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REAL-TIME MEASUREMENT OF POWER SYSTEM FREQUENCY

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Abstract: Most digital algorithms for measuring frequency have acceptable accuracy if voltage waveforms are not distorted. However, due to non-linear devices, e.g. semiconductor rectifiers, electric arc furnaces, the voltage waveform in a power supply system can include higher harmonics with order of magnitude some percent of the fundamental.

This paper presents an algorithm for the rapid and accurate measurement of the line frequency based on digital filtering and dynamic parameter estimation method. First the voltage waveform, taken from a voltage transformer, is filtered using a lowpass IIR filter. Then the line frequency is obtained by comparing the filtered voltage with a mathematical model using an optimization procedure. The model's parameters are varied until an adequate match is obtained with the filtered voltage. The method uses digitized samples of input voltage and requires approximately 1/4 cycle of recorded data.

Keywords: Power system frequency measurement, Digital filtering, Dynamic parameter estimation

1 INTRODUCTION

Frequency is an important operating parameter of a power system for analyzing irregularities in its operation and for improving its operating security.

An electric power system is required to work at a constant frequency; but generation-load mismatches and disturbances cause the line frequency to deviate from its nominal value.

Frequency sensitive relays are generally used to detect these conditions and to maintain the system frequency within pre-fixed limits. These relays measure time durations between successive zero crossing of the input voltage and determine frequency from these measurements.

Usually frequency deviations in the range of two to three percent only are allowed for short durations of time.

Most digital algorithms for measuring frequency have acceptable accuracy if voltage waveforms are not distorted [1]. However, due to non-linear devices, e.g. semiconductor rectifiers, electric arc furnaces, the voltage waveform in a power supply system can include higher harmonics with order of magnitude some percent of the fundamental [2]-[6].

The paper presents an algorithm for the rapid and accurate measurement of the line frequency based on digital filtering and dynamic parameter estimation method. First the voltage waveform, taken from a voltage transformer, is lowpass digital filtered. Then the line frequency is obtained by comparing the filtered voltage with a mathematical model using an optimization procedure. The model's parameters are varied until an adequate match is obtained with the filtered voltage. The method uses digitized samples of input voltage and requires approximately 1/4 cycle of recorded data. The parameters that affect the performance of the algorithm are essentially the data window size and the sampling rate. The frequency at a power system bus is usually not required to be measured during a fault transient: we consider therefore that the system frequency does not change during a data window used for measurement.

The effect of the quantization error of the input voltage introduced by a 12-bit A/D converter has been included in the analysis.

Mathematical development of the algorithm is presented and the effects of key parameters that affect the performance of the algorithm are discussed. A representative set of test results obtained with computer simulation are presented.

The nominal value of the line frequency considered in this paper is 50 Hz, but a value of 60 Hz can be used as well.

2 MODEL OF THE SYSTEM VOLTAGE

The input voltage in a power supply system is normally composed of a fundamental frequency component and harmonics with order of magnitude some percent of the fundamental.
Let us assume that the voltage waveform taken from a voltage transformer has the following form:

$$v(t) = \sum_{m=1}^{M} V_m \sin(m\omega t + \theta_m) \tag{1}$$

where:
v(t) is the instantaneous value of the voltage at time t,
M is the highest order of the harmonic component present in the signal,
V_m is the amplitude of the sinusoid m,
$\omega = 2\pi f$ is the fundamental radian frequency of the system,
t is time in seconds,
θ_m is the phase of sinusoid m.
In the proposed approach the algorithm to work assumes that the voltage waveform is a sinusoid of a single frequency; therefore the voltage waveform is first filtered using a lowpass IIR filter. Aim of this filtering operation is to improve the accuracy of the frequency measurement.

3 ESTIMATION ALGORITHM

At the output of the filter algorithm we obtain a set of "n" samples of the fundamental component of the voltage.
The filtered voltage waveform can be approximated by a sinusoid

$$v(t) = V \sin(\omega t + \theta) \tag{2}$$

where V and θ are amplitude and phase of the fundamental, respectively.
By rewriting the voltage in terms of quadrature components

$$v(t) = A \sin \omega t + B \cos \omega t = v(A, B, \omega, t) \tag{3}$$

with

$$\begin{aligned} V &= \sqrt{A^2 + B^2} \\ \theta &= \tan^{-1}(B/A) \end{aligned} \tag{4}$$

Expanding v(t) in a Taylor series in the neighborhood of given values A_0 , B_0 , ω_0 of parameters A, B, ω gives

$$v(t) = v(t)_\alpha + \left[\frac{\partial v(t)}{\partial A}\right]_\alpha \Delta A + \left[\frac{\partial v(t)}{\partial B}\right]_\alpha \Delta B + \left[\frac{\partial v(t)}{\partial \omega}\right]_\alpha \Delta\omega \tag{5}$$

where the higher order terms of the expansion are ignored and $\alpha = (A_0, B_0, \omega_0)$.
The principle of operation of the estimation technique is based on the comparison between the real values of the filtered voltage and the estimation values of the model.
Thus the problem consists in determining the parameters A, B, α able to minimize the error between sampled values and estimation values.
The error, at instant t_S, is expressed as

$$e(t_S) = [\, v_r(t_S) - v(t_S)] \tag{6}$$

where $v_r(t_S)$ and $v(t_S)$ represent the sampled outputs of the real system after filtering and the model reference at time t_S.
The total square error is given by

$$E = \sum_{s=1}^{n} [\, v_r(t_s) - v(t_s)]^2 \tag{7}$$

Substitution of (5) into (7) yields

$$E = \sum_{s=1}^{n} \left\{ v_r(t_s) - v(t_s)_\alpha - \left[\frac{\partial v(t_s)}{\partial A} \right]_\alpha \Delta A - \left[\frac{\partial v(t_s)}{\partial B} \right]_\alpha \Delta B - \left[\frac{\partial v(t_s)}{\partial \omega} \right]_\alpha \Delta\omega \right\}^2 \tag{8}$$

The total square error is minimized by solving the partial derivatives of (8) relative to A, B, ω evaluated at A_0, B_0, ω_0

$$\frac{\partial E}{\partial A} = 0; \quad \frac{\partial E}{\partial B} = 0; \quad \frac{\partial E}{\partial \omega} = 0 \tag{9}$$

There results after rearrangement

$$\begin{aligned}
&\sum_{s=1}^{n} \left[\frac{\partial v(t_s)}{\partial A} \right]_\alpha \left\{ v_r(t_s) - v(t_s)_\alpha \right\} = \\
&\qquad \sum_{s=1}^{n} \left[\frac{\partial v(t_s)}{\partial A} \right]_\alpha \left\{ \left[\frac{\partial v(t_s)}{\partial A} \right]_\alpha \Delta A + \left[\frac{\partial v(t_s)}{\partial B} \right]_\alpha \Delta B + \left[\frac{\partial v(t_s)}{\partial \omega} \right]_\alpha \Delta\omega \right\} \\
&\sum_{s=1}^{n} \left[\frac{\partial v(t_s)}{\partial B} \right]_\alpha \left\{ v_r(t_s) - v(t_s)_\alpha \right\} = \\
&\qquad \sum_{s=1}^{n} \left[\frac{\partial v(t_s)}{\partial B} \right]_\alpha \left\{ \left[\frac{\partial v(t_s)}{\partial A} \right]_\alpha \Delta A + \left[\frac{\partial v(t_s)}{\partial B} \right]_\alpha \Delta B + \left[\frac{\partial v(t_s)}{\partial \omega} \right]_\alpha \Delta\omega \right\} \\
&\sum_{s=1}^{n} \left[\frac{\partial v(t_s)}{\partial \omega} \right]_\alpha \left\{ v_r(t_s) - v(t_s)_\alpha \right\} = \\
&\qquad \sum_{s=1}^{n} \left[\frac{\partial v(t_s)}{\partial \omega} \right]_\alpha \left\{ \left[\frac{\partial v(t_s)}{\partial A} \right]_\alpha \Delta A + \left[\frac{\partial v(t_s)}{\partial B} \right]_\alpha \Delta B + \left[\frac{\partial v(t_s)}{\partial \omega} \right]_\alpha \Delta\omega \right\}
\end{aligned} \tag{10}$$

Solution of system (10) gives the corrections ΔA, Δ B and Δω necessary for updating parameters A, B and ω for each iteration step. This recursive technique permits to obtain the unknown quantities with good accuracy.

Iterative methods are well known for their sensitivity to the initially guessed values of the unknowns. The initial values used for the model reference are determined as follows. The initial value of the radian frequency ω_0 is obtained using the first five samples of the input voltage [7]

$$\omega_0 = \frac{1}{T_s} \sqrt{\frac{3\left[2v(t_1 + 2T_s) - v(t_1) - v(t_1 + 4T_s)\right]}{2\left[v(t_1 + T_s) + 4v(t_1 + 2T_s) + v(t_1 + 3T_s)\right]}} \tag{11}$$

The initial values of A_0 and B_0 are determined solving the system:

$$\begin{aligned}
v(t_1) &= A_0 \sin \omega_0 t_1 + B_0 \cos \omega_0 t_1 \\
v(t_1 + T_S) &= A_0 \sin [\omega_0 (t_1 + T_S)] + B_0 \cos [\omega_0 (t_1 + T_S)]
\end{aligned} \tag{12}$$

4 COMPUTER SIMULATION RESULTS

The measurement algorithm has been verified by computer simulation to investigate the validity of this technique. The program generates a voltage that is sampled at preselected rate; its waveform, taking into account a realistic distortion in high voltage networks, is expressed by

$v(t) = \sin(\omega t + \theta) + 0.02 \sin(5\omega t) + 0.01\sin(7\omega t)$

he lowpass filtering operation is obtained with a fourth order IIR filter, with cutoff frequency of 53 Hz; the filter keeps the fundamental frequency component at 50 Hz and gets rid of the 5ω and 7ω harmonics. The frequency response is shown in Fig. 1.

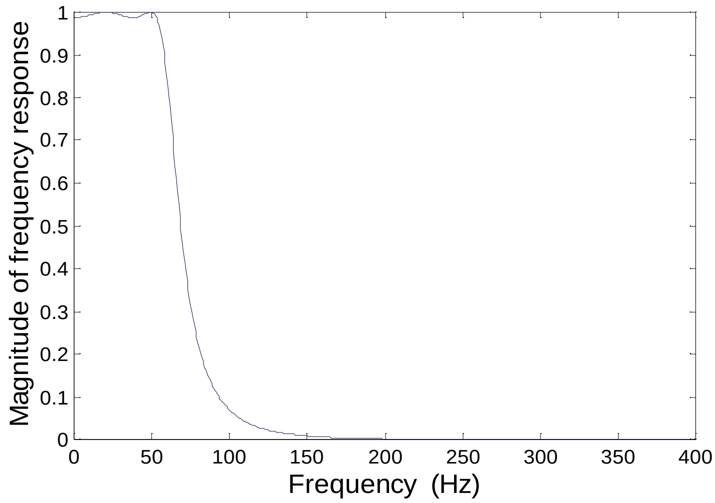

Figure 1. Frequency response of the filter

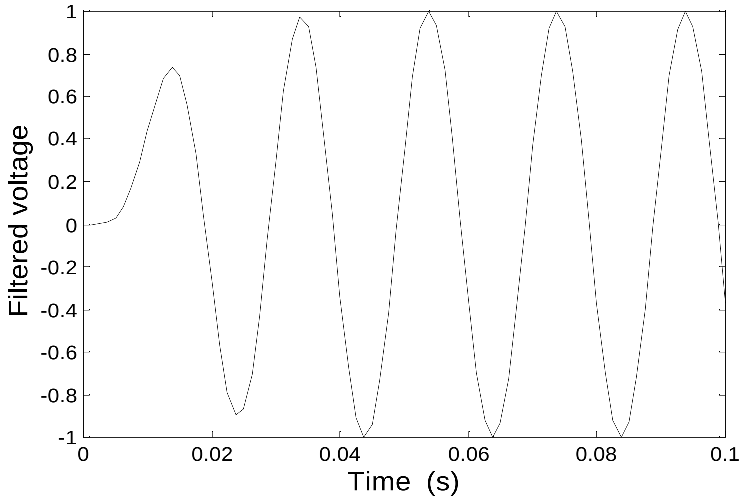

Figure 2. Filtered voltage

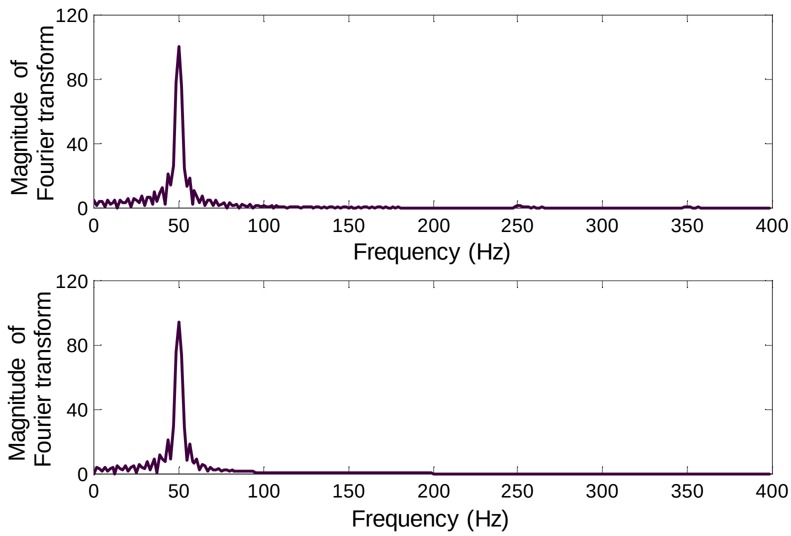

Figure 3. Frequency content of the voltage before and after filtering.

The voltage waveform after filtering is shown in Fig. 2. The delay introduced by the filtering operation is only 8.5 ms.

Fig. 3 shows the frequency content of the voltage before and after filtering.

The frequency of the signal has been varied from 48.00 to 52.00 Hz in steps of 0.01 Hz. Tests have been carried out with the sampling frequency set to 800 Hz. The elaboration for generating new values of A, B, and ω at each iteration step is carried out on a data window of n=4 samples of the filtered voltage; these values have been selected in order to increase the speed of convergence and to improve the accuracy.

The initial values of parameters ω_0 , A_0 , B_0 are estimated as mentioned in previous section; only three iteration steps are required to get to the convergence.
The frequency error is within 1.5 mHz. (absolute value) when the magnitude of the harmonics is not greater than 2% of the fundamental. Fig. 4 shows the results of frequency estimation

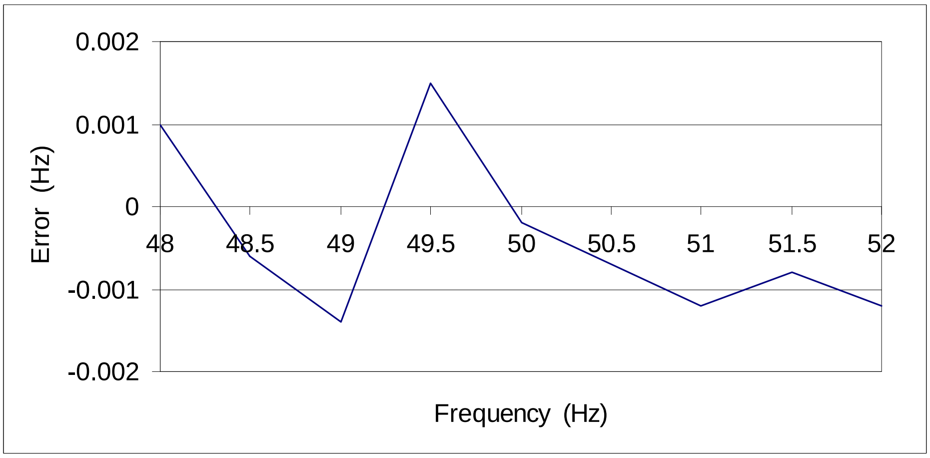

Figure 4. Results of frequency estimation for realistic distorted voltage waveform.

Figure 5 shows analogous results for heavy distorted voltage waveform
$v(t) = \sin(\omega t + \theta) + 0.12 \sin(5\omega t) + 0.06\sin(7\omega t)$

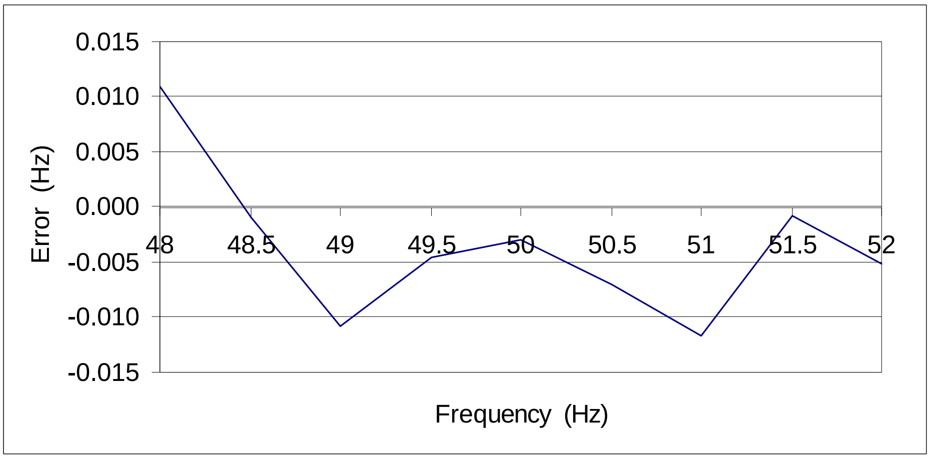

Figure 5. Results of frequency estimation for heavy distorted voltage waveform.

In this case the accuracy of the frequency estimate deteriorates to 12 mHz (absolute value); however this amount of distortion is not realistic in high voltage networks.

5 CONCLUSION

The paper has described an algorithm for the rapid and accurate measurement of the line frequency in the presence of harmonics. Most digital methods for measuring frequency have acceptable accuracy if the voltage waveforms are not distorted. In the proposed method the distorted voltage waveform is first filtered using a lowpass IIR filter. Then the line frequency is obtained by comparing the filtered voltage with a mathematical model using an optimization procedure. The model's parameters are varied until an adequate match is obtained with the filtered voltage. Useful estimates of frequency are obtained using about 1/4 cycle of recorded data. Sampling rate and data window size are critical parameters that affect the performance of the algorithm. Computer simulation tests confirmed the validity of the proposed technique.

This technique is useful in designing digital meters and relays that need to measure frequency very quickly and accurately.

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