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SURFACE FITTING BASED ON OPTICAL 3-D MEASUREMENT

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Abstract: The aim of reverse engineering is to create a computer-aided design (CAD) model of an existing physical object. This paper deals with technology for acquiring measured point data by use of an optical 3-D measuring system. This system is suited for measuring objects, at a high speed and in a high density, which have smooth shapes and character areas with steep slopes. Smooth curves are approximated by fitting non-uniform cubic B-spline curves with curvature taken into consideration. Free-form surfaces in higher-order continuity are derived from the fitted B-spline curves as the boundary curves of patches. A strategy for remeasurement is proposed to extract character areas in order to generate a series of free-form surfaces, and techniques are also proposed to connect fitted B-spline curves with surfaces by use of pseudo vertices. The results of the experiments show that the proposed methods are effective for generating free-form surfaces.
Keywords: Optical 3-D measurement, Free-form surface, Fitting

1 INTRODUCTION

In style production, designers can create a new design and/or modify the design of an existing physical object by putting the data about its shape into a computer and modifying them. The industrial term for this process is reverse engineering [1][2]. In reverse engineering, it is important to measure the shape of a physical object at a high speed and in a high accuracy. For this purpose, a non-contact measurement sensor is used, because the sensor utilizes laser light and high-speed and high-density measurement is possible with it.

Here is the procedure of reverse engineering. An original physical object is made of wood or clay, its shape is measured with the non-contact 3-D digitizing system and the coordinate measuring machine (CMM) [3], and the measured point data are fed into a CAD system to be processed for further downstream activities.

The authors propose a new measurement strategy and an algorithm for fitting free-form surfaces onto the measured point data about a physical object which have character areas with steep slopes. The shape of the physical object is measured in a high density with the optical 3-D measuring system. Non-uniform cubic B-spline curves are fitted onto measured point data on measuring lines by use of curvatures. Free-form surfaces are generated with Coons patches in a C^2 continuity. To make a quality CAD model, patches should be small in the character areas, so a strategy for remeasurement is proposed to extract character areas. In order to generate a series of free-form surfaces, techniques are also proposed for connecting fitted B-spline curves with the free-form surfaces by use of pseudo vertices. Inspection experiments with the measured point data obtained thus confirm that those methods are available.

2 MEASURING STRATEGY WITH OPTICAL 3-D MEASURING SYSTEM

When the shape of a physical object is measured, the resulting measured point data are generally arranged in the form of regular grids. The pitch of measured point data must be minimized in character areas so as to produce surfaces of high quality. In a conventional method, too many measured point data are obtained and too much time is spent on both measurement and modeling. Figure 1 shows the optical 3-D measuring system. The system is composed of a 3-D digitizing laser sensor, a CNC machine tool, a 3-D shape measuring controller, a personal computer which controls and monitors measuring condition, and an engineer workstation for processing huge numbers of measured point data. The 3-D digitizing laser sensor has a measuring range of 20 mm and a resolution of 11 ∝m/pixel, and it can read the displacement of a target surface, within the accuracy of ± 50 µm, sloping steeply up to ± 70 degrees without causing a dead angle called a shadow effect [4]. With this system, shape measurement of about 2000 point/min is possible at a speed faster than 4 m/min. The reason is that in this system, the sensor is automatically moved up and down by servomechanism so that the light spot to be detected may be located within the measuring range of the sensor while the CNC machine tool is traversing.

Figure 2 shows the measured point data obtained in a boundary high-density measuring method. When the shape of a physical object is measured in this method, the resulting measured point data are dense on measuring lines. The pitch of the measuring lines is set relatively longer than the pitch of the measured point data. The method adopts large meshes composed of measuring lines which are obtained from dense measured point data. The method can help decrease the number of measured point data.

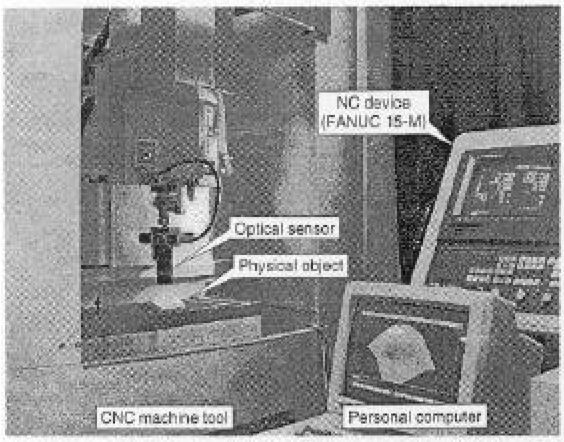


Figure 1. Optical 3-D measuring system

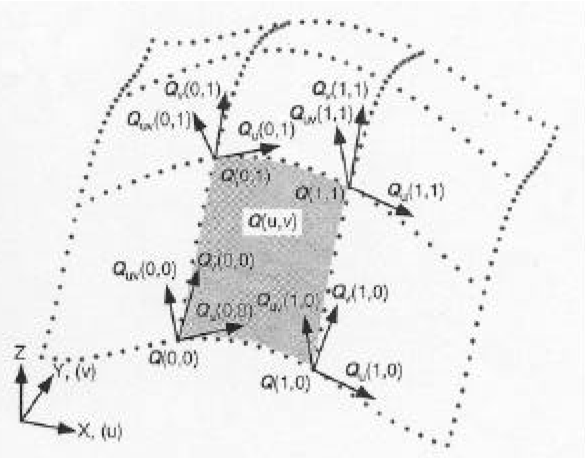


Figure 2. Measurement in a boundary high-density measuring method

3 PRODUCING FREE-FORM SURFACES FROM MEASURED POINT DATA

Free-form surfaces are produced in the following steps: (a) with the curvature taken into consideration, a non-uniform cubic B-spline curve is fitted onto the high-density measured point data on a measuring line; (b) free-form surfaces are produced by use of Coons patches, and their boundary curves become fitted cubic B-spline curves.

3.1 B-spline curve fitting

An m order ($m-1$ degree) parametric B-spline curve is given by

$$\mathbf{C}(t) = \sum_{i=0}^{n} N_{i,m}(t)\mathbf{V}_i \tag{1}$$

where the $n+1$ is the number of control points and $\mathbf{V}_i\,(i = 0,1,\cdots,n)$ are the control points in three dimensions. $N_{i,m}(t)$ are evaluated usually by calculating the values of the basis function from the well known Cox-de Boor algorithm[5], and are defined as

$$N_{i,1}(t) = \begin{cases} 1 & (x_i \le t < x_{i+1}) \\ 0 & (\text{otherwise}) \end{cases} \tag{2a}$$

$$N_{i,m}(t) = \frac{t - x_i}{x_{i+m-1} - x_i} N_{i,m-1}(t) + \frac{x_{i+m} - t}{x_{i+m} - x_{i+1}} N_{i+1,m-1}(t) \tag{2b}$$

The sequence values x_j $(j = 0,1,\cdots,m+n)$ are the elements of a knot vector satisfying the relation $x_j \le x_{j+1}$, and knots are chosen so that $0.0 \le x_j \le 1.0$. In Equation (1), the $\mathbf{V}_i\,(i = 0,1,\cdots,n)$ are the unknown control points necessary to fitting a B-spline curve onto the known measured point data. Control points are calculated from the B-spline inverse translation performed in the least square method which uses measured point data $\mathbf{P}_k(x_k, y_k, z_k)\,(k = 1,2,\cdots,q)$ and data points $\mathbf{C}(t_k)$ $(k = 1,2,\cdots,q)$ supposed to lie on a resulting B-spline curve. If a data point lies on the B-spline curve, then it must satisfy Equation (1). The results are a set of simultaneous linear algebraic equations written in matrix form as follows:

$$[\mathbf{C}] = [\mathbf{N}][\mathbf{V}] \cong [\mathbf{P}] \tag{3}$$

When the number of measured point data is q, measured point data $[\mathbf{P}]$ and data points $[\mathbf{C}]$ are $q \times 3$ matrix, $[\mathbf{N}]$ is $q \times (n+1)$ coefficient matrix of the B-spline basis functions and $[\mathbf{V}]$ is $(n+1) \times 3$ matrix of the unknown defining control points. In general, $[\mathbf{N}]$ is not a square matrix. Control points are calculated from the B-spline inverse translation based on the least square method. Therefore, the control points, obtained by solving $[\mathbf{V}]$ in the normalized Equation (3), are given by

$$[\mathbf{V}] = \left([\mathbf{N}]^T [\mathbf{N}]\right)^{-1} [\mathbf{N}]^T [\mathbf{P}] \tag{4}$$

In this curve fitting, the parameters t vary from 0.0 to 1.0 along the curve $\mathbf{C}(t)$, and these parameter values can be assigned from the chord-lengths of measured point data. A knot must be placed on the character area of a form under consideration, because the knot vector affects curve fitting onto the measured point data. The knot vector is obtained in the "Equivalent curvature area dividing method"[6]. Curvatures are calculated from high-density measured point data, in accordance with the curve theory. This method enables us to obtain a non-uniform knot vector set in a high curvature area where knots lie in high density.

3.2 Production of free-form surfaces with Coons patches

Free form surfaces are produced by using Coons patches. Coons patches are generated on a region bounded by the fitted cubic B-spline curves on the measuring lines.

A Coons patch, defined from two-dimensional parametric uv-space, is given by[7]

$$\mathbf{Q}(u,v) = \mathbf{Q}_A(u,v) + \mathbf{Q}_B(u,v) - \mathbf{Q}_C(u,v) \tag{5}$$

where patch $\mathbf{Q}_A(u,v)$ is calculated from the blending functions for u direction, the boundary curves for v direction $(\mathbf{Q}(0,v), \mathbf{Q}(1,v))$ and the u-tangent vectors $(\mathbf{Q}_u(0,v), \mathbf{Q}_u(1,v))$; patch $\mathbf{Q}_B(u,v)$ is calculated from the blending functions for v direction, the boundary curves for u direction $(\mathbf{Q}(u,0), \mathbf{Q}(u,1))$ and the v-tangent vectors $(\mathbf{Q}_v(u,0), \mathbf{Q}_v(u,1))$; patch $\mathbf{Q}_C(u,v)$ is calculated from the blending functions for u and/or v direction, the four position vectors $(\mathbf{Q}(u,v) : u = 0,1; v = 0,1)$, and the twist vectors $(\mathbf{Q}_{uv}(u,v) : u = 0,1\ ; v = 0,1)$. In this paper, the boundary curves are defined as the fitted cubic B-spline curves onto the measured point data. The tangent vectors are defined as the first derivative of the fitted B-spline curves. The blending functions are also defined as 5-degree Hermite polynomials in order to generate continual surfaces of C^2 continuity. The twist vectors are calculated in the Adini method which takes adjacent patches into consideration´[8].

3.3 Remeasurement strategy and storing patch structures

The patch size affects the accuracy of surfaces produced from measured point data. The accuracy of produced surfaces having large-size patches become low in the character areas that are the changed forms of a physical model. To produce quality surfaces, the patch size must be minimized. But when the size becomes small, there will be more point data to be measured, and this will cause as much trouble as in the conventional method. Therefore, in patch domains having large fitting errors on the produced surfaces, a physical model is remeasured and divided into four new domains. The character areas are extracted by use of the four extended patches[9].

For example, a patch (Patch(9)) which is the object of remeasurement is divided into four smaller patches ((9-a), (9-b), (9-c) and (9-d)), as shown in Figure 3. A uniform cubic B-spline curve is fitted onto the measured point data that are obtained by the remeasurement. Thus, the fitted B-spline curve is connected to the produced patch surfaces (e.g. Patch(4) and Patch(8)) in a connecting method which will be explained in 3.4. In the domains of subdivided patches, surfaces are produced in the generating method, as detailed in 3.2.

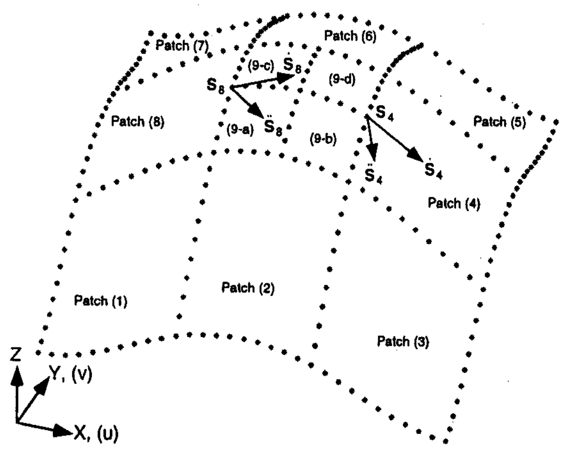


Figure 3. Remeasurement strategy

This remeasurement process is performed

recursively until the surface fitting error becomes within the tolerance of 0.10 mm. Subsequently, the produced surfaces are composed of hierarchical patches. Thus, the measured point data, the fitted B-spline curves, and the produced surfaces are arranged in the method of quadtrees[10].

3.4 Conditions of B-spline curve ends and connections with pseudo vertices

Remeasurement is performed to obtain measured point data on more measuring lines that can continue to extend, crossing other existing lines and dividing a low-accuracy domain into smaller domains. Hence a patch is subdivided into smaller ones. The uniform cubic B-spline curve is fitted onto the measured point data given by the remeasurement, and the B-spline curve needs to be capable of being connected to the produced patches.

The control polygon is composed of the control vertices that are given $(n+1)$ points, i.e., $\{\mathbf{V}_0, \mathbf{V}_1, \cdots, \mathbf{V}_n\}$, as shown in Figure 4. The uniform cubic B-spline curve segments are defined by

$$\mathbf{C}_i(t) = w_{i-2}(t)\mathbf{V}_{i-2} + w_{i-1}(t)\mathbf{V}_{i-1} + w_i(t)\mathbf{V}_i + w_{i+1}(t)\mathbf{V}_{i+1} \quad (i = 1, 2, \cdots, n-1; 0 \le t \le 1) \tag{6}$$

where $w_{i-2}(t) = \left(-t^3 + 3t^2 - 3t + 1\right)/6$, $w_{i-1}(t) = \left(3t^3 - 6t^2 + 4\right)/6$, $w_i(t) = \left(-3t^3 + 3t^2 + 3t + 1\right)/6$ and $w_{i+1}(t) = t^3/6$. It can be seen that those vertices can be used to generate the curve segments, specifically $\mathbf{C}_2(t)$, $\mathbf{C}_3(t)$, $\cdots$, $\mathbf{C}_{n-1}(t)$.

The two pseudo vertices $\mathbf{V}_{-1}$ and $\mathbf{V}_{n+1}$ create the additional curve segments $\mathbf{C}_1(t)$ and $\mathbf{C}_n(t)$ [11]. Since both additional curve segments are defined by the unmodified B-spline curve formulation, they are of a normal size. Further, by use of pseudo vertices techniques, it is possible to generate a curve consisting of n segments whose number is of practical convenience since the control polygon has the same number of segments.

The position vector $\mathbf{C}_1(0)$, the first derivative vector $\dot{\mathbf{C}}_1(0)$ and the second derivative vector $\ddot{\mathbf{C}}_1(0)$ at the initial end of the curve $\mathbf{C}_1(t)$ are given by

$$\begin{cases} \mathbf{C}_1(0) = \left(\mathbf{V}_{-1} + 4\mathbf{V}_0 + \mathbf{V}_1\right)/6 \\ \dot{\mathbf{C}}_1(0) = \left(\mathbf{V}_1 - \mathbf{V}_{-1}\right)/2 \\ \ddot{\mathbf{C}}_1(0) = \mathbf{V}_{-1} - 2\mathbf{V}_0 + \mathbf{V}_1 \end{cases} \tag{7}$$

$\mathbf{C}_n(1)$, $\dot{\mathbf{C}}_n(1)$ and $\ddot{\mathbf{C}}_n(1)$ at the terminal end of the curve $\mathbf{C}_n(t)$ are given by

$$\begin{cases} \mathbf{C}_n(1) = \left(\mathbf{V}_{n-1} + 4\mathbf{V}_n + \mathbf{V}_{n+1}\right)/6 \\ \dot{\mathbf{C}}_n(1) = \left(\mathbf{V}_{n+1} - \mathbf{V}_{n-1}\right)/2 \\ \ddot{\mathbf{C}}_n(1) = \mathbf{V}_{n-1} - 2\mathbf{V}_n + \mathbf{V}_{n+1} \end{cases} \tag{8}$$

The uniform cubic B-spline curve $\mathbf{C}(t)$ is connected to the patches $\mathbf{S}_A$ and $\mathbf{S}_B$, as shown in Figure 4. Connecting conditions are the C^2 continuity, as defined by

$$\mathbf{P}_A = \mathbf{C}_1(0),\ \dot{\mathbf{P}}_A = \dot{\mathbf{C}}_1(0),\ \ddot{\mathbf{P}}_A = \ddot{\mathbf{C}}_1(0),\ \mathbf{P}_B = \mathbf{C}_n(1),\ \dot{\mathbf{P}}_B = \dot{\mathbf{C}}_n(1) \text{ and } \ddot{\mathbf{P}}_B = \ddot{\mathbf{C}}_n(1) \tag{9}$$

where the position vector is $\mathbf{P}_A$, the first derivative vector is $\dot{\mathbf{P}}_A$ and the second derivative vector is $\ddot{\mathbf{P}}_A$ on the patches $\mathbf{S}_A$, and $\mathbf{P}_B$, $\dot{\mathbf{P}}_B$ and $\ddot{\mathbf{P}}_B$ on the patch $\mathbf{S}_B$. Thus, the control vertices and the pseudo vertices of the uniform cubic B-spline curve are given by

$$\begin{cases} \mathbf{V}_{-1} = \mathbf{P}_A - \dot{\mathbf{P}}_A + \ddot{\mathbf{P}}_A/3 \\ \mathbf{V}_0 = \mathbf{P}_A - \ddot{\mathbf{P}}_A/6 \\ \mathbf{V}_1 = \mathbf{P}_A + \dot{\mathbf{P}}_A + \ddot{\mathbf{P}}_A/3 \end{cases} \tag{10}$$

$$\begin{cases} \mathbf{V}_{n-1} = \mathbf{P}_B - \dot{\mathbf{P}}_B + \ddot{\mathbf{P}}_A/3 \\ \mathbf{V}_n = \mathbf{P}_B - \ddot{\mathbf{P}}_B/6 \\ \mathbf{V}_{n+1} = \mathbf{P}_B + \dot{\mathbf{P}}_B + \ddot{\mathbf{P}}_B/3 \end{cases} \tag{11}$$

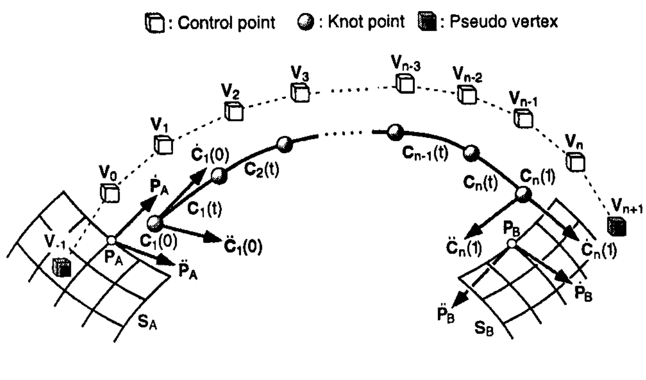


Figure 4. Connection of a B-spline curve with patches

4 AN EXAMPLE

Figure 5 shows the measured point data of a physical model having character areas. The conditions for the data are summarized in Table 1. Their pitches are 0.5 mm on the parameters, and the size of a patch to be produced is 4.0 mm×4.0 mm at the first measurement.

As shown in Figure 6, free-form surfaces are produced from the measured point data obtained at the first measurement and at the remeasurement. Fitted non-uniform cubic B-spline curves are represented by solid lines on the produced surfaces. The areas which must be remeasured are defined as the domains that possess surface-fitting errors larger than 0.10 mm.

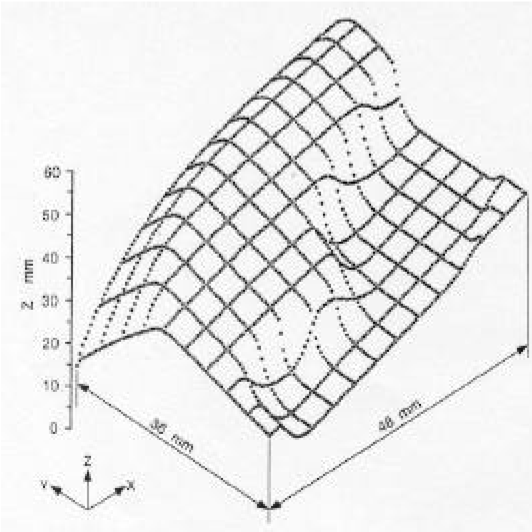


Figure 5. Measured point data of a physical model

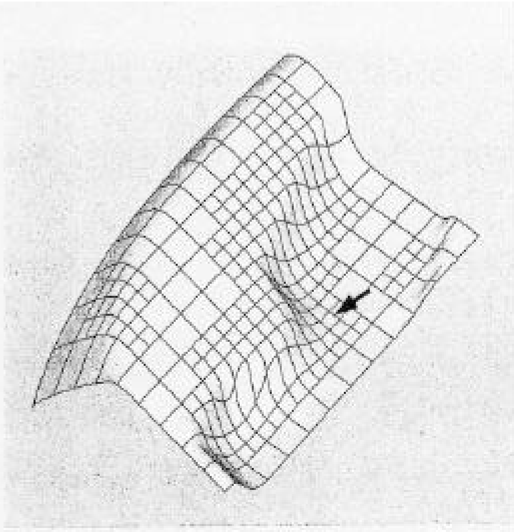
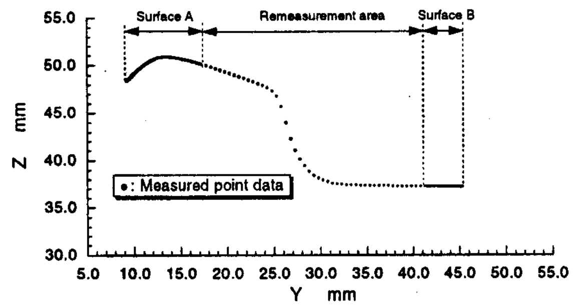
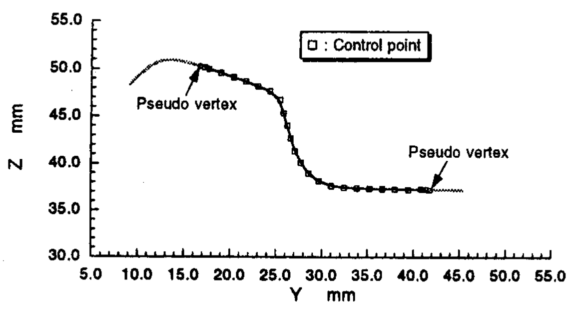


Figure 6. Produced free-form surfaces

Figure 7 (a) and (b) show an example of a fitted uniform cubic B-spline curve represented by an arrow head in Figure 6. Figure 7(a) shows the point data from the remeasurement and the produced surfaces (Surface A and Surface B). The pitch of measured point data is defined as the sampling length 0.5 mm, and the number of data obtained is 47 points. Figure 7 (b) shows the fitted uniform cubic B-spline curve by use of pseudo vertices. The standard deviation of curve-fitting errors for the fitted B-spline



(a) Produced surfaces and remeasured point data



(b) Fitted B-spline curve by use of pseudo vertices

Figure 7. Fitting a B-spline curve onto remeasured point data

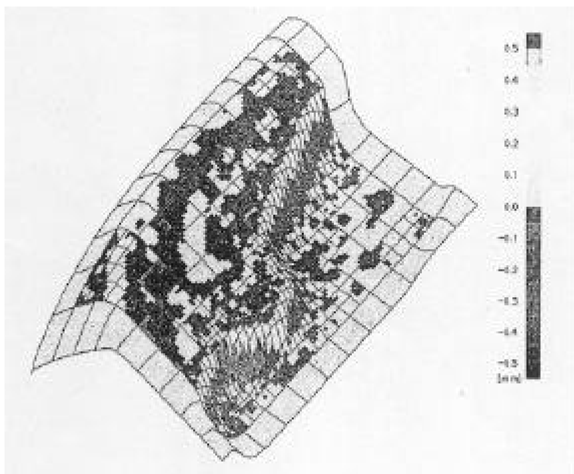


Figure 8. Produced free-form surfaces

curve is 0.0278 mm when 24 control points are used. A B-spline curve is fitted onto the remeasured point data and also is connected to the produced surfaces.

Table 1. Conditions of measured point data (at the first measurement)

Direction	u	v
Measuring size mm	48	36
Pitch of the measured point data mm	0.5	0.5
Pitch of the measuring lines mm	4.0	4.0
Number of the measured point data	97	73
Number of the measuring lines	10	13
The number of measured point data	1919	
The number of patches	108	

As shown in Figure 8, free-form surfaces can be produced by use of Coons patches. Fitted non-uniform and/or uniform cubic B-spline curves are represented by solid lines on the produced surfaces. The surface-fitting errors have been mapped on produced patch surfaces. In character areas, a patch is subdivided into four smaller ones, and the process is repeated three times. The standard deviation of surface-fitting errors on each patch is less than 0.10 mm.

5 CONCLUSION

The present studies propose a new strategy for high-density measurement with an optical 3-D system and new techniques for constructing free-form surfaces of C^2 continuity from measured point data. The results of the evaluation tests with the new strategy and the new techniques show that quality surfaces can be produced by use of the proposed new strategy and techniques for measurement and model production.

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