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THE ALGORITHM OF THE GEARS MEASUREMENT WITH A CO-ORDINATE MEASURING MACHINE (CMM)

J. Chajda and M. Grzelka

Division of Metrology and Measuring Systems
Poznañ University of Technology, Poznañ, Poland

Abstract: In the article, the algorithm and the method of the cylindrical gear measurement with a Co-ordinate Measuring Machine have been presented. The CMM with appropriate software enables to measure quickly and accurately both simple details like corpses and complicated ones like gears. All geometrical parameters of a gear may be measured, and program simulating single flank gear testing enables the complex analysis of the gear and the pair of gears.

Keywords: complex gear measurement, Co-ordinate Measuring Machine, gear measurement accuracy, algorithm of the gear measurement

1   INTRODUCTION

Though gears are no more as widely applied as before, e.g. in machine tools, in most cases they still are in use. However, the high accuracy is required, therefore in a car gear-boxes gears are grinded, and for aircraft gears are lapped with special ceramic lapping tools. $\Rightarrow$ *QUALITY REQUIREMENS FOR GEARS INCREASE.*

Methods of the gear measurement develop. The measurement of the elementary deviations is not a 21st century technology. $\Rightarrow$ *SELECTION FOR MONTAGE PURPOSES MAY BE CARRIED OUT ONLY ON BASIS OF THE CINEMATIC DEVIATION MEASUREMENT (SINGLE FLANK TESTING).*

Nowadays in mass production is applied co-ordinate way of measurement. $\Rightarrow$ *CO-ORDINATE MEASURING MACHINES ARE COMMONLY USED.*

Hence, it is necessary to work out a method for the measurement of the elementary deviations of gears by means CMM which is based on metrological analysis of the measurement model with next step calculating cinematic and dynamical deviations.

2   WAYS OF THE DEVIATIONS MEASUREMENT WITH A CMM

The gear measurement by means a CMM with a pulse probe is the cheapest method. It does not require additional, expensive equipment like a turn-table, scanning probe. However, it is needed to apply an accurate measuring algorithm and meteorological analysis of the measuring method.
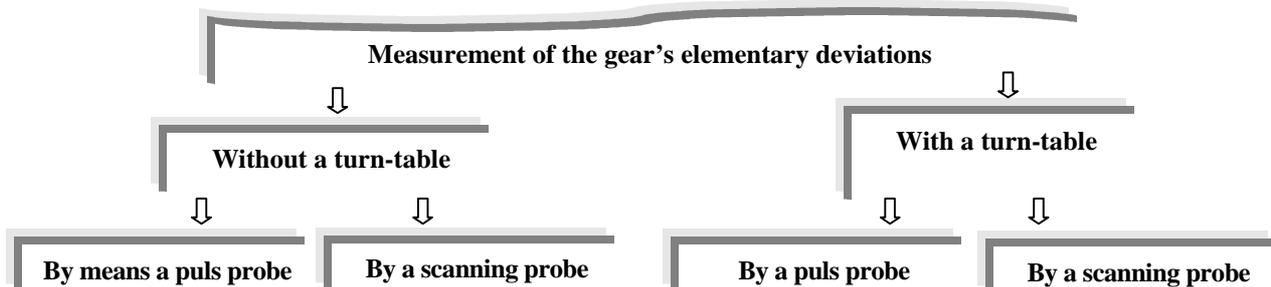
Measurement of the gear's elementary deviations

Without a turn-table		With a turn-table	
By means a puls probe	By a scanning probe	By a puls probe	By a scanning probe

Table 1.

3   ALGORITHM OF THE ELEMENTARY DEVIATIONS MEASUREMENT WITH A CMM

Before the measurement the analysis of the method and a necessary additional equipment should be carried out. During the measuring process the program leads the operator step-by-step though appropriate procedures according the algorithm.

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3.1   Analysis of the main and additional manipulations

3.1.1 The choice of the probe configurations

The configuration of the probes depends on the measuring task, method, type of the detail and equipment of the CMM. The measurement is complex, i.e. all of possible geometrical parameters of the gear are measured. Configuration of the probes is chosen paying attention to the gear type (is it inner or outer gear, cylindrical or conical and so on). The discussed algorithm, have been worked out for outer cylindrical gears with simple teeth with a module 6 to 12. The measurement have been carried out with an electrical-contact probe.

3.1.2 Calibration of the probes

Before any measurement with a CMM it is necessary to calibrate a probe in order to find out the diameters and distances between its centers. For this purposes is measured a standard detail with known diameter. In case we use several probes during the measuring process, we should calibrate each of them. The calibration should be carried out with highest accuracy, because any inaccuracy will put an error into further measurement.

3.1.3 Fastening

It is very important to fasten the gear in appropriate way depend on the gear type. The position must be stable and ensure contact of the probe with a gear (see the recommendations in next point).

3.1.4 Co-ordinates of the gear

Every CMM has got its own co-ordinates defined by producer. Ka¿da wspó³rzê dnœœciowa maszyna pomiarowa ma w³asny uk³ad wspó³rzê dnych zdefiniowany przez producenta. To make easy the process of the computations, analysis and results presentation it is recommended to define co-ordinates for each measured gear. The gear must be put inside measuring space of the CMM (fig. 1) and measurement should be carried out in the gear co-ordinates.
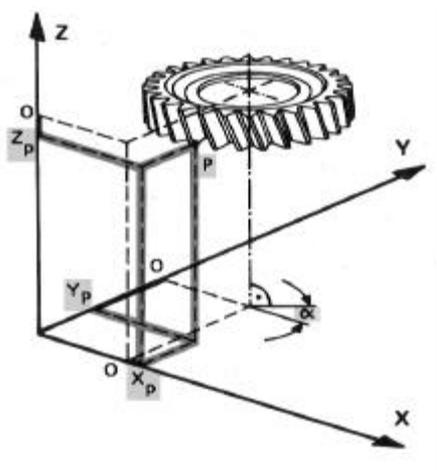

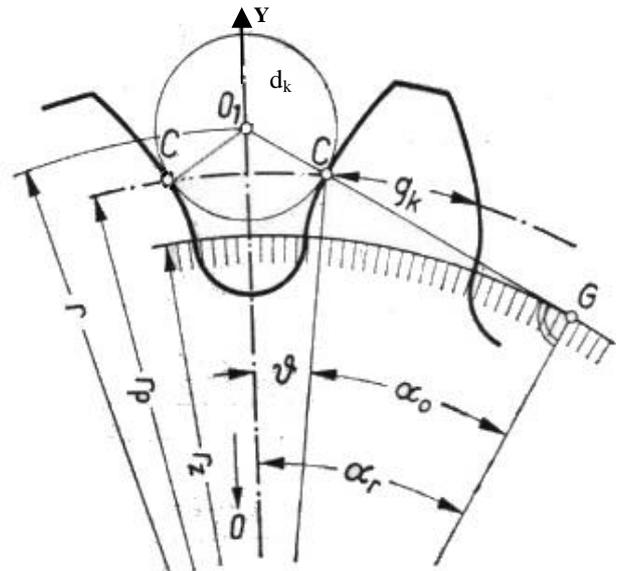**Figure 1.** Gear in the CMM co-ordinates

Figure 2. The gear with a measuring cylinder

Mainly it should be recommend: the Z-axis must be an axis of the turning of the gear, the flat X-Y must be defined that way to put a gear in the negative values of Z, X-axis should be defined using element of the flank flat of the gear, e.g. using an additional cylinder between the teeth.

The definition of the Y-axis should be discussed (X normal to Y in the 0 point). It was taken that the additional cylinder with a diameter d_k should be fastened to a CMM table (fig. 2). Its highness should be enough to measurement of involutes it touches (e.g. it should not be higher than 1/5 of the gear width), and its diameter should be close to the common cylinders used for gears measurement.

After measurement of the cylinder and pointing out its axis the gear is put in the way shown in the fig. 2. The measurement of the gear orifice gives an Y-axis and, hence, X-axis. This way the gear co-ordinates are pointed out according the recommendations.

$$d_k = 2\,r_p\left(\cos a_r - \sin a_0\right)$$

$$\hat{a}_r = \hat{a}_0 + \hat{J}$$

$$\hat{J} = \frac{m_0\,p - \hat{g}_k}{2\,r_p} \qquad \hat{J}^0 = \frac{180\,\hat{J}}{p}$$

3.1.5 Parameters of the gear

To define required parameters of the gear may be used data basis of the gear. It should be input: name of the object (number), number of teeth, module, type of teeth, pressure angle, direction of tooth, width of the gear, tooth highness coefficient, correction coefficient. The software computes geometrical values and output them in the screen or in the printer.

3.2 The algorithm of measurement and analysis steps for cylindrical gears

3.2.1 Co-ordinates
The master cylinder to fasten a gear is to be chosen according the formula:

$$d_k = 2\,r_p\left(\cos a_r - \sin a_0\right)$$

Then the master cylinder is fastened on the table.
The circle on the cylinder is measured and its center's co-ordinates $O_w(x_w, y_w)$ are calculated (it is recommended to apply min. 16 measuring points on the circle).
The gear is fastened on the CMM's table in such way, that its teeth connect with a master cylinder.
Then the diameter of the orifice of the gear is measured (it is recommended to apply min. 16 measuring points on the circle) and its center coordinates $O_o(x_o, y_o)$ are calculated. This point becomes a center of the gear's co-ordinates.
Next the side flat of the gear is measured and is considered as basic co-ordinates flat X-Y.
The line connecting the centers of the gear orifice $O_o(x_o, y_o)$ and of the master cylinder $O_w(x_w, y_w)$ is defined and then projected on the basic flat X-Y.
The axis Z is defined as a line including the central point $O_o(x_o, y_o)$ and normal to the flat X-Y.

3.2.2 Definition of the way of a probe and co-ordinates of the start points for each measurement point

The points $O_1, O_2, O_3, ..., O_z$, should be defined as a points from which starts a measuring movement to the measuring points (fig. 4).

$$r_p' = \frac{r_z}{\cos a_{rp'}} \qquad a_{rp'} = a_0 + J \qquad J^0 = \frac{90}{z}$$
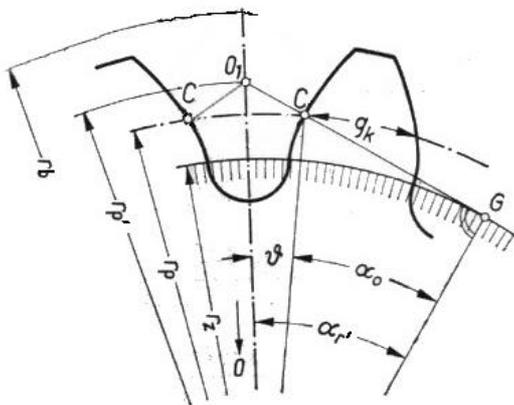
The angle between following points is 360°/z.

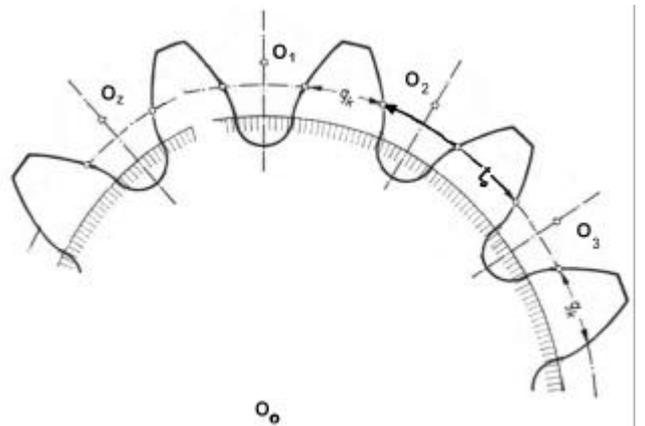**Figure 3.** The „safe" movement diameter r_b

Figure 4. Start points O_{1-z} on the orifice r_p

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The radius of "safe" movement of the probe r_b (fig. 3) is defined in the horizontal flat this way to make impossible the contact with gear material and to ensure the constant position in Z-axis. It is recommended to define the "safe" radius at least a probe diameter d_{kulki} larger then outer gear diameter.

$$r_b \geq \left(r_p + m \right) + d_{kulki}$$

3.2.3 Tooth thickness and circular pitch measurement

These two measurements are closely connected between one another in the CMM measurements. To point out these characteristics are used the same point. In the software, it is possible to divide them, but this article treat them together to make the description more clear.

First the probe is moved to the point O_1 (of course, all the start points coordinates are calculated before). The Z co-ordinate of the center of the probe is –B/2 (as the Z-axis is directed down, and the measurement is carried out in the flat in the middle of the gear width).

Then is carried out the first measuring movement along the vector:

$$\left[x_o + \left(\cos \left(\arcsin \left(\frac{x_p}{\sqrt{x_p^2 + y_p^2}} \right) + \arccos \left(\frac{r_b}{\sqrt{x_p^2 + y_p^2}} \right) \right) \right) ; \; y_o - \left(\sin \left(\arcsin \left(\frac{x_p}{\sqrt{x_p^2 + y_p^2}} \right) + \arccos \left(\frac{r_b}{\sqrt{x_p^2 + y_p^2}} \right) \right) \right) \right]$$

This formula is valid only for first quarter of the co-ordinates and the first measuring point on the left flank. The software contains formulas for every particular measurement, but it is not necessary to put them in this article. The movement of the probe along vector ends when the contact with the gear is reached. Co-ordinates of the probe center in the contact moment are input into the computer. There are two possibilities to treat these data. The first way is to calculate immediately the value (taking into consideration appropriate correcting coefficient). The second way is to put data in the memory and treat them after the whole measurement is finished.
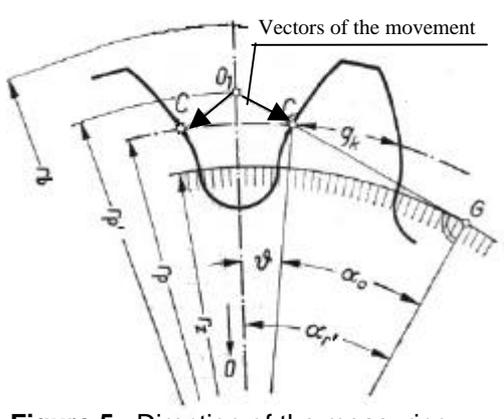

Figure 5. Direction of the measuring movement from the start point O_1
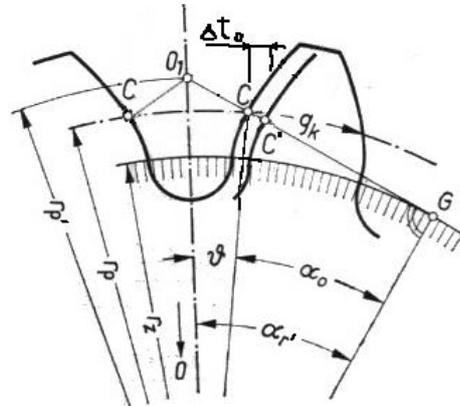

Figure 6. Influence of the pitch deviation on the measuring point's co-ordinates

After the contact is reached the probe returns to the start point and moves from there to the right flank of the next tooth. When the contact is reached and data transmitted, the probe returns to the start point O_{1-n} once more. Then the probe moves to the "safe" radius and goes to the next start point clockwise. The alternative way of measurement is to collect measurements on the left flank around the gear and then collect points on the right flanks. However, the first way enables to have two measurements from one start point, whish shortens the time of whole operation. It is only needed to prepare appropriate way of data record to make it possible to identify and use them correctly later.

Treated data are corrected in two steps. The first step is that co-ordinates of the probe's center are projected on its outer diameter along the movement vector:

$$\left[x_o + \left(r_{kulki} \cos \left(\arcsin \left(\frac{x_p}{\sqrt{x_p^2 + y_p^2}} \right) + \arccos \left(\frac{r_b}{\sqrt{x_p^2 + y_p^2}} \right) \right) \right) ; \; y_o - \left(r_{kulki} \sin \left(\arcsin \left(\frac{x_p}{\sqrt{x_p^2 + y_p^2}} \right) + \arccos \left(\frac{r_b}{\sqrt{x_p^2 + y_p^2}} \right) \right) \right) \right]$$

The result is not a contact point, because every gear has its own deviation depending on accuracy. Therefore the second step is carried out. It is clear that circular pitch and tooth thickness are defined

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on the pitch circle as well as their deviations. In the algorithm, direction of the movement and the later correction of the co-ordinates of the probe's center to the contact point responses to the direction of the contact line (correction I). The fig. 6 shows that the measuring point will move under the pitch circle because of the deviation. It is incorrect in the pitch and tooth thickness measurement, so the second correction is necessary. It should put a measuring point to a pitch circle along a radius. The second step calculates co-ordinates of the 1st correction and the line connecting it with the zero point. Along this line may be moved corrected point.

All the measuring points are corrected in two steps: both left and right flank points. The difference is in change of the direction in the 1st correction. The analysis and digital simulation was carried out. On the basis of the corrected points put on the pitch circle should be calculated the angle between them. For the circle pitch there may be calculated both left and right pitch. With the angles, it is easy to calculate the pitch.

Then are calculated deviations by means comparison the measuring and corrected points with theoretical values: pitch deviations (left and right) and cumulated pitch deviation. The results are presented as a numbers file or as a graph depends on customer requirements.

3.2.4 Measurement of the profile and topography of the tooth

The operator chose the appropriate number of measured teeth. It is recommended, however, to put under analysis at least 4 teeth, each in other co-ordinates' quarter. The choice is automatic by program or manual by entering a tooth number. Then the program points out the way of the probe to the appropriate start points and measuring points. The accuracy of the measurement depends on the measuring points' number. Because of the pulse measurement, it is necessary to optimize the points' number. It depends on module that influence on the tooth highness. It is recommended to choose about 10 to 16 points.

The profile is measured on the contact height of the tooth. The distribution of the points may be based on the radius of start point (distance between the gear center and the start point) as shown in the fig. 7. The direction of the movement vector and the co-ordinate's correction has been discussed above. The data is recorded and then measurement repeated for the right flank. The analysis of the results collected during the tooth profile measurement is not finished yet. The profile deviations depend also on the basic circle radius deviation Δe_{zi} and involute angle α_0. The tooth profile is measured in the space between 1/5 B from upper and lower side. All the start points should lay on the pitch circle in

Figure 7. Distribution of the measuring points on the start point O_1 radius

the different section of the gear. The number of measuring points depends on the gear width. It is enough to chose 7 points (6 uniformly distributed on the tooth width + point of the pitch measurement). The operator should be able to change the number of points, but the program has to check if this number will work appropriately. The direction of the movement vector from the start point to the measuring point is the same, of course, the Z-axis must not be changed. The collected data are corrected (1st and 2nd correction) and compared to the line parallel to the Z-axis. Any deviation from this line is an error, presented as a numbers or graphs.

Topography of the tooth is the sum of the tooth profiles (fig. 8). The algorithm is repeated for both methods. The radial run-out is determined by computing coordinates of the center of the inscribed circle between two teeth and its distance from the gear center. For each couple of gears are known co-ordinates of the start point O_{1-z} and two measuring points. The final result is the radial run-out and the vector of the maximum deviation direction.
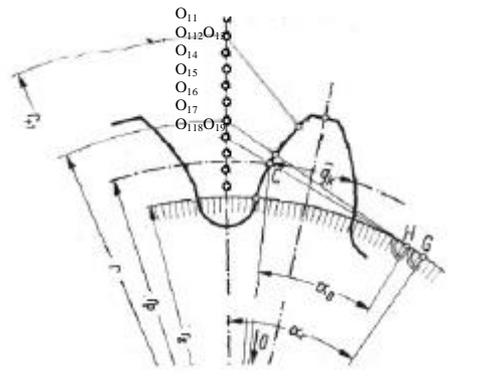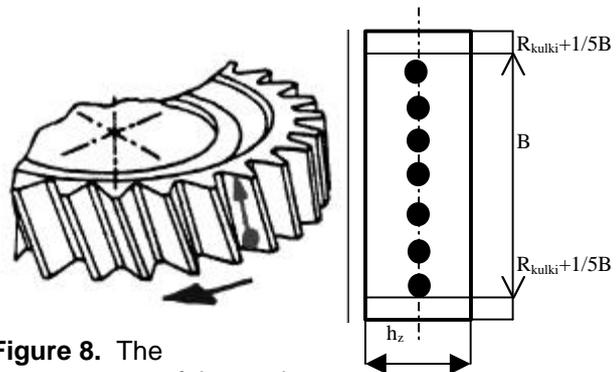

Figure 8. The measurement of the tooth

4   CONCLUSIONS

The measurement of the elementary deviations of the gear as a pitch, tooth thickness and profile deviations may be used for calculation of *the cinematic and dynamic deviations*. The tooth thickness and profile deviations may be used for *the selection of the appropriate couples of gears depending on required backlash and load in the gear box.*

For better deviations' analysis it is possible to visualize them in the graph of topography (tooth profiles) or of distribution of the deviations on the whole gear.

It is possible to calculate cinematic and dynamic deviations on basis of the elementary geometrical deviations.

The measurement of elementary deviations and calculating of the cinematic deviations had lead to the requirements of the CMM and its pulse probe. The algorithm of its analysis depending on accuracy of the gear have been worked out in the Division of Metrology and Measuring Systems of Poznan University of Technology and adapted to a SOFTIOS software. Verification of this method is to be carried out by means modernized Co-ordinate Measuring Machine KEMCO 600.

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AUTHORS: Prof. Dr. Eng. J. CHAJDA, Mgr. Eng. M. GRZELKA, Division of Metrology and Measuring Systems, Institute of Mechanical Technology, Poznan University of Technology, ul. Piotrowo 3, 60-965 Poznan, Poland, Tel. (+4861) 6653569, Fax (+4861) 6653570, E-mail: jan.chajda@put.poznan.pl and miroslaw.grzelka@put.poznan.pl