MAPPING OF REMOTE SENSING MEASUREMENTS WITH INTRINSIC RANDOM FUNCTION OF ORDER K

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Abstract: Implementation of mapping refered to "localized" quantities from Remote Sensing (i.e.) is based on known information processing, on a set of bi-dimensional Space coinciding normally with the same mapping. An usual representation of interested objects is " contouring", that allows to illustrate the curve, place of points that have the same value. This representation is obtained by using known value interpolation of quantities of interest. The widespread approaches to generate interpolating surfaces, using personal computer, through Intrinsic Random Function in Remote Sensing are considered in this work.

Keywords: Remote Sensing Measurements, Intrinsic Random Function of Order k, Geostatistics, Point Estimation.

1   INTRODUCTION

The widespread techniques in absolute are based on the interpolation of values at nodes of a regular mesh to which we apply a generation operation of "contour". Such techniques can be characterized in function of: *selection criteria of neighbourhood* and *estimation algorithm of point value*. Value estimation at grid nodes is provided by using a data set called " *neighbourhood* " (or influence area). The neighbourhood may either coincide with all of available information or to be limited to a restricted data set, whose data are reasonably near to the node to be estimated. Data recovered from Remote Sensing face this kind of problem [1], especially if they concern territorial ones like temperatures at soil, soil roughness, moisture, etc..to be calculated where they are not known. So we use IRFk's, *Intrinsic Random Functions of Order k* [2] to solve the problem with less errors. It is important to underline that the application of reduced *neighbourhood* is a strict choice only if all the used model (spatial autocorrelation, etc.) is specifically limited to the considered area of the neighbourhood itself. To choose the estimation algorithm, it is obvious to see that not all the interpolators are usable as "exact estimators" because they do not "*honor*" data, we mean they do not give back as estimated value in a known point the true value.

2   THEORETICAL ASPECTS

The main problem in producing contour – maps, is however to produce accurate maps, we mean the accuracy of estimated value of grid. We remember that values at grid nodes are the estimation, not the true values, and they are affected by *estimation errors*. Moreover they are generated from data that are direct or indirect measures of quantities to be mapped that are also affected by *measure errors*. A general problem of estimation and contouring is, in particular, that of using indirect information that are not direct measures of the quantity to be mapped, but measures or information of other variables that are correlable to it. When the number of nodes increases [3], it becomes more difficult to deal with data because of matrix dimensions involved. To solve the problem, we prefer to use *Form Factor Technique* that permits to take into account additional information of diverse meaningfulness, without appealing to a priori correlation analysis. In practice the correlable quantity, that must be " known" to at least to estimation grid nodes, is assumed as a *form parameter*, of the main quantity trend.

2.1   IRFk APPLICATION TO KNOWN POINTS

Let us consider a remote sensed measure $\ddot{e}$ that attributes weight $\ddot{e}_i$ to x points of n dimensional Space R_n. A measure (by definition) is said to be " authorized to order K" if it annuls polynoms of junior or equal degrees to k, so if

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$$\sum_{i=1}^{m} \mathbf{1}_i \, x_i^l = 0 \qquad l = 0, k \tag{1}$$

A generalized increment of order k (GIk) is a linear combination

$$Z(\mathbf{1}) = \sum_i \mathbf{1}_i Z(x_i) \tag{2}$$

where $\ddot{e}$ is an "authorized" discrete measure at order k. Expression "authorized linear combination" is a synonim of Gik. A simple way to build a Gik is to take the difference between a value and a correct linear estimation of this value calculated by assuming a polynomial trend of degree k. In fact

$Z_0^* = \sum_i \mathbf{1}_i Z_i$ is a correct estimator of $Z_0 = Z(x_0)$ if only if : $\sum_i \mathbf{1}_i x_i^l = x_0^l \quad l = 0, k$

and then $\left[\sum_i \mathbf{1}_i Z_i - Z_0\right]$ is a Gik. A **Random function Z(x) is intrinsic of order k** if, for any authorized measure $\ddot{e}$, the random function

$$Z_1(x) = \sum_i \mathbf{1}_i Z(x_i + x) \tag{3}$$

is weakly stationary in $x\hat{I} \; R_n$ and has a mean equals to zero. In general the function

$$K(h) = \sum_{p=0}^{k} (-1)^{p+1} b_p |h|^{2p+1} \tag{4}$$

is a model of generalized covariance of order k [4] under some constraints for coefficients b_p. When the assumptions of stationarity are limited to order k – generalized increments, to characterize the correlation structure of $Z(x)$ we use a new function called **Generalized Covariance Function**, has indicated in (3) and also signed as $K(h)$. With the increase of k the class of IRFk (Intrinsic Random Function of order k) expands, as well as the class of Generalized Covariance Function models.

2.2 IRFk application to a known function

Suppose that $f(x)$ represents concentration or value of an element at a specific point x, which may be a vector (x_1, x_2, x_3) in 3D-space. The average m for a volume v satisfies:

$$m = (1/v)\int_v f(x)dx \tag{5}$$

Suppose that M is the assemblage average for the entire block of study with volume V;
Then:

$$M = (1/V)\int_V f(x)dx = (1/V)\sum m_i v_i \tag{6}$$

Where the m_i are for smaller blocks with volumes v_i. On the basis of $f(x)$ and V, we can define two intrinsic functions:

Covariance: $\quad g(y) = (1/V)\int_V [f(x) - M][f(x+y) - M]dx \tag{7}$

Semivariogram: $\boldsymbol{g}(y) = (1/2V)\int_V [f(x+y) - f(x)]^2 dx \tag{8}$

The variance is $g(0)$; we derive immediately:

$$\boldsymbol{g}(y) = g(0) - g(y) \tag{9}$$

An important problem is to determine the covariance $\boldsymbol{s}(m_1, m_2)$ of averages m_1 and m_2 for two volumes v_1 and v_2.

$$\boldsymbol{s}(m_1, m_2) = (1/V)\int_V (m_1 - M)(m_2 - M)dx$$

$$= \frac{1}{Vv_1 v_2} \int_V \int_{v_2} [f(x+x_1) - M dx_1 \int_{v_2} [f(x+x_2) - M]dx_2 dx$$

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$$\frac{1}{Vv_1v_2}\int_{v_1}dx_1\int_{v_2}dx_2\int_{V}[f(x+x_1)-M][f(x+x_2)-M]dx \tag{10}$$

From Equation 7, we obtain:

$$\boldsymbol{s}(m_1,m_2)=\frac{1}{v_1v_2}\int_{v_1}dx_1\int_{v_2}g(x_2-x_1)dx_2$$

or, when y represents the distance between all possible pairs of points in the two volumes:

$$\boldsymbol{s}(m_1,m_2)=\frac{1}{v_1v_2}\int_{v_1}dx_1\int_{v_2}g(x)dx_2 \tag{11}$$

Thus σ(m₁,m₂) is an average of the covariance function g(y). When v₁ and v₂ coincide, it reduces to:

$$\boldsymbol{s}^2(m)=\frac{1}{v^2}\int_{v}dx_1\int_{v}g(y)dx_2 \tag{12}$$

representing the variance of average concentration or value m [5] for a block with volume v.

3   SPATIAL MEASUREMENT MAPPING APPROACH

In many practical cases we do not known the function that describes the spatial distribution trend, so we have to retrieve spatial data distribution by using an accurate interpolation. We have chosen to do it by means of Intrinsic Random Function of order k. We have used an appropriate software [6] to build the spatial distribution of measured and integrated liquid water in Rome atmosphere in year 1997. The algorithmic approach we have adopted is described below:

Input/reading and preliminary analysis of available data, in this stage we input all data concerning measured and integrated liquid water with their spatial locations. The software executes a preliminary analysis by giving a weight to each data of the selected neighborhood ;

*Order k identification of Random Function (RF), w*e select the order k on the basis of the previous step. The algorithm calculates the average order and the rms for each order k over all the processed area;

Identification of Generalized Covariance function model, in this case the algorithm , with a loop cycle on the recognition areas, acquires data barycentre locations, data number internal to the area, selected data number and data maximum distance from barycentre;

Identification of variogram function model, whether the order k is equal to zero, it may be suitable to operate by using Stationary Geostatistics. The autocorrelation function to be identified is then the variogram. The variogram identification provides to compute experimental variograms and to select variogram model;

Evaluation of a predefined grid nodal variables with Kriging test, the algorithm, through Kriging test [7], assigns   a value of a regionalized variable to nodes of a regular grid. Hence it is possible to determine the following quantities: the average of experimental errors of evaluation, Rms and standardized Rms with the relative evaluation variance;

Editing of map results, all the evaluation operations conclude with the graphic representation of results, that, we remember, are constituted by two value grids: the first one relative to evaluated values and the second one relative to evaluation variance. In Figure 1 we see a 2D-mapping of contour line and a 3D-mapping with a net surface. In Figure 2, we have a graph that represents the reconstruction of liquid water after processing. The liquid water is expressed in mm.
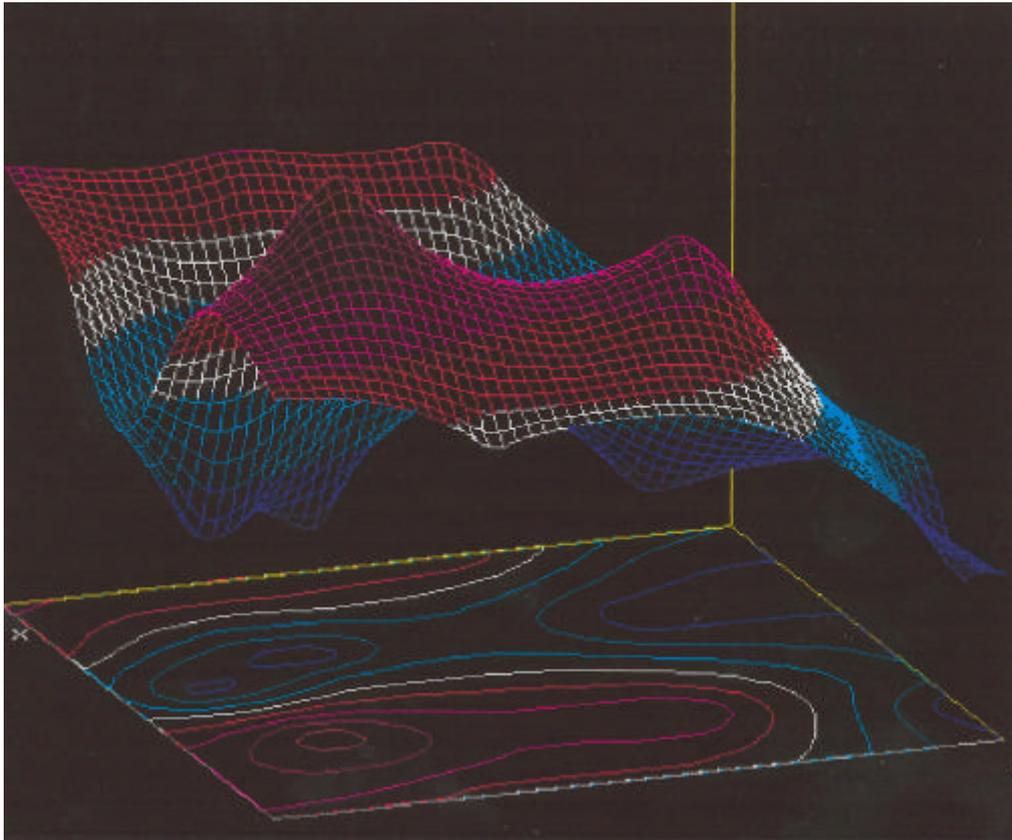
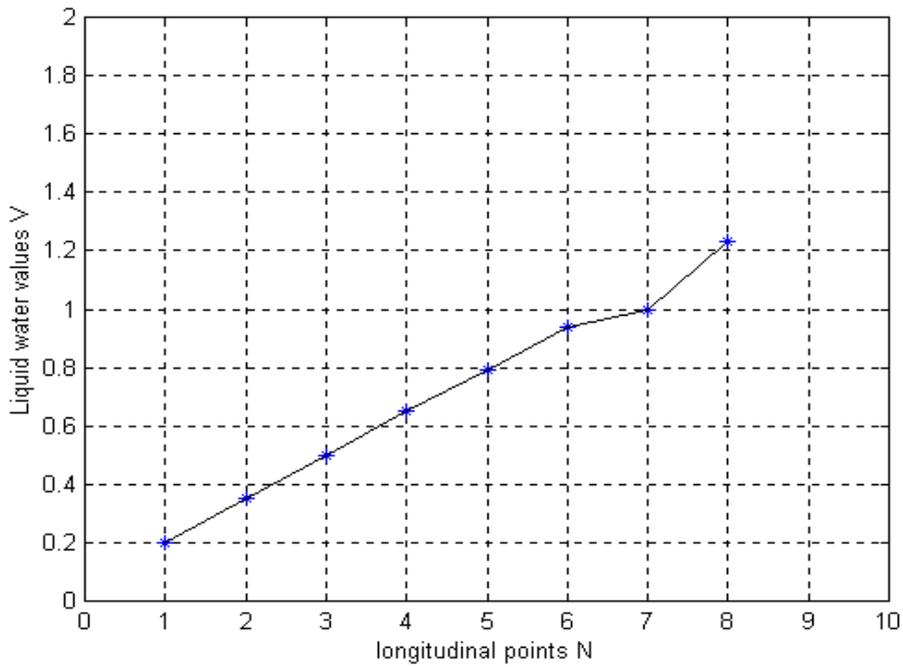**Figure 1**. 2D & 3 D contour map of measured and integrated liquid water

Figure 2. Cross section of liquid water values vs longitudinal points

4   REMARKS

We have presented a good technique of processing localized data from Remote Sensing by using Intrinsic Random Functions of order *K* (IRFk's). We have used IRFk's to evaluate the amount of liquid water in the atmosphere. IRFk's overcome limitations of numerical radiative transfer model interpolation data [9]. The numerical radiative transfer model has applications in analysis and design of passive remote-sensing systems and development of precipitation-parameter-retrieval algorithms using microwave data. The advantage of IRFK's ,in particular, Kriging technique, is to obtained a correct value of liquid water. In Kriging procedure [10]   the value at any point is estimated by a weighted linear combination of the nearby indicators. The weights are chosen so that the resulting estimate is unbiased and has a minimum estimation variance. An other goal of this work has been to show the possibility of recovering data in spatial locations where we do not get them by means of measurements.

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