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PARAMETERS SELECTION IN MULTIPOINT FORM MEASUREMENT

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Abstract: In the paper some ways of straightness and roundness measurements with multipoint methods are presented. They are based on workpiece surface measurement in limited number of regularly dislocated points and on reconstruction of an actual profile. For this purpose different interpolation methods can be applied: linear function, polynomial, trigonometric series, third order spline interpolation. The main point when using these methods is to choose a number of measuring points correctly. It depends on character of examined surface, applied interpolation method and acceptable measuring error. These methods can be used for coordination machines with touch trigger probe. Comparison tests were performed using a device for straightness and roundness referenceless measurements.
Keywords: straightness, roundness, CMM

1 INTRODUCTION

One of the most difficult problems taking place when analyzing responsible machine elements is form measurement. These difficulties come from the fact, that a specialized device ensuring accurate reference execution or a coordinate measuring machine with scanning probe must be used [1,2]. Measurements taken on such devices are usually time-consuming and require laboratory conditions. Yet, if tested workpieces are manufactured in big lots and in stable technological processes, they possess features characteristical for these processes and repeatable in statistical meaning. In these cases form measurements can be simplified using multipoint methods.

Multipoint methods base on measurements of workpiece shape deviations in small number of points and creation an actual profile with one of interpolation methods [3]. These research, regarding runout, roundness and straightness measurements were conducted in Division of Metrology and Measuring Sciences at Poznan University of Technology.

To establish to which workpieces multipoint methods can be used it is necessary to determine characteristical features of form profile, analyzing in statistical meaning a batch of elements on accurate devices, measuring full form profile with referenceless method.

Form measurements with multipoint methods can be taken also on coordinate measuring machines still more often used in industry. In more simple and cheaper configuration they are equipped with a touch trigger probe. In this case a probe touches the analyzed workpiece in a small number of points regularly dislocated on the tested area, what can shorten time of measurement in relation to traditional experiments and eliminate a necessity of purchase expensive specialized devices.

2 METHODICS OF MEASUREMENT RESULTS ELABORATION

Measured profile coordinates dislocated regularly on a profile, can be elaborated in many ways. First it is necessary to remove trend (in case of straightness) and than to compute a least mean square element (straight line, circle), that is a base to determine form errors. Finally, basing on the results performed as distances from the mean square element form profile between sampling points must be evaluated.

The simplest way of interpolation is the assumption of linear coordinate change between measuring points. In case of straightness it is a segment line, in case of roundness profile is a composition of arcs. Lets then make an interpolation using spline functions of the first order. Profile y-coordinates can be obtained from equation:

$$l_i(x) = ty_i + (1-t)y_{i-1} \tag{1}$$

where: $i = 1,2,\dots n$ n - n- number of measuring points

$$t = \frac{(x - x_{i-1})}{h_i} \quad \text{dla} \quad x \in [x_{i-1}, x_i]$$

$$h_i = x_i - x_{i-1}$$

and $\quad l_{i+1}(x_i) = l_i(x_i) \quad \text{dla} \quad i = 1, 2, ..., n-1$

The second way of profile approximation is polynomial interpolation. This can be done using for example Lagrange interpolation formula:

$$L_m(x) = \sum_{i=1}^{m} y_i \prod_{j=0}^{m} \frac{(x - x_j)}{(x_i - x_j)} \tag{2}$$

where: $\quad i, j = 0, 1, 2, ..., m \quad \text{and} \quad i \neq j$

In case of straightness m=n, i.e. the number of sampling points, in case of roundness on the other hand to obtain proper connection of profile in the first and the last point, the number of interpolation points m was increased below the first as well as above the last one (using coordinates changing in cycles), and for profile approximation only its middle part was applied.
The third method of profile approximation is application of trigonometric interpolation. Coordinates can be computed from formula:

$$T_n(x) = \frac{A_0}{2} + \sum_{k=1}^{n} \left[A_k \cos(kx) + B_k \sin(kx) + g\, \frac{A_{m+1}}{2} \cos[(m+1)x] \right] \tag{3}$$

where: $\quad g = 0; \quad m = \frac{n}{2}$ - for even n,

$g = 1; \quad m = \frac{(n+1)}{2}$ - for odd n

A_k, B_k - trigonometric polynomial coefficients

The last way of creating profile between measuring points is third order spline interpolation. In the case of straightness we additionally assume that in boundary profile points – the first and the last one – the second derivative is equal to zero, what means that the curve changes into a straight line behind a measuring segment. In case of roundness it is necessary to assume that the first and the last point has the same coordinate as well as the first and the second derivative in order to maintain profile continuity. The coordinates can be than computed from formula:

$$q_i(x) = t\, y_i + (1-t)\, y_{i-1} + h_i\, t(1-t) \left[(k_{i-1} - d_i)(1-t) - (k_i - d_i)t \right] \tag{4}$$

where: $\quad i = 1, 2, ..., n \qquad h_i = x_i - x_{i-1}, \qquad d_i = \frac{(y_i - y_{i-1})}{h_i}$

$$t = \frac{(x - x_{i-1})}{h_i} \quad \text{for} \quad x \in [x_{i-1}, x_i]$$

k_i - coefficients of spline function equation

and $\quad q_{i+1}(x_i) = q_i(x_i) \quad \text{for} \quad i = 1, 2, ..., n-1$

$$q'_{i+1}(x_i) = q'_i(x_i)$$

$$q''_{i+1}(x_i) = q''_i(x_i)$$

3 EXPERIMENT SETUP

Shape deviations measurement was performed on a specialized device for referenceless straightness and roundness deviations measurements. A number of cylinder liners used in different engines was tested (MWM, Volvo, Dural Dur, Peugeot, Mercedes, Ursus, Case). From a primary research material obtained that way workpieces that can be used for further multipoint measurements were selected.

During straightness measurements profile was recorded with 0,05 mm step. These data after averaging in neighboring points represent a profile using 240 – 256 points. Allowable coordinate measurement error is 1 µm / 100 mm.

During roundness measurement profile was sampled in 1024 points, and after averaging in 4 neighboring points we obtain 256 coordinate values, from which profile is obtained and deviations are computed. Roundness deviation measurement error does not exceed 0,35 µm.

The results of measurements (form profiles) were printed as well as transmitted to a computer as input data for simulation research perform in further stage of analysis.

Simulating multipoint straightness and roundness measurements using a computer, profile was divided into 3 to 30 equal segments. For every division actual profiles were recreated using all the four described methods of interpolation. Form error calculation was made from 1000 coordinate values obtained from interpolation, and least mean square elements just like in actual measurements were taken as reference. Thus the method was tested in details as well as profile representation ability basing on a limited number of points.

The next research stage enabling for a deeper analysis of error occurance reasons in measurement by means of multipoint methods, was additionally generating random errors in interpolation nets. Comparing calculations of form errors from the same input profiles with or without generating random errors their influence on form errors range can be assessed.

4 RESULTS AND DISCUSSION

Workpieces for which multipoint methods can be applied have regular shapes with small number of vertexes and low dominating harmonics. For good approximation of such profiles in case of straightness it is enough to divide a sampling length into 4 – 5 parts, and in case of roundness into 8 – 10 parts. Differences in form error values calculated from this number of divisions in relation to actual values are not bigger than about 8 – 10 %.

On figures 1 and 2 examples of straightness and roundness profiles on cylinder liners selected for analysis were presented as well as their recreation obtained by means of linear function and spline for 5 (straightness) and 8 (roundness) interpolation nets.

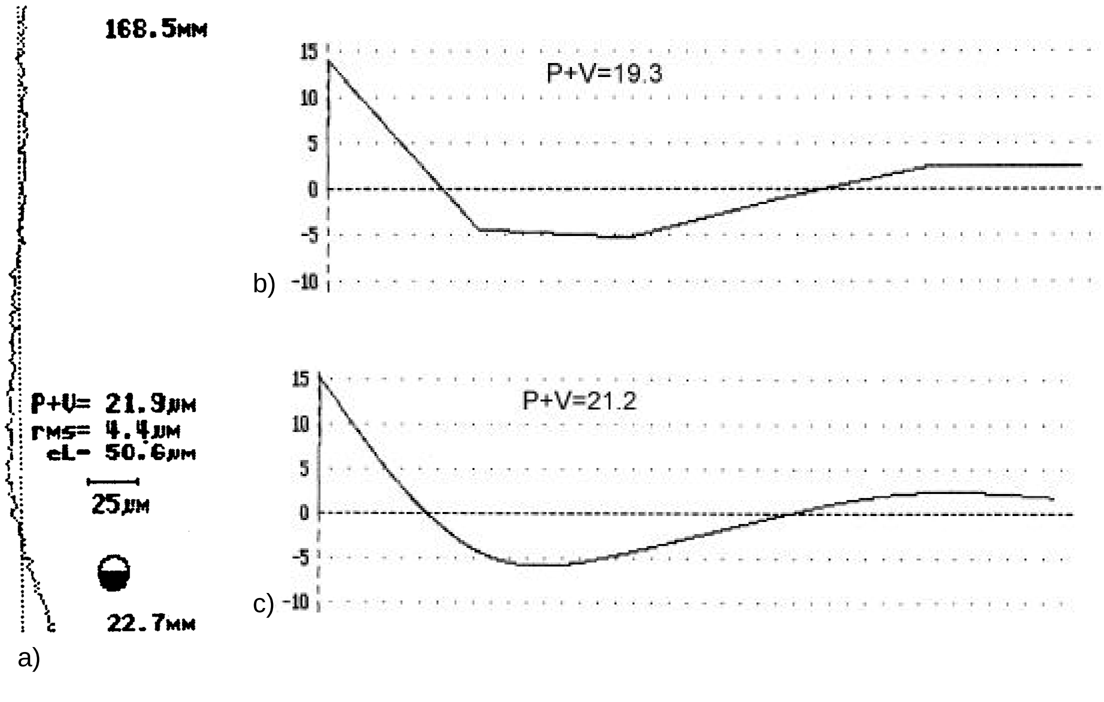

Figure 1. A comparison of actual straightness profile (a) with its recreation by linear function (b) and spline (c) when measuring in 5 points.

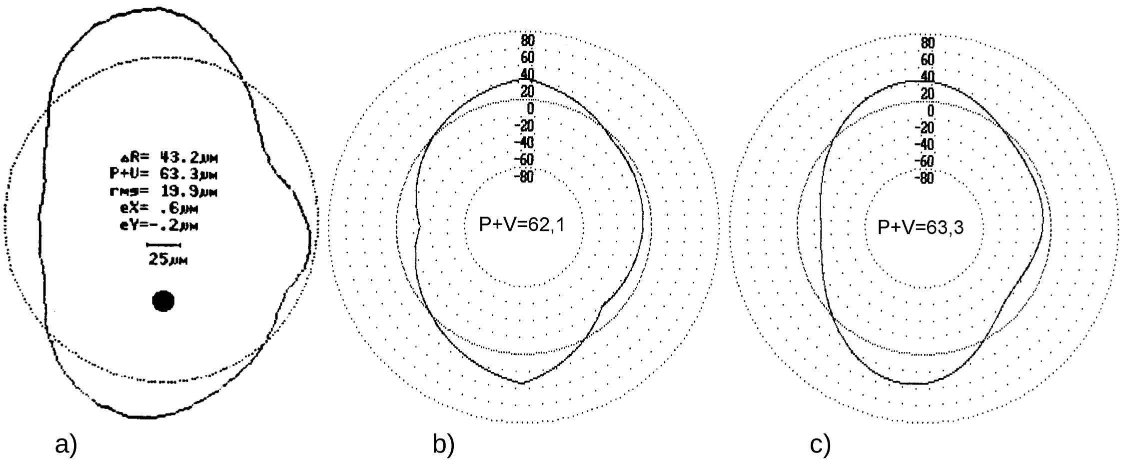

Figure 2. A comparison of actual roundness profile (a) with its recreation by linear function (b) and spline (c) when measuring in 8 points.

In table one, straightness error values for all types of interpolation and different number of sampling length division was presented.

Table 1. Straightness error values of cylinder liners from Fig. 2 computed by means of linear (L), polynomial (W), trigonometric series (T) and spline (S) interpolation for sampling length division into 3 – 30 parts.

n	L	W	T	S
3	19.0	21.6	22.0	20.5
4	20.5	22.4	25.6	22.5
5	19.3	21.1	23.2	21.2
6	20.5	21.0	22.9	21.0
8	20.9	21.3	21.9	21.3
12	21.1	23.7	23.3	21.7
20	21.3	-----	22.7	21.5
30	21.7	-----	22.7	21.8

Analyzing computations of form errors by means of multipoint methods one can say, that the best approximation in relation to actual values are obtained for third order spline function, i.e. for the smallest number of sampling length divisions it already shows good results. This interpolation gives also the smallest deviations when number of divisions n increases.

However, we also observe asymetry of form error values distribution computed with spline function, shifted some 4% towards smaller values. Thus, to obtain more reliable results it is necessary to multiply obtained values by 1,04.

In case of polynomial interpolation, for division number bigger than 10, we observe Runge phenomenon, what means appearance of high extreme values between interpolation nets, especially around the first and the last point.

It is than necessary to consider form errors computations with interpolation polynomial function for straightness measurement up to 8 divisions and for roundness and flatness up to 12 divisions.

Comparing form error values for three types of interpolation versus results obtained from third order spline interpolation the following rules can be observed:

– values obtained from linear function are approximately 1 - 5 % smaller for straightness and 1 – 4 % smaller for roundness,
– values obtained from polynomial function are approximately 2 - 6 % greater for straightness and 0 – 5 % greater for roundness
– values obtained from trigonometric series are approximately 5 - 10 % greater for straightness and 1 – 4 % greater for roundness

These rules are related to the number of divisions n where spline interpolation did not differ more than 10% in relation to actual values bearing in mind this 1,04 coefficient described above.

Using computer simulation of measurements with random errors generation in sampling points, absolute value of form errors can be determined. These functions are the following: when form deviation values P+V are small, equal to about 15 µm and values of generated random errors are +/- 5 µm distributions in form deviations are asymetrical form 0 to 8 µm, whereas for P+V increasing to 100 µm the distributions are symetrical, equal to +/- 8 µm with approximately linearly changed lower boundary. It is than possible to say that random errors in particular points generate form deviation computation errors some 60% bigger than they are.

To express relation between form profile character, measurement errors and number of divisions n a parameter called relative mean square form profile wavelength λ_{qw} was taken. It is computed the same way as a well-known roughness parameter λ_{qw} divided by sampling length. It can be expressed using the following formula:

$$l_{qw} = \frac{1}{l_k} 2p \frac{R_{qk}}{\Delta_{qk}} \tag{5}$$

where: $R_{qk} = \sqrt{\frac{1}{l_k}\int_0^{l_k} y^2(x)dx}$, $\Delta_{qk} = \sqrt{\frac{1}{l_k}\int_0^{l_k} y'^2(x)dx}$,

l_k - sampling length, $y(x)$ - form profile coordinate.

This parameter is a function of form profile smoothness. The bigger is the number of peaks and valleys the smaller values it has. Relative mean square wavelength was computed for form profiles after initial smoothing by third order polynomial function, in order to eliminate random errors. For roundness profiles λ_{qw} was included between 0,18 to 0,48 whereas for straightness from 0,25 to 0,80. Taking as reference form deviation measurement error of 10% it is possible to present graphically a relation between number of sampling length divisions and relative mean square wavelength λ_{qw}. For example fig. 3 shows this relation for straightness, for two types of interpolation: linear and spline. It is noticeable that this relation is approximate and averaged. It can happen that properly computed form errors are obtained from much smaller number of sampling length divisions or the opposite. This relation is than the smallest number of divisions n when proper results can be obtained, though – as it is clear from the above mentioned facts – λ_{qw} is not fully satisfying for appropriate determination of necessary number of divisions n.

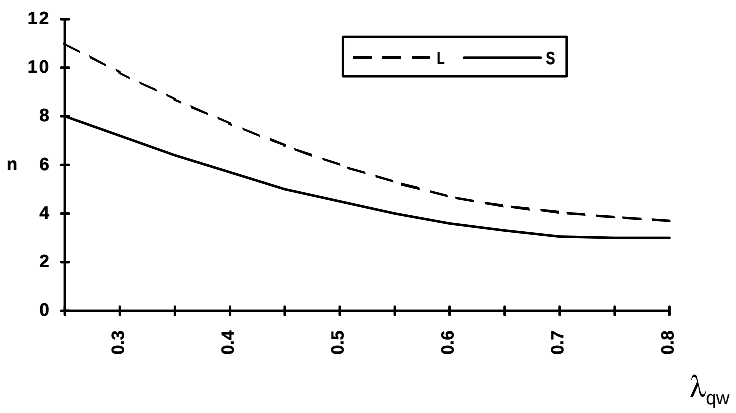

Figure 3. The smallest number of sampling length divisions n versus relative mean square wavelength λ_{qw} for straightness deviation measurement error up to 10%, for linear (L) and spline (S) interpolation

5 CONCLUSIONS

From the results shown above it is visible that application of multipoint methods for form error measurements can give good results for workpieces with regular profiles with low dominating harmonics and small number of peaks. They must also be characterized by big value of relative mean square wavelength l (above 0,15). In this case number of sampling length divisions will be smaller than 12 – 24 points, depending on type of expected form error.

Because workpieces shapes must be statistically repeatable their production should take place in stable technological processes in big lots.

The best representation of actual profile between sampling points are obtained for third order spline function. Comparing with other ways of interpolation it is the most stable when number of points n increases, show profile properly for the smallest number of points and computation algorithm is relatively fast.

Multipoint methods can be well applied on coordinate measuring machines with touch trigger probe because at one setting of workpiece measurement of all: dimensions, location and form errors can be computed. Moreover small number of sampling points does not increase time of measurement.

It is also necessary to remember about proper selection of measuring devices for their own random errors shouldn't influence much the result of measurement. As it can be seen the result of form deviations computation has error 60% bigger than random deviations in particular sampling points. If we assume a maximum error of 10% of the tolerance field the random error of measuring device cannot exceed 6% of workpiece tolerance field.

REFERENCES

[1] *Fertigungsmesstechnik. Handbuch für Industrie und Wissenschaft*. Herausgegeben von H.J. Warnecke und W. Dutschke. Springer – Verlag, Berlin, Heidelberg, New York – Tokyo 1984.
[2] Westkämper E., Michel S., Gente A., Lange D.:*Prozessnahe Rundheitsmeßtechnik mit 3-Punktmesßystemen.* Internationales DAAAM Symposium, TU KRAKÓW 1995, P. 363-364.
[3] Cellary A., Swornowski P., Wieczorowski M.: *Pomiary b³êdów walcowoœci na maszynie wspó³rzêdnoœciowej z ograniczon¹ liczb¹ punktów styku*. Politechnika Czêstochowska, Konferencje 13, XXVIII Miêdzyuczelniana Konferencja Metrologów MKM'96 vol I, p. 72 - 77, Czêstochowa 1996r.

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