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OPTICAL AMPLITUDE EVALUATION OF SQUARE-WAVE PRESSURE

T. Kobata and A. Ooiwa
Mechanical Standards Section, Mechanical Metrology Department
National Research Laboratory of Metrology
1-1-4 Umezono, Tsukuba, Ibaraki, 305-8563, Japan

Abstract: We have developed a system for the amplitude measurement of periodic square-wave pressure by measuring the change in refractive index of air. Using a differential interferometer and a pressure cell whose capacity was less than 5 cm^3, the change in the optical path difference between the reference and measurement paths was measured when square-wave pressure was applied in the cell. A rotating valve was used to vary the pressure in the cell. For the measurement in the amplitude range from 10 kPa to 100 kPa as gauge pressure, the pressure amplitude and the optical path difference were simultaneously measured as time series. In this study, the transient change in temperature of the medium due to the rapid change in pressure and the results of the optical amplitude measurement were examined.
Keywords: Square-wave pressure, Interferometer, Refractive index

1 INTRODUCTION

In order to develop precise quantitative calibration techniques for evaluating the dynamic characteristics and performance of pressure measurement devices, we have investigated two techniques. One is for generating precise dynamic pressure, and the other is for the accurate evaluation of the generated dynamic pressure. These techniques are mutually related. To generate precise dynamic pressure, we have developed a dynamic pressure generator using a novel rotating valve [1], [2]. The generated waveform is square and the pressure amplitude and fundamental frequency can be changed. Until now, we measured the generated pressure using several different pressure transducers and evaluated the amplitude spectrum of the measured waveform using frequency analysis. On the other hand, in order to measure the generated dynamic pressure accurately, a technique to measure the change in refractive index of the medium has been developed [3]. For quasi-static pressure measurement, the pressure difference and the optical path difference were compared in the range from 0 kPa to 100 kPa as gauge pressure. The experimental results revealed an excellent linear relationship between them. Moreover, their ratio was compensated by the temperature of the medium and determined with good reproducibility.

In this study, a system for the amplitude measurement of the square-wave pressure using an interferometric method has been developed. The measurement equations and method are described and the experiment results are shown and discussed in this paper.

2 MEASURMENT EQUATIONS

The refractive index of dry air $(n\text{-}1)_{tp}$ at a temperature t of a medium and the absolute pressure p is given as follows [4]:

$$(n-1)_{tp} = \frac{(n-1)_s\, p}{A(1-Bt)} \times \left[1+(C-Dt)p\right] \quad , \tag{1}$$

where A, B, C and D are constants. The value of $(n\text{-}1)_s$ is the refractivity of standard air and calculated from the dispersion formula at a specific wavelength. In this paper, the differences in the refractive index due to water vapour and carbon dioxide level are not considered since the effects of these parameters are relatively small.

By disregarding the term of higher order with respect to p in equation (1) and transforming the equation, the following equation is obtained.

$$p = \frac{A(1+Bt)(n-1)_{tp}}{(n-1)_s} \tag{2}$$

In equation (2), if the temperature does not change during the measurement, the refractive index $(n\text{-}1)_{tp}$ is proportional to the absolute pressure p. By taking the ratio of the equation for the state of pressure p and temperature t, to the equation for the state of the atmospheric pressure p_0 and atmospheric temperature t_0,

$$\frac{p}{p_0} = \frac{(1+Bt)(n-1)_{tp}}{(1+Bt_0)(n-1)_{t_0p_0}} \quad . \tag{3}$$

In equation (3), the ratio of the refractive index is transformed into the following equation (4).

$$\frac{(n-1)_{tp}}{(n-1)_{t_0p_0}} = 1 + \frac{(n-1)_{tp} - (n-1)_{t_0p_0}}{(n-1)_{t_0p_0}} = 1 + \frac{\Delta n}{(n-1)_{t_0p_0}} \tag{4}$$

The refractive index difference Δn is proportional to the optical path difference d between the reference and measurement paths. That is, it is $\Delta n = d / L$, where L is the effective length of the cell. The following relations are introduced, $p = p_0 + \Delta p$, and $t = t_0 + \Delta t$, where Δp and Δt are the changes in pressure and temperature of the medium, respectively. From these relations and equation (4), equation (3) is transformed to equation (5).

$$\frac{p_0 + \Delta p}{p_0} = \frac{[1 + B(t_0 + \Delta t)]}{(1 + Bt_0)}\left[1 + \frac{d}{L(n-1)_{t_0p_0}}\right] \tag{5}$$

When there is no transient temperature change, $\Delta t = 0$, the following equation is obtained by substituting equation (2) with equation (5).

$$\Delta p = \frac{A(1+Bt_0)d}{L(n-1)_s} \tag{6}$$

From equation (6), Δp was obtained by measuring d and t_0 when $\Delta t = 0$.

When there is a transient temperature change, it is difficult to determine even with a temperature sensor with a good response. However, the temperature change can be calculated by the following equations using the output of the pressure transducer with a good response. In equation (5), $L(n\text{-}1)_{t_0p_0}$ is constant if p_0 and t_0 are constant. Since the value of d for the state at $\Delta p = p_a$ and $\Delta t = 0$ can be experimentally obtained as d_a, equation (5) is approximately transformed as follows.

$$\frac{p_0 + \Delta p}{p_0} = \frac{[1 + B(t_0 + \Delta t)]}{(1 + Bt_0)}\left[1 + \left(\frac{p_a}{p_0}\right)\cdot\left(\frac{d}{d_a}\right)\right] \quad , \tag{7}$$

where p_a and d_a are the amplitude of the square-wave pressure and the amplitude of the optical path difference, respectively. In order to obtain the transient temperature change Δt, equation (8) is calculated.

$$\Delta t = \frac{1}{B}\left[\frac{p_0 d_a (p_0 + \Delta p)(1 + Bt_0)}{p_0(p_0 d_a + p_a d)} - 1\right] - t_0 \tag{8}$$

3 SYSTEM

Figure 1 shows the experimental system that measures the change in the optical path difference by square-wave pressure. This system was modified from the previous system [3]. The major modifications are as follows:

(1) The pressure cell was replaced with a cell with a smaller volume. The reason to reduce the capacity of the pressure cell is (a) to generate square-wave pressure with a short rise time, and (b)

to quicken the recovery of the temperature after the adiabatic change. The material of the housing is aluminium and two cylindrical pressure cells are enclosed in the housing. The diameter of each cell is 20 mm, and the length *L* of each cell is approximately 12 mm. The capacity of each cell including the piping volume is less than 5 cm^3. These cells are closed with two optical windows with a diameter of 50 mm at both ends. The optical windows are coated with an antireflection coating. Four O-rings are used between the window and the housing to prevent air leakage. Two pressure cells are used, one as a reference cell and the other as a measurement cell. The pressure in the reference cell is equalized to the atmospheric pressure. The pressure in the measurement cell is varied by a rotating valve as described below.

(2) A commercially available rotating valve was used to vary the pressure in the cell. The valve has two input ports and one output port. The stabilized pressure source is connected with one of the input ports and the other is left open to the atmospheric pressure. The pressure in the output port can be controlled according to the electric signal. In this system, the valve is driven by the periodic control signal from the signal generator.

(3) The shut-off valve type of pressure controller was replaced by the servo valve type in order to increase the flow rate. When compressed air flowed into the pressure cell, the supply pressure from the pressure controller changed temporarily. In order to decrease this influence, the type of the pressure controller was changed.

The laser and a differential interferometer are commercial system. The interference signal detected by a remote sensor attached to the interferometer was fed to the receiver via a fiber-optic cable and converted to a counting number by the electrical interface. The counting number was sampled every sampling period and stored in the trace memory. The sampling frequency could be arbitrarily changed by a signal generator. The sampled data was fed into a computer via a digital interface. Using this system, the resolution of the length measurement was 1/1024 of the wavelength. Since the wavelength was about 633 nm, the resolution was about 0.6 nm as obtained from the manufacturer's specifications.

The pressure variation in the cell was measured using a semiconductor transducer. The resonant frequency of the transducer is above 3 kHz. The atmospheric pressure was measured by an absolute pressure gauge. The average temperature in the pressure cell was measured using a thermistor. A computer can be used to control these devices. The optical path difference and the pressure variation in the measurement cell were measured simultaneously as time series, and all measured data were recorded in the computer.

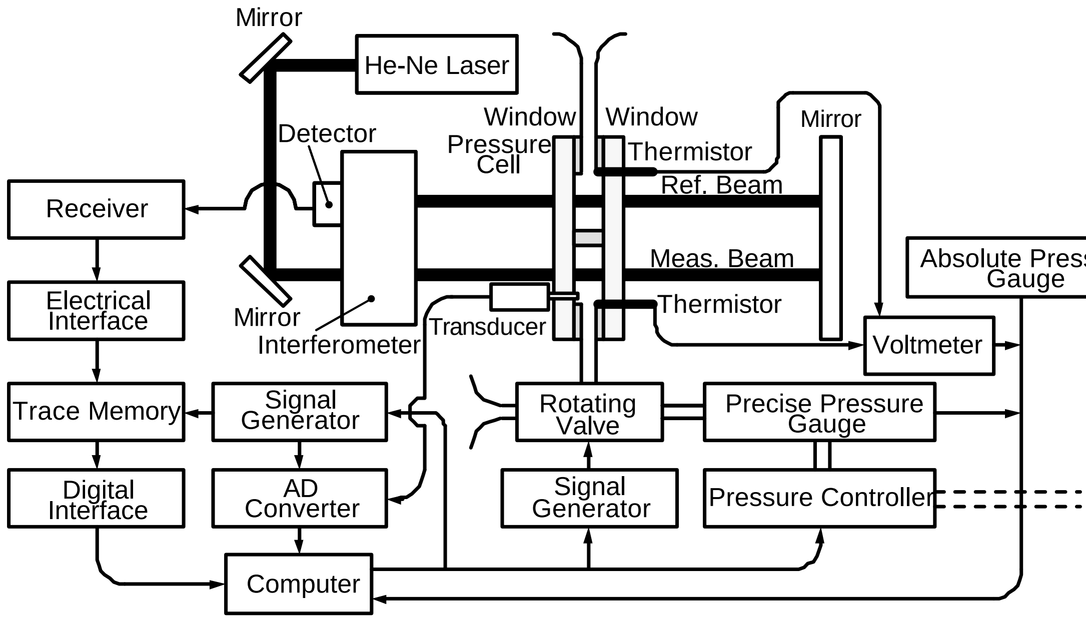

Figure 1. Experimental system

4 RESULTS AND DISCUSSION

Figure 2 shows the waveforms obtained from the experiment. The amplitude of the square-wave pressure p_a is about 100 kPa in Fig. 2(a) and 10 kPa in Fig. 2(b), and the fundamental frequency f_0 is 125 mHz. The sampling number N_s and sampling frequency f_s for each set of data are N_s = 4096 points and f_s = 128 Hz, respectively. Each figure shows three periods. The upper row figures show the waveform of the optical path difference d measured using the differential interferometer. The middle row figures show the waveform of the pressure change Δp measured using the pressure transducer. The lower row figures show the temperature change Δt calculated from equation (8) using the two above mentioned time series d and Δp. The values of p_a and d_a necessary for calculating equation (8) were obtained from the amplitude of corresponding waveforms. The values of p_0 and t_0 were acquired from the measurements.

From these waveforms, it is observed that Δt changes rapidly when Δp changes rapidly. This momentary temperature change can be understood as a type of adiabatic change and is fundamentally calculated from Poisson's equation as $\{(p_0+\Delta p)/p_0\}^{\gamma-1} = \{(t_0+\Delta t)/t_0\}^{\gamma}$ where γ is the ratio of the specific heat of the medium. The values of the temperature change obtained from equation (8) qualitatively agree with those obtained from Poisson's equation. After a rapid change, the temperature gradually recovers back to the atmospheric temperature. The time required for temperature recovery was about two seconds from the waveforms Δt, as shown in Figure 2, after which time, Δt and d are almost constant until the next rapid change. On the basis of these results, in the following experiment, the amplitude measurement using this interferometric system was performed when Δt = 0 in a steady state after about two seconds of rapid change. Therefore, the fundamental frequency of square-wave pressure was set below 125 mHz to obtain the plateau of the waveform d.

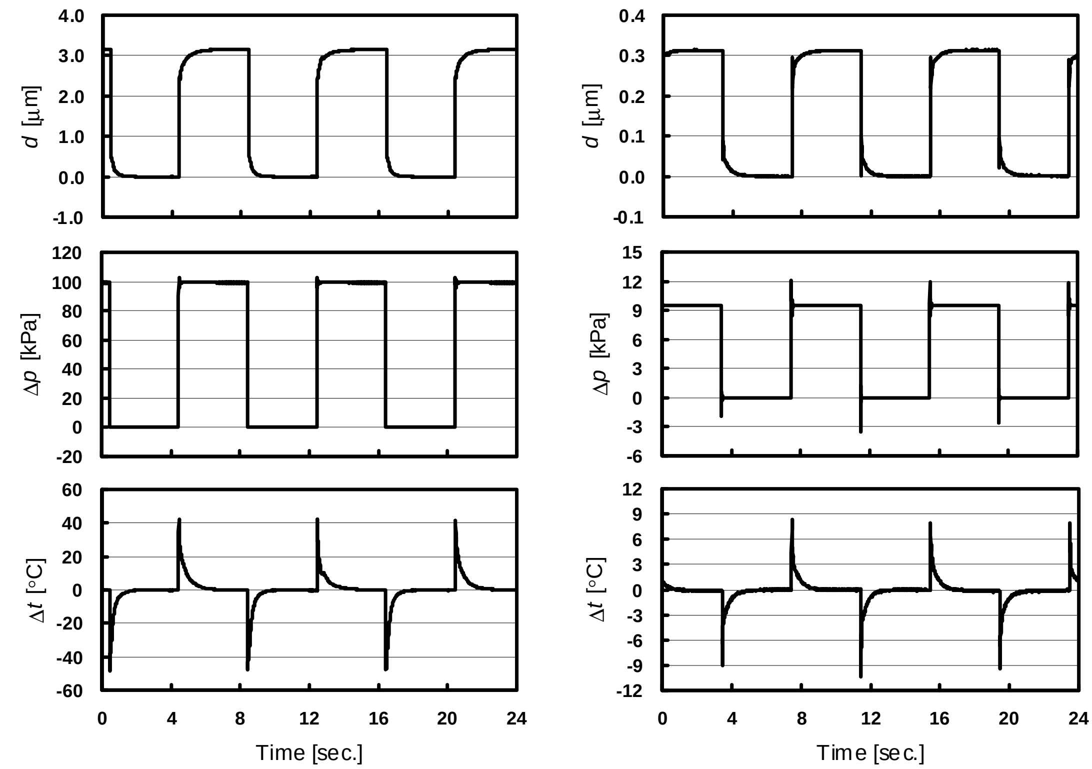

Figure 2. Waveforms obtained from measurements, upper row: d, middle row: Δp, bottom row: Δt, p_a = (a) 100 kPa, (b) 10 kPa , f_0 = 125 mHz.

Figure 3 (a) shows the relationship between the amplitude of square-wave pressure Δp, and optical path difference compensated by atmospheric temperature, $d(1+Bt_0)$. The pressure amplitude was increased from 10 kPa to 100 kPa in steps of 10 kPa. As shown in Figure 3 (a), $d(1+Bt_0)$ and Δp have a good linear relationship. Therefore, the pressure amplitude can be obtained using equation (6) and the measured values of the optical path difference and the atmospheric temperature. The slope of the line is about 29 Pa/nm, which is nearly equal to the value obtained from $A/\{L(n\text{-}1)_s\}$ in equation (6). Since the resolution of d is about 0.6 nm/count, the resolution of the pressure measurement of this system was calculated to be about 20 Pa/count, which corresponds to about 0.2 % of the full range of 100 kPa. Figure 3 (b) shows the deviation from linearity of $d(1+Bt_0)$ as a function of the pressure amplitude. The average value and the standard deviation are shown with the square mark and the error bar, respectively. As shown in Figure 3 (b), the maximum deviation of the average value was below 0.02 %, which was better than that of the transducer used for the measurement. The standard deviation was from about 0.05 to 0.08% over the entire range. The main reason for this comparatively large standard deviation is the rough measurement resolution.

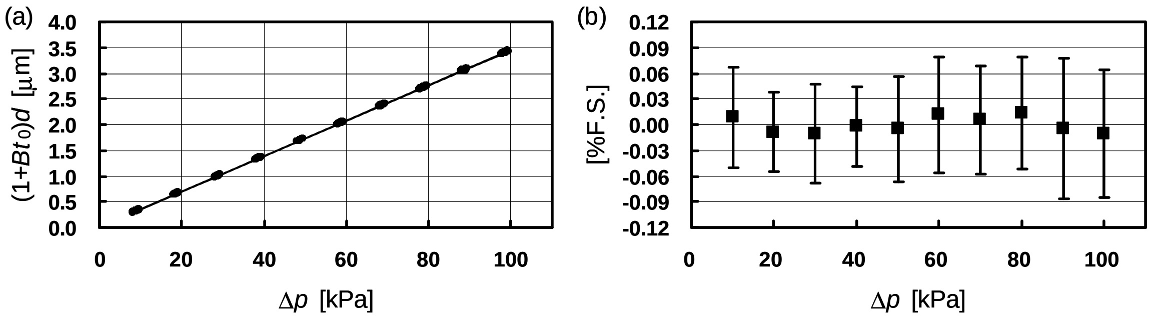

Figure 3. Amplitude measurement of square-wave pressure, (a) relationship between pressure difference Δp and optical path difference compensated by atmospheric temperature $(1+Bt_0)d$, (b) average and standard deviations from linearity of data are shown with respect to (a).

5 CONCLUSION

To evaluate the dynamic characteristics and performance of pressure measurement devices, we have developed interferometric techniques for the amplitude measurement of square-wave pressure by measuring the change in refractive index of the medium. Using a differential interferometer and a pressure cell whose capacity is less than 5 cm^3, the change in the optical path difference between the reference and measurement paths was measured when square-wave pressure was applied in the cell. A rotating valve was used to change the pressure in the cell. For the measurement in the amplitude range from 10 kPa to 100 kPa as gauge pressure, the changes in the optical path difference and pressure were measured as time series and the change in temperature of the medium was calculated. The momentary temperature change due to a rapid change in pressure can be understood as a type of adiabatic change. From the amplitude measurement, the good linear relationship between pressure amplitude and optical path difference compensated by atmospheric temperature was shown. In order to measure dynamic pressure using the developed interferometric method, further detailed evaluation of the variation in temperature of the medium is necessary.

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AUTHORS: Dr. Tokihiko KOBATA and Dr. Akira OOIWA, Mechanical Standards Section, Mechanical Metrology Department, National Research Laboratory of Metrology, 1-1-4 Umezono, Tsukuba, Ibaraki, 305-8563, Japan, Phone Int +81 298 61 4024, Fax Int +81 298 61 4020, E-mail: kobata@nrlm.go.jp