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RECONSTRUCTION OF THE OSCILLATING PRESSURE SIGNAL

J. Cieplucha, L. Horodko and Z. Kazimierski
Institute of Turbomachinery
Technical University of Lodz, Poland

Abstract: An iterative method for the reconstruction of the oscillating-pressure measuring signal at the inlet of the small diameter cylindrical tube is presented. This method requires a good mathematical model of the flow phenomena in the tube. The theoretical model of the phenomena and its experimental correction are described. The limitations and the application range of this reconstruction method are discussed.
Keywords: Pressure and Vacuum Measurement; Reconstruction of Distorted Signals

1 INTRODUCTION

Measurements of unsteady pressure (or other physical quantities transduced to pressure) are often performed by means of a pneumatic signal line transmitting the pressure from a measuring point to an electrical transducer. An oscillating component of the inlet pressure signal is distorted in this line [1, 2]. A distortion depends on dimensions and configuration of the line, gas parameters and a frequency of oscillations. A reconstruction of the inlet pressure signal on the basis of the measured output pressure shape is the subject of this paper.

2 MATHEMATICAL MODEL OF THE PNEUMATIC SIGNAL LINE

An assumption that pneumatic phenomena in a thin tube can be described by means of differential equations of the laminar compressible boundary layer forms the basis of the mathematical model [3] of the pneumatic part of the measuring path. These are equations of conservation of motion and energy, continuity of flow, state of gas, and, additionally, a constant pressure in the cross-section of the tube is assumed.

$$r\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial x}+m\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{1}$$

$$\frac{\partial r}{\partial t}+u\frac{\partial r}{\partial x}+v\frac{\partial r}{\partial r}+r\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}\right)=0 \tag{2}$$

$$r\,c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=l\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+m\left(\frac{\partial u}{\partial r}\right)^2 \tag{3}$$

$$p = r\,RT \tag{4}$$

$$\frac{\partial p}{\partial r}=0 \tag{5}$$

A solution to this equation system is usually achieved by means of the perturbation method in the form of power series, in which the value of the 'small parameter' e depends on the amplitude of the inlet signal oscillation. This solution allows for obtaining both the linear and non-linear approximation of the mathematical model of the pneumatic signal transmission. The solution formula for the pressure is:

$$p(x,t)=p_0+e\;p_1(x,t)+e^2 p_2(x,t)+\ldots \tag{6}$$

For the inlet pressure signal containing the constant component p^{0}_{in} and the harmonic component with the frequency *n*, the amplitude p^{1}_{in} and the phase $j_{in}=0$, one can obtain the linear approximation of the model expressed in dimensionless quantities:

$$\hat{p}_z(x,t) = 1 + e\ \hat{p}_{1z}(x)\exp(i\ 2p\ N\ t) \tag{7}$$

The complex quantity $\hat{p}_{1z}(x)$ for ξ=1 is the equivalent of the transfer function *G* of the pneumatic line.

3 EXPERIMENTAL VERIFICATION OF THE MATHEMATICAL MODEL

Experiments [4] proved a good agreement with the linear mathematical model for small, acoustic amplitudes (*e =0.00065*) of the input pressure signal. The investigations carried out at the Technical University of Lodz [5, 6, 7] for greater amplitudes of ε (*e =0.03 ¸ 0.15*) have shown still good compatibility of the theoretical and experimental phase of the transfer function of the pneumatic line, however the magnitude |G| = f(N) of the experimental transfer function depends on the relative amplitude of oscillations ε, the wave damping number D and, in general, is considerably smaller than its theoretical equivalent (see Fig. 1).

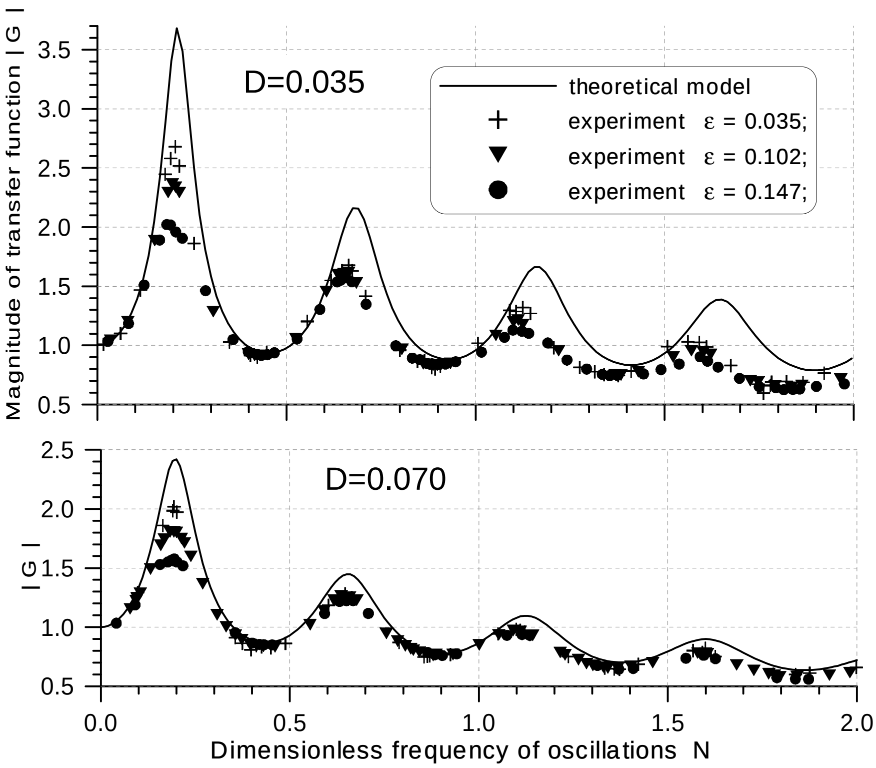

Figure 1. Comparison of the magnitude of the theoretical and experimental transfer function of a pneumatic signal line made in the form a single straight tube

The absence of the mathematical formula of the turbulence effect in the theoretical model is supposed to be a reason of this phenomenon. In order to test this supposition, a dependence between the transfer function and the Reynolds number was investigated, because the Reynolds number describes the laminar to turbulent transition. Figure 2a shows a disagreement between the empirical and theoretical transfer function *D|G|* versus the Reynolds number: there is no clear dependence between the quantities under investigation. A modification of the Reynolds number with respect to the amplitude *e* and the frequency *N* of the input signal oscillations:

$$Re_X = Re\,(1 + A_n\,N + B_e\,\varepsilon)\,/\,\varepsilon \tag{8}$$

has led the investigation to success (see Fig. 2b). The relation between *D|G|* and Re_X can be described by means of the formulas:

$$\Delta|G| = a\,(Re_X - Re_{Xkr})^2 + d \quad \text{for} \quad Re_X \ge Re_{Xkr} \tag{9}$$

$$\Delta|G| = d \quad \text{for} \quad Re_X < Re_{Xkr}$$

The quantities Re_{Xkr}, a, d could be obtained by means of the least squares method, while the determination of A_n, B_e requires a calibration of the pneumatic line. The calibration should be made carefully in a wide range of frequencies and amplitudes of the pressure signal.

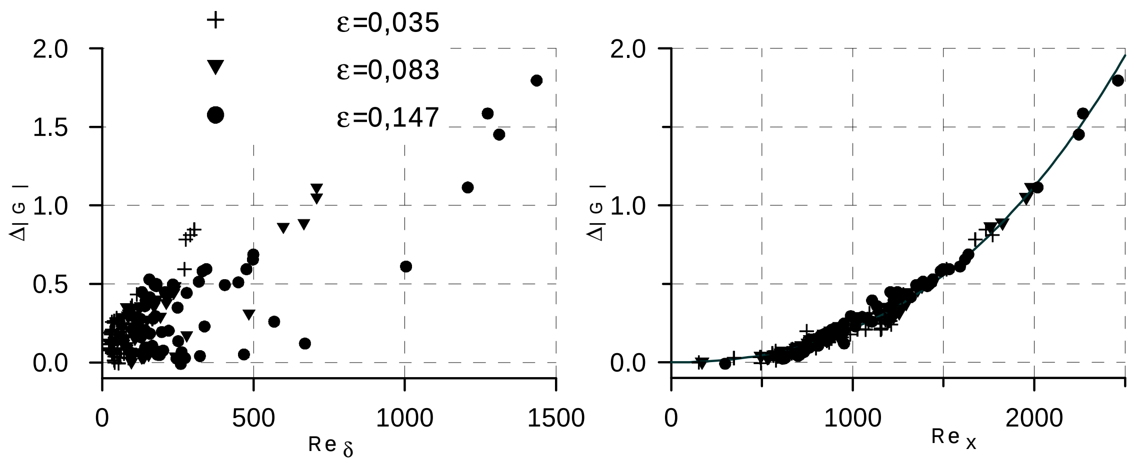

Figure 2. Difference between the theoretical and experimental transfer function versus the Reynolds number (a) and the modified Reynolds number (b).

4 RECONSTRUCTION OF THE INLET PRESSURE SIGNAL

The corrected model of the flow phenomena determines the output signal p_{out} by means of the parameters of the unknown inlet signal p_{in}. However, an iterative procedure allows one to reconstruct the input signal shape from the measurement results obtained with a transducer mounted at the end of the tube. In the first approximation, the transfer function is assumed to be equal to the transfer function of the plain mathematical model. This enables one to compute an approximation of the amplitude of the first input harmonic, which is used to modify the Reynolds number, and then it is used to compute a new approximation of the transfer function of the pneumatic path. This algorithm of calculation of the amplitude, Reynolds number and transfer function should be repeated to obtain the assumed accuracy. Two examples of the reconstructed input signals $p_{in\,rec}$ are shown in Figure 3.

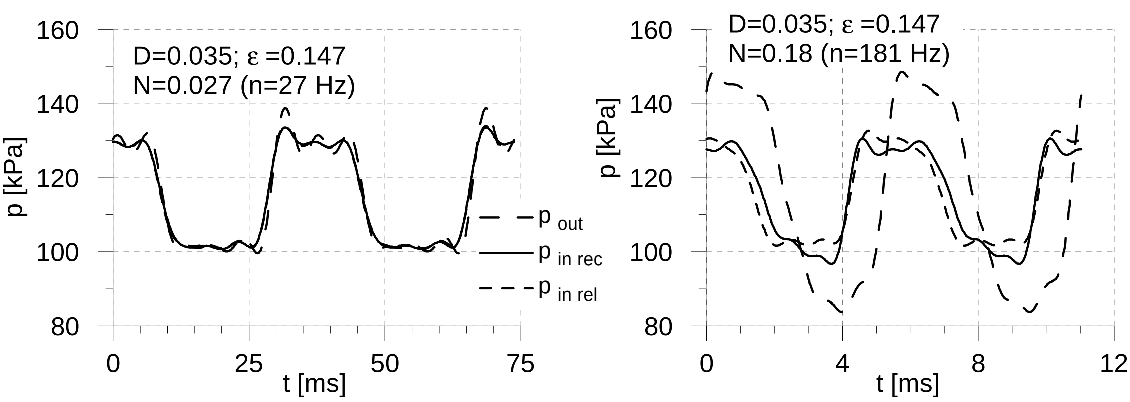

Figure 3. The reconstructed inlet pressure signal $p_{in\,rec}$ and its comparison with the original input signal $p_{in\,rel}$

5 CONCLUSION

The reconstruction method of the inlet pressure signal yields very good results for frequencies significantly smaller than the frequency of the first resonance of the tube. For the frequency equal to the first tube resonance, the effect of the reconstruction is still satisfactory. However, the reconstruction of signals of a higher frequency gets worse. This is the reason why the method should be enhanced by a new mathematical model of flow phenomena, taking into account effects of gas turbulence on the basis of the non-linear theory.

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APPENDIX

1. Nomenclature

a mean sound velocity, m/s
c gas velocity
i = $\sqrt{-1}$, imaginary unit
L tube length, m
n frequency of oscillations, Hz
p pressure, Pa
R internal tube radius, m
Re = *cd* /*n*, Reynolds number,
t time, s
T temperature, K
d = $(n / p\, n)^{0.5}$, Stoke's layer thickness
m dynamic viscosity, kg/ms
n kinematic viscosity, m^2/s
x dimensionless axial co-ordinate, related to L
r density, kg/m^3

sub- and superscripts:
in inlet cross section (for ξ = 0)
ou outlet cross section (for ξ = 1)
0, 1,... ,k order of harmonics

2. Dimensionless transmission parameters:

D = $(m / a^0_{in}\, r^0_{in} R)\,(L/R)$, wave damping number
N = $n\, L / a^0_{in}$, frequency of oscillations
e = p^1_{in} / p^0_{in}, relative oscillation amplitude
G transfer function

AUTHORS: Ass. Prof. D.Sc. Jan CIEPLUCHA, Dr. Longin HORODKO and Prof. D.Sc. Zbyszko KAZIMIERSKI, Institute of Turbomachinery I-10, Technical University of Lodz, 219/223 Wolczanska St, 93-005 Lodz, Poland, Phone Int. +0048 42 6312364, Fax Int. +0048 42 6361383
E-mail: jancie@ck-sg.p.lodz.pl