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ABOUT VIBRATIONS OF COMPOSITE BAR IN PLAN MOTION

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Abstract: The vibrations analysis has become one of the most important parameters to be considered during the machines designing process, especially in the case of high velocities. The use of the composite materials that posses high resistance and low specific weight, represents a solution to the above mentioned problems. In this paper, the mathematical model for composite bars presenting constant section, is described by a equation. The dynamic response is obtained using the Laplace transform and the sinuses finite Fourier transform of the spatial variable. The experimental treatment are made with a BK2515 vibration analyzer, the measures being in the time mode (0…12.5) sec and in the frequency mode, in the interval (0…1000) Hz.
Keywords: Composite bar, Vibrations, Plan Motion

1 INTRODUCTION

Nowadays, under the pressure of industry it is necessary to develop composite parts more and more quickly and consequently, determine their mechanical behavior in still shorter times. The structural elements (bars) for automatic use or aviation, based on composite materials can cause certain problems in case of strong vibrations of the engine.

The mathematical modeling of the mechanical comportment is the quickest method to appreciate the behavior of the composite structure. Due to the application of the mathematical modeling, it was possible to demonstrate the external mechanical vibrations on the process parameters of the composite bars. Many of the dynamic studies have as object the vibrations of the structures made of composite materials. Some of the authors like Mead [1], Mead and Markus [2], Di Taranto [3], Yin and others [4] have made a theory based only on the transversal inertia of the tangential tensions from the section. Other authors like Di Sciuva [5], Yan and Dowell [6] have based on these effects too in the dynamic study of the composite bars.

More complex studies have been made by Miles and Reinhall [7] the ones who accepted a variation of the transversal deflection from the section, variation which determines the apparition of some compression efforts on the thickness, the final conclusion being that such mechanisms of solicitation have influence only at high frequencies.

2 THEORETICAL CONSIDERATIONS

The composite bars depending on the rapport between length and width, can be considered as Euler-Bernoulli or Timoshenko bars. In this paper, the mathematical model implies the following simplifying hypothesis:

- there are no distributed forces and external torque on the outside surface of the bar;
- no supplementary contact points or other shock generator factors will appear during the movement;
- initially, the bar is considered not to be tensioned;
- the considered section will remain plan, but it will not maintain the perpendicularity on the bar neuter axe.

Although the considered section may not remain plane during the deformation, it will be approximated to a plane, this approximation being made so as the considered tensions and deformations will be as closer to reality as possible.[8].

So, the rotation inertia of the section towards all the axes of the referential system will be taken into account and a medium value of the tangential tensions from the section will also be calculated.

The deduction of the mathematical model of the composite bar vibrations can be made using either Hamiltons' varying principle or the fundamental theorems from the classical mechanic.

The mathematical model for composite bars presenting constant section, is described by the equation:

$$[E]\{\ddot{q}\}+[G]\{\dot{q}\}+[L]\{q\}+[V]\{q\}_{,11}=\{H\} \qquad (1)$$

where: {q} represents the column matrix of the system output (spatial deformations).

The [E], [G], [L] and [V] matrices depend on the elastic and dimensional characteristics of the section and they also depend on bar rototranslation elements.

The [H] matrix includes the external solicitations and the inertial forces which influence the bar. The form and the meaning of every element on these matrix are presented in [9]. Also, the equation [1] and every component relation are presented in [10].

The solution of the mathematical model is made in the following hypotheses , which are valid for the time range of the study:

-the elements of composite bar's rototranslation are constants;

-the deformations and the accelerations of the bar at the extremities are nulls.

For the bars in plan motion, the vibrations equations can be separated by the torsion vibrations and the vibrations perpendicular on movement plan.

For each of these vibrations cases, the movement equations have the form given by equation (1), where:

- for vibrations in plan motion:

$$\{q\}=\begin{Bmatrix} u_1 \\ u_2 \\ q_{3,1} \end{Bmatrix};\quad [E]=\begin{bmatrix} <r\ A> & 0 & 0 \\ 0 & <r\ A> & 0 \\ 0 & -<r\ A> & <r\ I_3> \end{bmatrix};\quad [G]=\begin{bmatrix} 0 & -2<r\ A>w & 0 \\ 2<r\ A>w & 0 & 0 \\ -2<r\ A>w & 0 & 0 \end{bmatrix}$$

$$[V]=\begin{bmatrix} -<E\ A> & 0 & 0 \\ 0 & -<AG_2> & 0 \\ 0 & 0 & -<E\ I_3> \end{bmatrix};\quad [L]=\begin{bmatrix} -<r\ A>w^2 & -<r\ A>e & 0 \\ <r\ A>e & -<r\ A>w^2 & <AG_2> \\ -<r\ A>e & <r\ A>w^2 & -<rI_3>w^2 \end{bmatrix} \qquad \textbf{(2)}$$

$$\{H\}=\begin{Bmatrix} p_1-<r\ A>(a_{01}-w^2x_1 \\ p_2-<r\ A>(a_{02}+e\ x_1) \\ m_{3,1}-p_2+<r\ A>(a_{02}+e\ x_1) \end{Bmatrix}$$

- for torsion vibrations and the vibrations perpendicular on movement plan:

$$\{q\}=\begin{Bmatrix} u_1 \\ q_{1,1} \\ q_{3,1} \end{Bmatrix};\quad [E]=\begin{bmatrix} <r\ A> & 0 & 0 \\ 0 & <r\ I_1> & 0 \\ 0 & 0 & -<r\ I_2> \end{bmatrix};\quad [G]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2w<r\ I_2> \\ 0 & 2w<r\ I_2> & 0 \end{bmatrix}$$

$$[V]=\begin{bmatrix} -<AG_3> & 0 & 0 \\ 0 & -<GI\ > & 0 \\ 0 & 0 & -<E\ I_2> \end{bmatrix};\quad [L]=\begin{bmatrix} 0 & 0 & -<AG_3> \\ 0 & -<r\ I_2>w^2 & <r\ I_2>e \\ 0 & <r\ I_2>e & -<rI_2>w^2 \end{bmatrix} \qquad \textbf{(3)}$$

$$\{H\}=\begin{Bmatrix} p_3 \\ m_{1,1} \\ m_{2,1}+p_3 \end{Bmatrix};$$

In this case, the displacements and the bending moments on the end of composite bar are equal with

$$\{q\}|_{x_1=0}=\{q\}|_{x_1=l}=\{0\};\quad \{q\}_{,11}|_{x_1=0}=\{q\}_{,11}|_{x_1=l}=\{0\} \tag{4}$$

The dynamic response is obtained using the Laplace transform and the sinuses finite Fourier transform of the spatial variable :

$$\{q\}=\frac{2}{l}\sum_{n=1}^{\infty}L^{-1}\left\{\left[s^2[E]+s[G]+[L]-a_n{}^2[V]\right]^{-1}\left\{\{\overline{H}\}^*+[E]\{s\}\{q_0\}^*+\{q_1\}^*\right\}+[G]\{q_0\}^*\right\}\cdot\sin\frac{npx_1}{l} \tag{5}$$

where: s is the complex variable Laplace;
{H} is the Laplace transform of the {H} matrix;
$\{q\}^*$ is the sinuses finite Fourier transform of the {q} expression;
L^{-1} means the inverse of Laplace transform.

$$\{q_0\}=\{q\}|_{t=0};\{q_1\}=\left\{\dot{q}\right\}|_{t=0} \tag{6}$$

In the most general case, the cinematic parameters of bar movement are not constants. The solutions are approximate. In plan motion the following solutions are accepted:
- for longitudinal vibrations:

$$u_1(x_1,t)=\frac{2}{l}\sum_{n=1}^{\infty}\{\left(f_1^*(n)+\frac{w^2}{w^2-w_n^2}\cdot\frac{(-1)^{n+1}l}{a_n}\right)\cdot\cos\left(t\sqrt{w_n^2-w^2}\right)-\left(\frac{w^2}{w^2-w_n^2}\cdot\frac{(-1)^n l}{a_n}+\frac{g_1^*(n)}{\sqrt{w_n^2-w^2}}\right)\cdot$$

$$\sin\left(t\sqrt{w_n^2-w^2}\right)+\frac{1}{\sqrt{w_n^2-w^2}}\int_0^t\sin\left(\sqrt{w_n^2-w^2}(t-t)\right)\cdot\left[\frac{p_1^*(n,t)}{<rA>}-\frac{1+(-1)^{n+1}}{a_n}a_{01}(t)\right]dt\}\cdot\sin a_n x_1 \tag{7}$$

- for transversal vibrations:

$$u_2(x_1,t)=\frac{2}{l}\sum_{n=1}^{\infty}\{\left(f_2^*(n)\right)\cdot\cos\left(t\sqrt{\Omega_n^2-w^2}\right)+\left(\frac{g_2^*(n)}{\sqrt{\Omega_n^2-w^2}}\right)\cdot\sin\left(t\sqrt{\Omega_n^2-w^2}\right)-\frac{1}{(<rA>+a_n^2<rI_3>)\sqrt{\Omega_n^2-w^2}}$$

$$\int_0^t\sin\left(\sqrt{\Omega_n^2-w^2}(t-t)\right)\cdot\left[m_{3,1}^*(n,t)-p_2^*(n,t)+\frac{<rA>}{a_n}(a_{02}(t)(1+(-1)^{n+1})+(-1)^{n+1}el)\right]dt\}\cdot\sin a_n x_1 \tag{8}$$

where: ω is the angular velocity and ε is the angular acceleration;
<ρA>; <ρI_3> are the massic characteristics of the bar section;
a_{01} ,a_{02} are the acceleration components of its own triedre origin;
<EA>; <EI_3> are the bar rigidities at traction and, respectively, at bending.
ω_n Ω_n are the pulsation of the longitudinal vibrations, and, respectively, the pulsation of the transversal vibrations;

$$f_1(t)=u_1(0;t)\qquad g_1(t)=\dot{u}_1(0;t)$$
$$f_2(t)=u_2(0;t)\qquad g_2(t)=\dot{u}_2(0;t) \tag{9}$$

The numerical results are obtained for a length composite bar L=0.45m, which is distributed in a mechanism, having the conduction element with the length 0.038m, that is rotating with constant angular velocity equal with 25 * 3.1415 s^{-1}

The composite bar with the transversal rectangular section 0.005 x 0.012 [m^2], respectively, 0.012 x 0.012 [m^2] has an epoxidic resin matrix, reinforced with glass fibers in a longitudinal way, having a 50% the volume percentage of the reinforced. Fig. 1 presents the section structure of the bar.

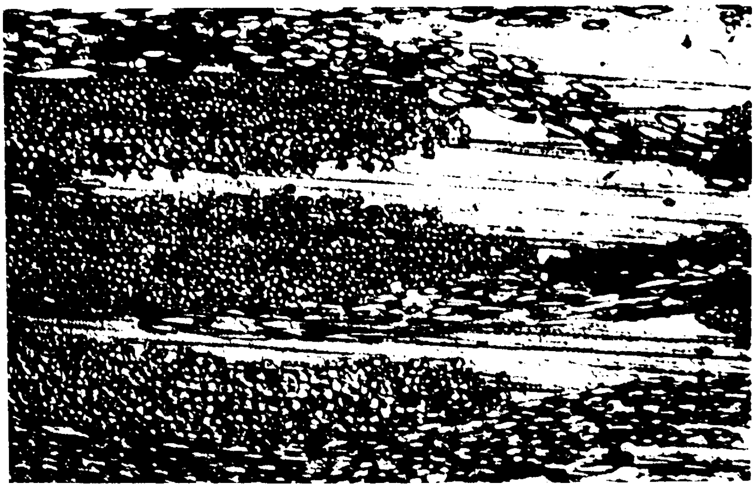


Figure 1. The material structure in a section of bar

Fig. 2 presents the variations of the transversal deformation u(t) in the middle of the first bar, and fig. 3 presents the variations of the transversal deformation u(t) in the middle of the second bar.

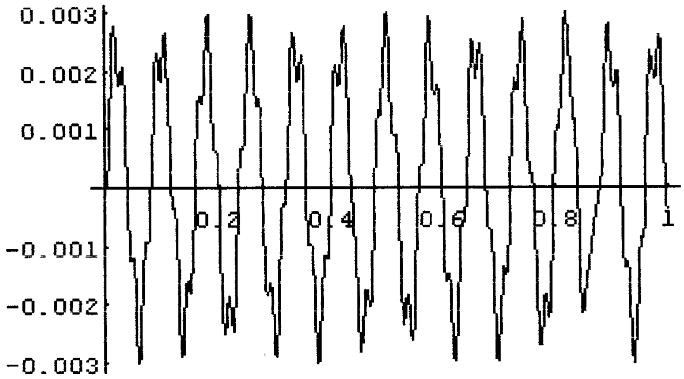


Figure 2. The variation of the transversal deformation u(t), in the middle of the bar 1 (numerical solution)

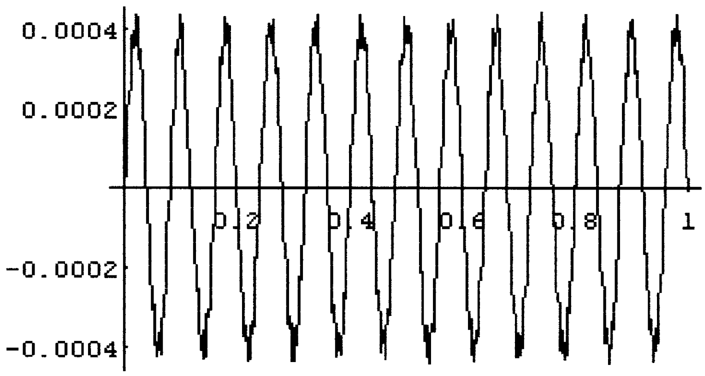
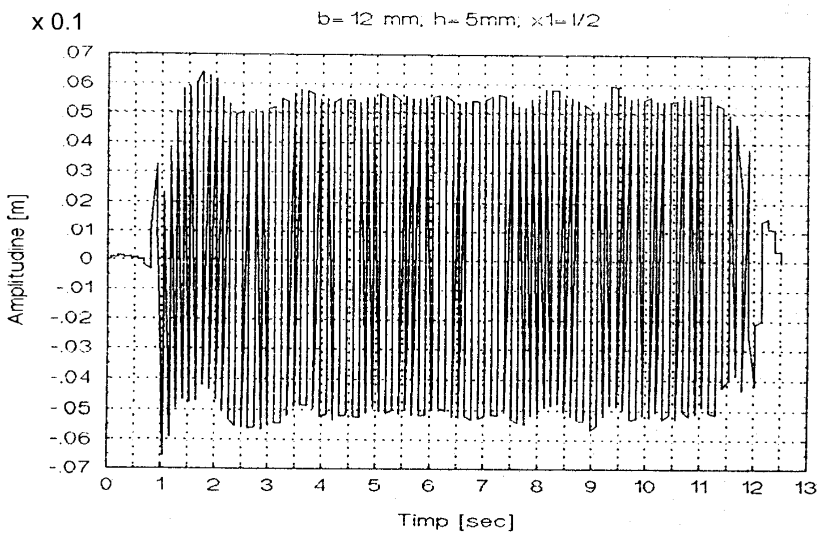


Figure 3. The variation of the transversal deformation u(t), in the middle of the bar 2 (numerical solution).

The experimental treatment are made with a BK2515 vibration analyzer, the measures being in the time mode (0…12.5) sec and in the frequency mode, in the interval (0…1000) Hz. The mechanical signal displacement was processed by the piezo-electrical transducer type 4391 and to unload the memory are used the software BK7616 and the interface functions IEEE-488. Fig. 4 and fig. 5 present the transversal vibrations measured in the middle of the bar 1, respectively, in the middle of the bar 2.

Fig 4. The variation of the transversal deformation u(t) , in the middle of the bar 1 (experimental reply)



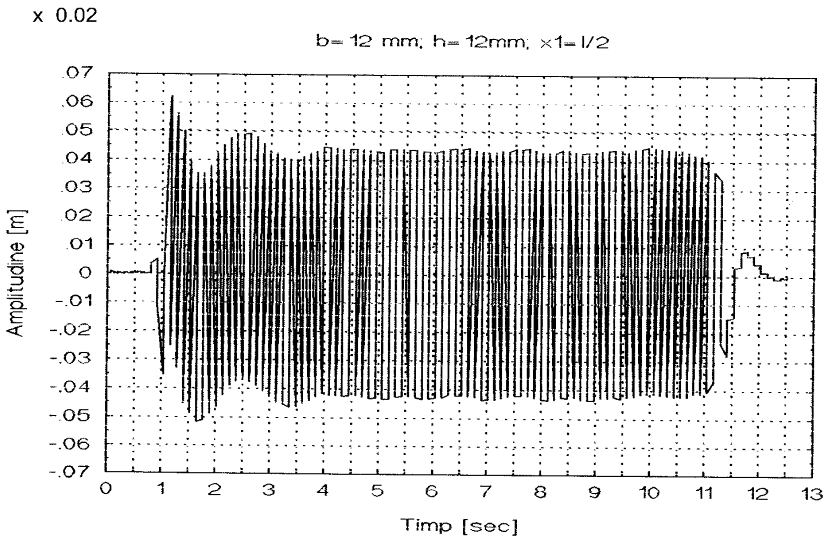


Figure 5. The variation of the transversal deformation u(t) in the middle of the bar 2 (experimental reply)

3 CONCLUSIONS

The theoretical results and their comparing with the experimental measured one, it shows the next conclusions:

The pulse of longitudinal vibrations depend only on the reinforced proportion.

The pulses of the transversal vibrations depend both on the reinforced proportion and on its spatial distribution in the bar section.

The pulses of transversal vibrations increase with the reinforced proportion .

The dynamic response, measured with a BK 2515 vibration analyzer, is in concordance with the one deduced analytically, the errors covering the interval of 1%-10%.

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