XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

DISCRETE LINEAR FILTERS FOR METROLOGY

M. Krystek [1]***, P.J. Scott*** [2] ***and V. Srinivasan*** [3]
[1] Physikalisch-Technische Bundesanstalt, Braunschweig, Germany
[2] Taylor Hobson Ltd., Leicester, U.K.
[3] IBM Corporation and Columbia University, New York, U.S.A.

Abstract: Linear filters are used quite extensively in the field of dimensional metrology and surface texture for smoothing data collected from measurements. Past application of linear filters was largely limited to the Gaussian and 2RC filters. But recent advances in spline and wavelet theories have opened the way for more flexible and powerful linear filters that are getting more attention in industry. This paper describes these advances in linear filters and how they are influencing the development of ISO filter standards.
Keywords: Metrology, Linear Filters, ISO Standards

1 INTRODUCTION

For the past four years, a technical group of international experts in the International Organization for Standardization (ISO) has been working on a series of ISO standards for geometrical filters [1]. This group has identified linear filters and morphological filters as the two major types of geometrical filters that require standardization. Technical details on morphological filters can be found in [2]. This paper deals with linear filters.

Linear filters are popular in industrial metrology. They are used mainly in the form of 2RC and Gaussian filters [3]. Recently other linear filters based on advances in spline and wavelet theories have been proposed and these have attracted industrial attention. These new filters enhance the toolset available to engineers in attacking problems of increasing complexity in profile and surface metrology. The ISO effort is directed towards issuing a series of technical specifications of these linear filters by first treating them under a unified, basic concept level [4] and then defining various specializations [5,6] illustrated with examples.

In Section 2 we give a brief introduction to linear filters. This is quickly followed in Section 3 by a more detailed description of discrete linear profile filters and their recent specializations - spline and wavelet filters. Section 4 offers a summary and some concluding remarks.

2 LINEAR FILTERS

Intuitively, linear filters map each point on a profile or a surface to a weighted average of points in its neighborhood. Mathematically, a linear filter can be defined as a linear operator on functions, such as

$$y(x) = \int_{-\infty}^{+\infty} K(x,s)\, z(s)\, ds$$

where *z(x)* is the unfiltered input profile, *y(x)* is the filtered profile, and *K(x,s)* is the kernel function with finite or infinite support. We require *K(x,s)* be real, symmetric and space invariant. The kernel is the weighting applied to "average" the input function. Different choices of the kernel lead to different linear filters. If *K(x,s) = K(x-s)* the filtering is a convolution given by

$$y(x) = \int_{-\infty}^{+\infty} K(x-s)\, z(s)\, ds$$

We consider only those kernels that lead to the convolution form as shown above.

For practical applications in metrology, the input profile or surface comes from sampled data. In these cases, it is advantageous to employ discrete linear filters.

3 DISCRETE LINEAR PROFILE FILTERS

An extracted profile can be represented by a vector. The length n of the vector is equal to the number of data points. The sampling is assumed to be uniform, i.e., the sampling interval is constant. The i-the data point of the profile is the i-th component of the column vector

$$Z = (z_1\, z_2 \ldots z_i \ldots z_n)^T$$

A filtered profile is also represented discretely by a column vector Y of length n.

A linear filter is given by an $n \times n$ square matrix A so that the filtered profile is obtained by a simple matrix multiplication as $Y=AZ$, which is often called the filter equation[1]. Elements of the matrix A are determined by the kernel K. If the filter is phase correct, then A is symmetric, i.e., $a_{ij} = a_{ji}$. The sum of the matrix elements in each row equals unity, i.e.,

$$\sum_j a_{ij} = 1$$

If the kernel is of the form $K(x\text{-}s)$, each row of the matrix is a mere shift of the one above so that the matrix elements may be represented by only one row

$$a_{ij} = S_k\,, \quad k = i - j$$

The values s_k form a vector S of length n. This vector is a discrete representation of the weighting function (kernel). Usually, S contains lots of contiguous zeros at both ends and the length of the non-zero part of S is much smaller than n.

Example 1: The moving average filter is frequently used for easy smoothing of a profile (not necessarily an optimal method). The weighting function of length 3 is given by

(… 0 0 1/3 1/3 1/3 0 0 …)

and the matrix A contains rows that are merely one column shifts of the above row vector. The result is simply that each component y_i in the filtered output vector is the arithmetic average of the components z_{i-1}, z_i, and z_{i+1} of the unfiltered input vector. Note that the averaging is very local.

Example 2: The Gaussian filter of [3] corresponds to a continuous weighting function

$$s(x) = \frac{1}{\alpha \lambda_c} \exp\left[-\pi \left(\frac{x}{\alpha \lambda_c} \right)^2 \right]$$

of infinite support, where x is the distance from the center (maximum) of the weighting function, λ_c is the so called cut-off wavelength, and α is a constant given by $\sqrt{(\log 2)/\pi}$ = 0.4697.. Figure 1 provides an illustration of this weighting function. This continuous function is sampled at equal intervals of Δx to yield

$$S_k = \frac{1}{C} \exp\left[-\pi \left(\frac{\Delta x}{\alpha \lambda_c} \right)^2 k^2 \right]$$

where the normalization constant is

$$C = \sum_k \exp\left[-\pi \left(\frac{\Delta x}{\alpha \lambda_c} \right)^2 k^2 \right]$$

The matrix A for Gaussian filters contains rows that are merely one column shifts of the row vector defined by s_k. The result is that each component y_i in the filtered output vector is the weighted average of all components of the unfiltered input vector Z. However, due to the rapid decline to zero of the weighting function $s(x)$ beyond the cut-off wavelength in either direction from y_i, the result is that the averaging of Y is neither global, nor as local as in Example 1. Figure 2 shows the result of applying Gaussian filter superposed on an unfiltered input profile.

Further details on basic concepts of linear profile filters can be found in [4].

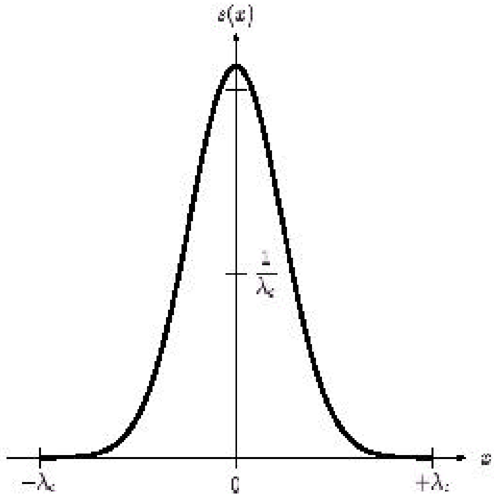

Figure 1. Gaussian weighting function.

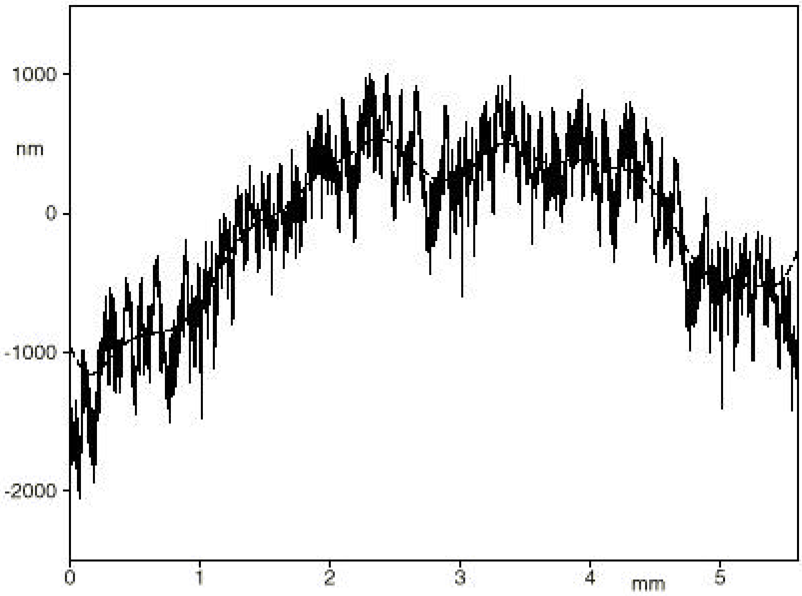

Figure 2. Output of the Gaussian filter superposed on unfiltered input profile, with λ_c=0.8 mm.

3.1 Spline Filters

As the name implies, spline filters are characterized by the use of spline functions to define the filtered output. Splines are low degree polynomials that can be pieced together to obtain very smooth functions that can be changed locally to suit the need. Recent experiments with spline filters have proven their flexibility and usefulness for industrial applications.

Spline filters are linear filters. However their weighting functions cannot be given in a simple closed form. Instead the filter equations are defined for the spline filters, and where necessary, a numerical calculation of the weighting function is provided.

<u>Example 3</u>: The filter equation for non-periodic open profiles using cubic spline is given by

$$(1 + a^4 Q)Y = Z$$

where Q is the *n´n* square matrix given by

$$Q = \begin{bmatrix} 1 & -2 & 1 & & & & & & \\ -2 & 5 & -4 & 1 & & & & & \\ 1 & -4 & 6 & -4 & 1 & & & & \\ & & \cdot & \cdot & \cdot & \cdot & & & \\ & & & \cdot & \cdot & \cdot & \cdot & & \\ & & & & 1 & -4 & 6 & -4 & 1 \\ & & & & & 1 & -4 & 5 & -2 \\ & & & & & & 1 & -2 & 1 \end{bmatrix}$$

and the parameter $\alpha = 1/(2\sin(\pi\Delta x/\lambda_c))$. If the sampling interval Δx is small enough, the weighting function *s(x)* can be approximated by the continuous function given by

$$s(x) = \frac{p}{l_c}\sin\left(\sqrt{2}\,\frac{p}{l_c}|x| + \frac{p}{4}\right)\exp\left(\sqrt{2}\,\frac{p}{l_c}|x|\right)$$

which is illustrated in Figure 3. It is instructive to compare this with the weighting function defined in Example 2 for the Gaussian filter and shown in Figure 1. Figure 4 shows the result of applying the cubic spline filter superposed on an unfiltered input profile.

Further details on spline filters can be found in [5].

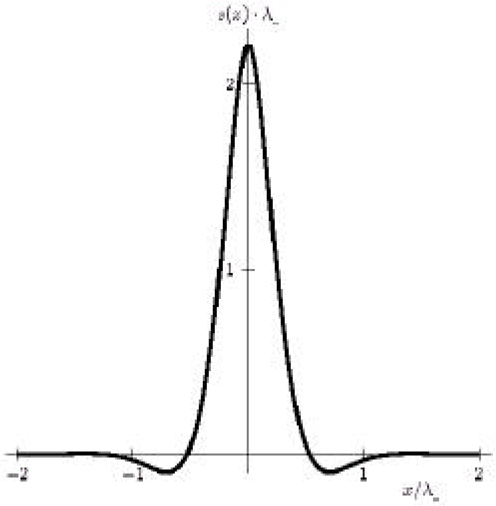

Figure 3. Weighting function for cubic spline filter.

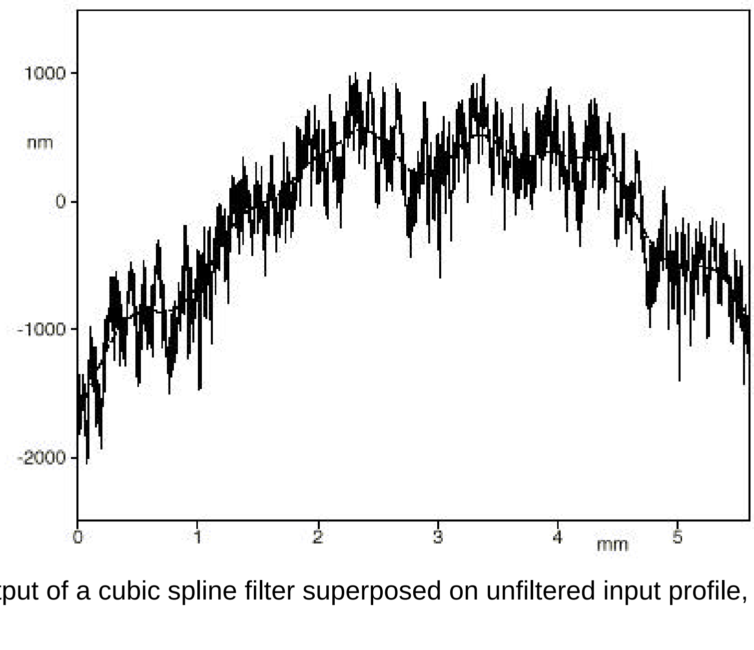

Figure 4. Output of a cubic spline filter superposed on unfiltered input profile, with λ_c=0.8 mm.

3.2 Wavelet Filters

Wavelets provide an attractive alternative to traditional Fourier analysis. In the field of metrology, their use has received recent attention because they can be used to perform multi-resolution analysis and they have good diagnostic capabilities.

Wavelet analysis consists of decomposing a profile into linear combination of wavelets $g_{a,b}(x)$, all generated from a single mother wavelet. A mother wavelet is a function of one or more variables which forms the basic building block for wavelet analysis. See Figure 5 for an example mother wavelet. Usually a mother wavelet integrates to zero and is localized in space with finite support.

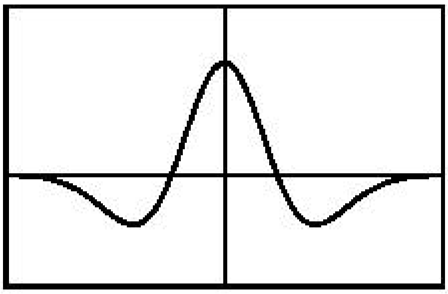

Figure 5. A mother wavelet.

If $g(x)$ is a mother wavelet, then the wavelet family is generated as $g_{a,b}(x) = a^{-1/2}\, g((x-b)/a)$ where a is the dilation (scaling) parameter and b is the translation parameter. This is similar to Fourier analysis, which decomposes a profile into a linear combination of sine waves, but unlike Fourier analysis wavelets can identify the location as well as the sclae of a feature in a profile. As a result they can decompose profiles where the small-scale structure in one portion of the profile is unrelated to the structure in a different portion, such as localized changes (e.g., scratches). Wavelets are ideally suited for non-stationary profiles. Basically, wavelets decompose a profile into building blocks of constant shape but of different scales.

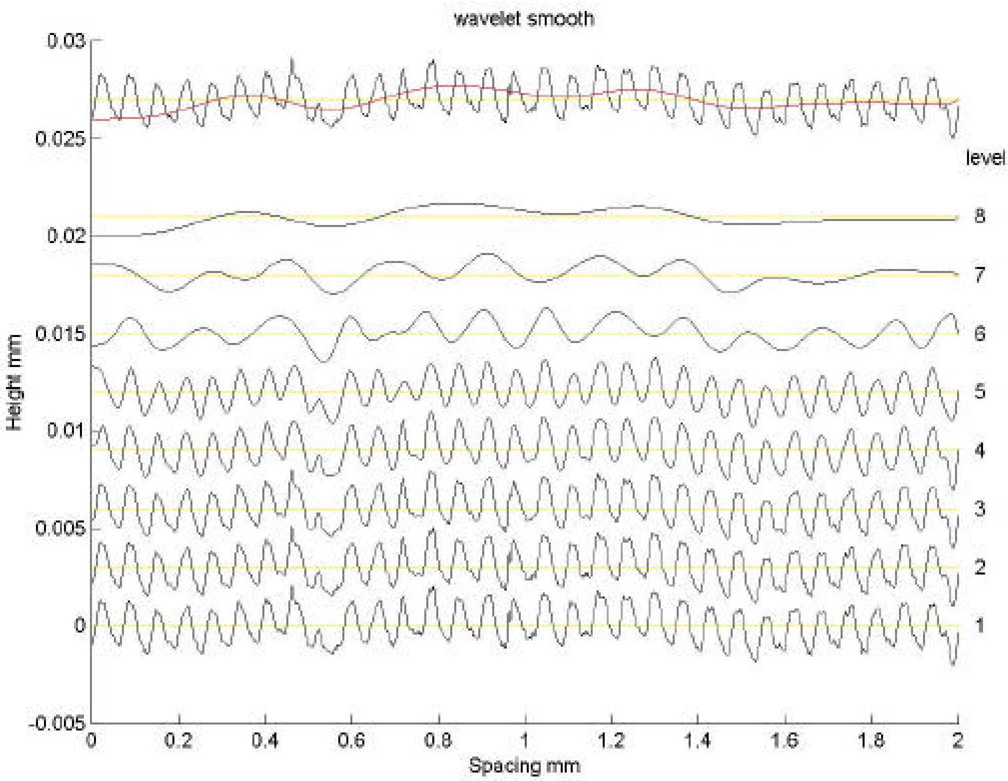

Figure 6. Successively smoothed profiles of a milled profile using (4-6) cubic spline wavelet.

The discrete wavelet filter is defined as

$$y_i(a) = \Delta x \sum_j z_j \, g_{a, j\,\Delta x}(j\,\Delta x)$$

with mother wavelet *g(x)*. The dilation parameter *a* is also restricted to discrete values. Typically consecutive values of *a* have a fixed ratio of 2. Of particular interest are spline wavlets, which are families of wavelets whose corresponding weighting functions are B-splines. Figure 6 shows a milled profile measured with a 5 µm tip stylus with successively smoothed profiles using a (4-6) cubic spline wavelet.

Further details on spline wavelet filters can be found in [6].

4 SUMMARY AND CONCLUDING REMARKS

In this paper we presented technical information on discrete linear profile filters that are being considered for ISO standardization. Ideas expressed here are extendable to discrete linear surface filters [7].

These linear filters include popular ones like the Gaussian filter and several of its variants. New linear filters such as spline filters and spline wavelet filters are being introduced in ISO metrology documents for the first time. Our intention is to introduce them first in the form of technical specifications. This will enable industry to try them on a wider scale and offer us an opportunity to evaluate its use. We envision that they will evolve into official ISO standards at a later date.

5 ACKNOWLEDGMENT AND A DISCLAIMER

We gratefully acknowledge the help and support of numerous colleagues in our national and international standards bodies. However, opinions expressed in this paper are our own and do not represent the official position of ISO or any of its member bodies.

REFERENCES

[1] V. Srinivasan, P.J. Scott and M. Krystek, ISO Standards for Geometrical Filters, *Proceedings of the 16th IMEKO World Congress*, Hofburg, Vienna, Austria, September 25-28, 2000.
[2] V. Srinivasan, Discrete morphological filters for metrology, in: P.H. Osanna, D. Prestrednik, N.M. Durakbasa (Eds.) *Proceedings of the 6th ISMQC IMEKO Symposium on "Metrology for Quality Control in Production"* (Vienna, Sept. 8-10, 1998), TU Vienna, Austria, 1998, pp. 623-628.
[3] ISO 11562:1996, Geometrical Product Specification (GPS) - Surface texture: Profile method - Metrological characteristics of phase correct filters, International Organization for Standardization, Geneva.
[4] ISO/TS 16610-2, Geometrical Product Specifications (GPS) - Data extraction techniques by sampling and filtration - Part 2: Basic concepts of linear profile filters. (Under preparation)
[5] ISO/TS 16610-3, Geometrical Product Specifications (GPS) - Data extraction techniques by sampling and filtration - Part 3: Spline filters. (Under preparation)
[6] ISO/TS 16610-4, Geometrical Product Specifications (GPS) - Data extraction techniques by sampling and filtration - Part 4: Spline Wavelets. (Under preparation)
[7] ISO/TS 16610-9, Geometrical Product Specifications (GPS) - Data extraction techniques by sampling and filtration - Part 9: Basic concepts of linear surface filters. (Under preparation)

AUTHORS: Dr. Vijay SRINIVASAN (Corresponding Author), IBM Corporation, M/S P535, Building 3, 522 South Road, Poughkeepsie, NY 12601-5400, U.S.A., Phone: ++914-433-6372, Fax: ++914-432-9608, E-mail: vasan@us.ibm.com , Michael KRYSTEK, Physikalisch-Technische Bundesanstalt, Braunschweig, Germany and Paul J. SCOTT, Taylor Hobson Ltd., Leicester, U.K.