XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

SQUARE WAVEFORM FOR CALIBRATION: UNCERTAINTY ANALYSIS

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Abstract: On the market are available many data acquisition systems not specifically designed for measurement application. When used for this particular purpose, although the digital stage would be adequate, the analog front-end is often source of unacceptable uncertainty. A new opportunity offered by modern powerful digital processor hosted on data acquisition systems consist in employing the processor for identify and compensate the non-ideal behavior of the analog front-end in real-time. The effectiveness of this method is greatly conditioned by the accuracy that can be achieved in the identification of the analog stage transfer function.

This paper aims to investigate the effect of uncertainty in the test signal generator used to produce well-tailored stimula for the identification process. The use of a square waveform as a test signal has been investigated. This signal can be easily generated by the acquisition system itself, at low cost, with a proper accuracy level.

Keywords: Measurement Science, Estimation of Uncertainty and Errors in Measurement, Evaluation of Measurement Results

1. INTRODUCTION

At present the performances of data acquisition systems, suitable for measurement applications, are generally limited by the analog front-end stage. For instance the presence of the anti-aliasing filter causes an unacceptable increase in global uncertainty [1]. The reason for that resides in the shape of the actual low pass filter transfer function that in the pass band has a response which is not flat in amplitude and linear in phase. Such a behavior of the anti-aliasing filter inevitably introduces higher uncertainty than the ADC quantization error.

Nevertheless the good time invariance of the analog filter characteristics allows introducing digital compensation techniques in the measurement process, that permits to greatly reduce the distortions due to the analog filter transfer function. This method is based on the introduction of a digital filter with theoretical transfer function reciprocal of the analog input filter one in the pass-band (see fig. 1) [1-5].

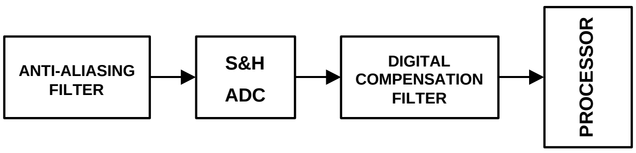

Figure 1. Typical structure of a digital acquisition system with digital compensation filter.

The accuracy in knowledge of the analog filter transfer function limits the compensation possibility of this methodology. It is extremely important thus to analyze the anti-aliasing filter transfer function estimation procedure, with respect to experimental measurement uncertainty propagation.

In this paper the authors present a study aimed to characterize the uncertainty of a signal generator suitable for identifying the parameters of the analog front-end stage. An example of transfer function estimation is shown together with a detailed analysis of the uncertainty propagation from the test generator to the estimated filter frequency response.

2. TEST SIGNAL CHARACTERISTICS

Considering the requirements of high accuracy and low cost for the compensated data acquisition and measurement system, the definition of a proper test signal for the analog front-end estimation is a critical point. Indeed auto-calibration is a mandatory feature for the system if high accuracy specification has to be met.

Periodic square waveforms are an attractive type of stimulus signal. Its discrete spectrum allows simultaneous sampling at many equally spaced frequencies of the analog front-end transfer function in one test only.

Both test periodic waveform and synchronous sampling signal can be derived from the system clock available on digital circuits of the apparatus using digital dividers as shown in figure 2. The synchronization between test signal and sampling clock guarantees spectrum analysis without any leakage effects.

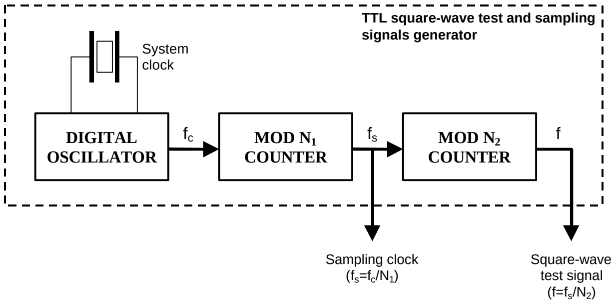

Figure 2. TTL square waveform test signal and synchronous sampling clock generator.

The resulting test signal is a TTL square waveform. From a calibration point of view the uncertainty of phase and amplitude of each harmonic of the square waveform spectrum is too high since it is deeply affected by single components characteristics dispersion as well as power supply and temperature effects. On the other hand considering the ratio of each harmonic with the fundamental the uncertainty of the calibration signal reach a level adequate for calibration purposes as shown in the next paragraph. Furthermore, compared with traditional sinusoidal signal generators, the proposed circuit has the clear advantage of higher accuracy and reproducibility of the test signal at the lowest possible cost.

3. GENERATOR UNCERTAINTY ANALYSIS

The amplitude decreasing of square waveform spectrum limits the number of test signal harmonics available. Moreover square waveform cannot be considered ideal because of dispersions into the amplitude spectrum. The spectrum dispersion can be reduced considering the ratio of each harmonic with the fundamental.

The covariance matrix of the amplitudes of the harmonic components of the input square waveform referred to the fundamental has been estimated by 128 acquisitions of one period of the signal. We analysed the effects of the ADC quantization on the covariance estimation, comparing the statistical analysis using 10, 14, and 15 bit ADC resolutions. Different resolutions in the comparison were obtained masking the same data set acquired with the maximum resolution available.

While using a 10 bit ADC the effects of quantization are relevant on the estimation, the improvement using a 15 bit ADC is negligible compared to the case of 14 bit ADC, as it can be noted in figure 3 and 4. Peak values are interference due to a non-perfect shielding of the acquisition system. Thus we can state that a 15 bit ADC acquisition channel have enough resolution for the statistical signal characterisation up to the 39th harmonic.

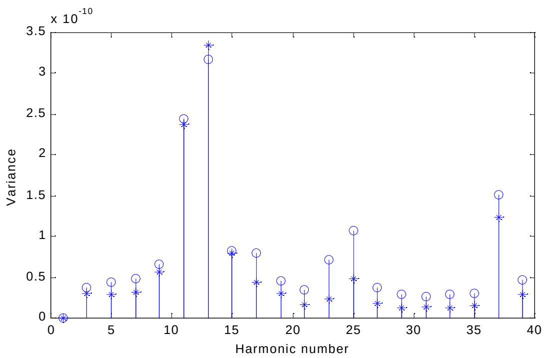

Figure 3. Stars represent estimated variances of the first harmonics using a 15 bit ADC and circles represent the variances estimated by a 10 bit ADC acquisition channel at 23°C.

As shown in figure 3, the harmonic amplitude uncertainty, evaluated at 23°C, is of the order of 10^{-5}. In order to evaluate the influence of ambient temperature on the spectrum stability the variance analysis was performed at 50°C as well. The result is shown in figure 5 where it appears the same order of magnitude for the maximum harmonic variance.

At 23°C the differences between the mean of vector **x** and the corresponding theoretical values are in the order of 10^{-6}. Those differences at temperature of 50°C are increased to 10^{-5}. These results show that each harmonic uncertainty is of the order of 10^{-5} in a range of temperatures from 23°C to 50°C.

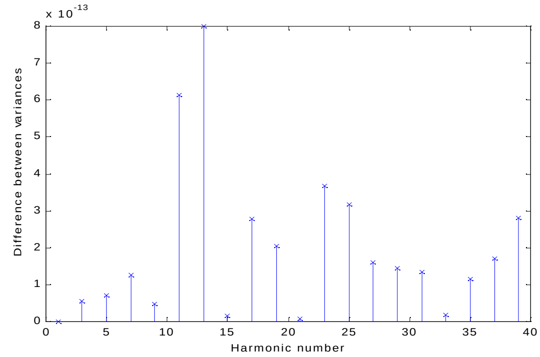

Figure 4. Differences between the variances estimated using a 14 bit ADC and those using a 15 bit ADC at 23°C.

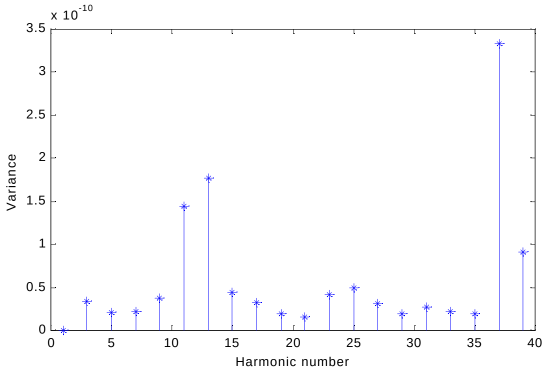

Figure 5 - Estimated variances of the first harmonics using a 15 bit ADC at T=50 °C.

Since the harmonics are produced from the generation of a square waveform they are all correlated. Estimation of correlation coefficients between harmonic components of different order at T=23 °C is shown in figure 6.

Figure 6 - Correlation coefficients between the first 20 odd harmonic components at 23°C.

4. EXAMPLE OF TRANSFER FUNCTION PARAMETRIC ESTIMATION

Let us consider a 4th order anti-aliasing analog filter, characterised by the following transfer function normalised to its gain G_1 at fundamental frequency of the test signal (with the approximation that it is equal to the DC gain):

$$H(s) = \frac{1}{as^4 + bs^3 + gs^2 + ds + 1} \tag{1}$$

We will consider the inverse value of the squared amplitude frequency response, that is an eight-degree polynomial of w and a four-degree polynomial of w^2:

$$\left|H(jw_i)\right|^{-2} = \left|aw^4 - jbw^3 - gw^2 + jdw + 1\right|^2 \tag{2}$$

The adopted estimation algorithm is a least squares estimation of model (2), linear in four parameters. Let the random variable x_i be the $(2i+1)^{th}$ harmonic component amplitude of the input square waveform referred to the fundamental and the random variable y_i the measured $(2i+1)^{th}$ harmonic amplitude of the filter output signal referred to the fundamental. Experimental amplitude frequency responses at $w=w_i$ are given by:

$$H_i = \frac{y_i}{\bar{x}_i} = \frac{\left|H(jw_i)\right|}{\bar{x}_i} x_i = \left|H(jw_i)\right| + e_i \qquad i=1,\ \ldots\ ,N \tag{3}$$

where $\bar{x}_i$ is the mean of the random variable x_i and e_i is a random variable the overall measurement error of the filter gain.

The previous normalisation provides an estimation procedure independent on test signal amplitude, but it does not allow the evaluation of G_1. On the other hand low pass filters show a flat frequency response at low frequencies. If the fundamental frequency is low enough, the gain G_1 can be measured with a simple and inexpensive DC test.

Expression (3) can be written in a matrix form as:

$$\mathbf{e} = \mathbf{Mx} + \mathbf{m} \tag{4}$$

where **M** is the diagonal matrix:

$$\mathbf{M} = diag\left\{\frac{\left|H(jw_1)\right|}{\bar{x}_1}, \frac{\left|H(jw_2)\right|}{\bar{x}_2}, \ldots, \frac{\left|H(jw_N)\right|}{\bar{x}_N}\right\} \tag{5}$$

and **m** is the column vector:

$$\mathbf{m} = -\left(\left|H(jw_1)\right|, \ldots, \left|H(jw_N)\right|\right)^T \tag{6}$$

The covariance matrix **C**[**e**] of **e** can be derived from the covariance matrix **C**[**x**] of **x** with the following relation [6]:

$$\mathbf{C}[\mathbf{e}] = \mathbf{M} \cdot \mathbf{C}[\mathbf{x}] \cdot \mathbf{M}^T \tag{7}$$

Let us define the experimental data vector **g** of components:

$$g_i = H_i^{-2} - 1 = \left|H(jw_i)\right|^{-2} + \hat{e}_i \qquad i=1\ ,\ \ldots\ ,N \tag{8}$$

Indicating with:

$$\mathbf{N} = diag\left\{\frac{-2}{\left|H(jw_1)\right|^3}, \frac{-2}{\left|H(jw_2)\right|^3}, \ldots, \frac{-2}{\left|H(jw_N)\right|^3}\right\} \tag{9}$$

an expression analogue to (7) can be obtained:

$$\mathbf{C}[\hat{\mathbf{e}}] = \mathbf{N} \cdot \mathbf{M} \cdot \mathbf{C}[\mathbf{x}] \cdot \mathbf{M}^T \cdot \mathbf{N}^T \tag{10}$$

For numerical stability, it is necessary to scale angular frequencies referring them to the maximum experimental angular frequency w_{MAX} ($w_n = w/w_{MAX}$, $w_{ni} = w/w_{MAX}$).
After some algebraic calculations, (8) can be expressed, using (2), as:

$$g_i = a_1\Omega_i^{\,4} + a_2\Omega_i^{\,3} + a_3\Omega_i^{\,2} + a_4\Omega_i\,\hat{e}_i \qquad i=1,\ \ldots\ ,\ N \tag{11}$$

where $\Omega_i = w_{ni}^{\,2}$ and the new four parameters are related to those appearing in (1) through the non linear system:

$$\begin{aligned} a_1 &= w_{MAX}^{\;8}\,a \\ a_2 &= w_{MAX}^{\;6}\left(b^2 - 2ag\right) \\ a_3 &= w_{MAX}^{\;4}\left(g^2 + 2ae - 2bd\right) \\ a_4 &= w_{MAX}^{\;2}\left(d^2 - 2eg\right) \end{aligned} \tag{12}$$

The linear model (11) can be expressed in matrix form as:

$$\mathbf{g} = \mathbf{V}\cdot\mathbf{a} + \hat{\mathbf{e}} \tag{13}$$

where the regression matrix **V** shows generalised Vandermonde structure:

$$\mathrm{V} = \begin{bmatrix} \Omega_1^{\,4} & \Omega_1^{\,3} & \Omega_1^{\,2} & \Omega_1 \\ \Omega_2^{\,4} & \Omega_2^{\,3} & \Omega_2^{\,2} & \Omega_2 \\ \vdots & \vdots & \vdots & \vdots \\ \Omega_N^{\,4} & \Omega_N^{\,3} & \Omega_N^{\,2} & \Omega_N \end{bmatrix} \tag{14}$$

The unknown parameters **a** can be estimated by generalised least squares solution of (13) [6]. This approach requires the **g** covariance matrix in order to evaluate both the solutions of (13) and its covariance matrix.

From an operative point of view – consistent with the industrial application of the proposed methodology – we can define a *nominal covariance matrix* $\mathbf{C}_{\hat{\mathbf{e}}}$ that is given by (10) considering the actual estimated transfer function equal to the nominal one and using experimental characterisation of the test signal to fill **C**[**x**]. Generalised least squares normal equations give the following estimation of the vector of parameters **a**= (a_1 , a_2 , a_3 , a_4)^T :

$$\hat{\mathbf{a}} = \left(\left(\mathbf{V}^T\cdot\mathbf{C}_{\hat{\mathbf{e}}}\cdot\mathbf{V}\right)^{-1}\cdot\mathbf{V}^T\right)\left(\mathbf{C}_{\hat{\mathbf{e}}}^{\;-1}\cdot\mathbf{g}\right) \tag{15}$$

Parameters in (1) can be derived from those estimated by (15) by numerical solution of the non-linear system (12), starting from nominal values.
The expression for the covariance matrix of the least squares solution [6] is:

$$\mathbf{C}_{\hat{\mathbf{a}}} = \left(\mathbf{V}^T\mathbf{C}_{\hat{\mathbf{e}}}^{\;-1}\mathbf{V}\right)^{-1} \tag{16}$$

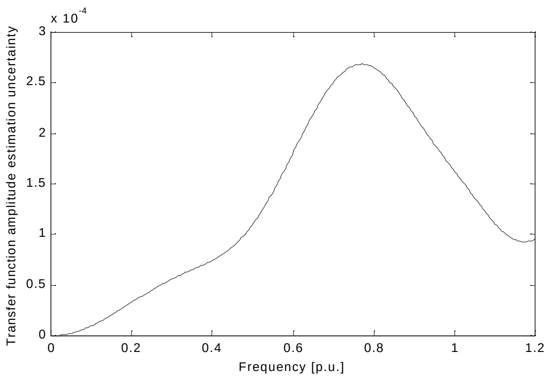

Figure 8. Expanded uncertainty band (coverage factor k=3) on the amplitude estimation of the transfer function, using 12 harmonic components.

The matrix $\mathbf{C}_{\hat{\mathbf{a}}}$ would be the covariance matrix of the parameters in the case that $\mathbf{C}_{\hat{\mathbf{e}}}$ were the data covariance matrix. From that expression, the variance of the amplitude and of the phase of the estimated transfer function can be derived. As an example figure 8 shows the expanded uncertainty for the estimation of the filter amplitude response considering a coverage factor k=3. It shows that the uncertainty level, in a frequency range up to 120% of the cut-off frequency, is smaller then $3 \cdot 10^{-4}$.

5. CONCLUSIONS

Recent developments in signal processing techniques open the way to a new approach towards the design of digital acquisition systems for measurements. The availability of a high performance Digital Signal Processors (DSP), able to carry out complex digital processing, pushes to employ the software procedures for the compensation of the real behaviour of the analog front-end devices. The DSP used for the above application usually are capable to sustain an over computational burden for data acquisition pre-processing.

Once that the actual analog front-end transfer function has been identified, a well-tailored pre-processing algorithm is introduced to compensate the roll-off amplitude and the phase non-linearity in the analog front-end pass band. The resulting increment of the acquisition system accuracy justify the further computational burden require to the DSP.

The process for the estimation of the analog transfer function parameters requires a reference signal generator suitable for an industrial environment, both in terms of accuracy and cost. In this paper we propose to generate such a signal, together with a synchronous sampling signal, directly from the system clock available on digital circuits of the apparatus using digital dividers. This solution has the clear advantage of the lowest possible cost.

Starting from the spectral properties of this kind of signal, the paper describes the procedure followed for the complete uncertainty characterisation of this signal generator, showing good performances in terms of accuracy and reproducibility of the test signal, in agreement with many industrial measurement requirements.

An example of anti-aliasing low-pass filter transfer function estimation has been reported, able to give both the estimated parameters of the filter transfer function and the uncertainty of the estimated amplitude frequency response.

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