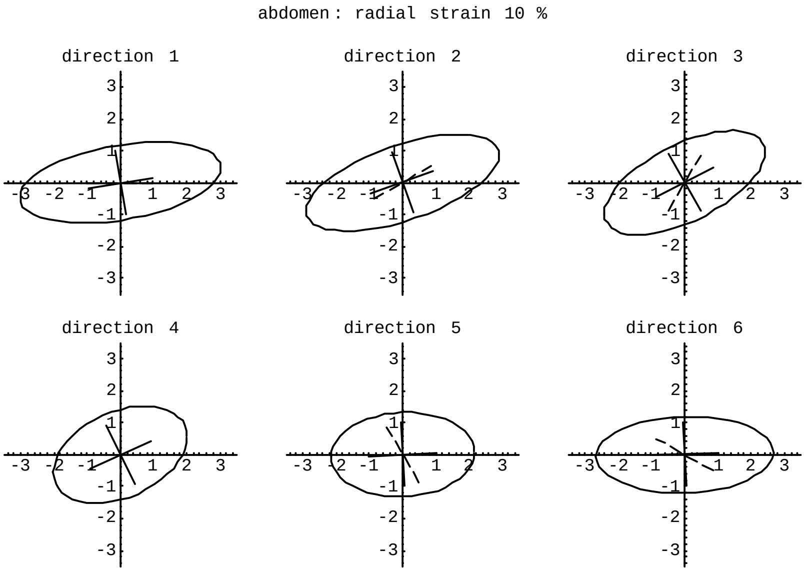

Figure 6.1: Relative magnitudes and orientations of the main stresses (—) as a function of the applied main strain directions (–––; dir 1, α=0°; dir 2, α =30°; dir 3, α =60°; dir 4, α =90°; dir 5, α =120°; dir 6, α =150°).

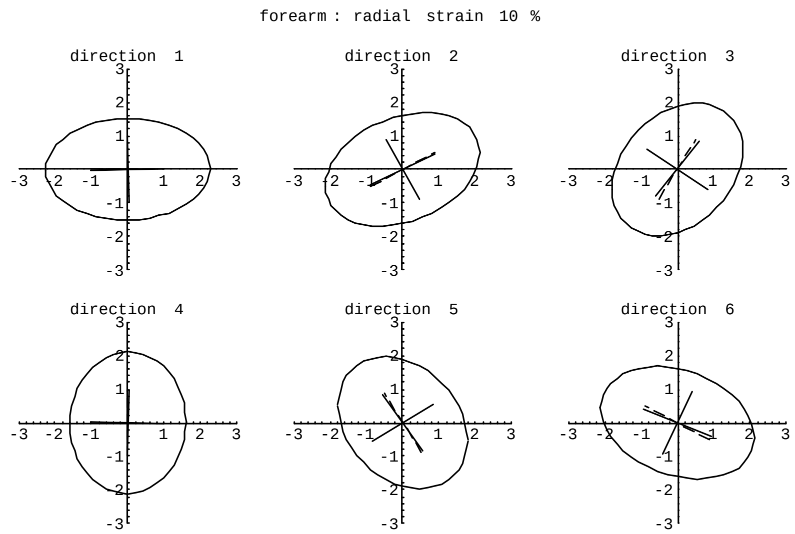

Figure 6.2: Relative magnitudes and orientations of the main stresses (—) as a function of the applied main strain directions (–––; dir 1, α =0°; dir 2, α =30°; dir 3, α =60°; dir 4, α =90°; dir 5, α =120°; dir 6, α 150°).

These results, obtained in this paper, show that even in cases where the real density distribution is not known (this is the usual situation) we are able to determine a theoretical density distribution $\lambda(\varphi)$, acting in the same way as the real one, for further investigation of the specimen.

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