A POSSIBLE IMPROVEMENT TO THE ISO-GUM

G. Fanti

Dipartimento di Ingegneria Meccanica, University of Padova, Italy

Abstract: The ISO-GUM (Guide for Uncertainty Measurement) is a milestone in the history of uncertainty analysis, but practical procedures have still not been completely defined. In order to clarify some operative aspects, this paper proposes a practical improvement to the GUM, presenting a step-by-step procedure that illustrates how to evaluate uncertainty, developing analysis in parallel with the measurement process of the parameter. Following the ISO-GUM baseline, the proposed procedure is subdivided into: a) modelling stage: before designing a measurement chain suitable for a certain parameter, the modelling of the physical system under analysis and of the measurement system used to acquire that parameter is examined; b) instrument design stage: the metrological characteristics of each component of the measurement chain are analysed, also considering the traceability of reference samples; c) advanced stage: all previously detected components are combined with those from test operating conditions, e.g., effects of time, interfering and modifying disturbances; d) correction stage: the systematic effects are accounted for; e) indirect measurement stage: uncertainty propagated to the result of an indirect measurement is obtained by means of sensitivity indexes.

Keywords: uncertainty, measurement, procedure.

1   INTRODUCTION

The ISO-GUM (Guide for Uncertainty Measurement) [1] contains some points which could be clarified: the experimenter using GUM in practice may encounter difficulties in applying a formally rigorous tool to procedures which are in themselves approximate. Operators who are required to associate uncertainty with indirect measurements may often encounter some problematic situations; although this is not the case when they must assign uncertainty to simple measurements, following a procedure established by guidelines, e.g., measurements of size with certified caliper. Problems arise when the parameter derives from the processing of several indirect measurements which, in turn, may have non-negligible systematic effects.

For example [2], one debatable aspect of the GUM is the treatment of correlated measurements. Standard uncertainty often includes that quote of systematic effects which has not been eliminated and which, in repeated measurements, should be characterised by a correlation coefficient of 1. As these effects are mixed with random ones, it is not easy to assign a suitable overall correlation coefficient to the parameter measured. As a practical result, often at a normative level [3], for the sake of simplicity a correlation coefficient of zero is assigned, producing an under-estimation of the uncertainty measurement.

This work proposes a procedure more detailed than that presented in § 8 of the ISO-GUM, presenting a step-by-step procedure which allows an uncertainty value to be assigned to the measured parameter not, as is frequently done, *a posteriori,* but by following the various phases of the measurement process.

2   DETAILED PROCEDURE TO EVALUATE UNCERTAINTY

Due to the multiple aspects which must be considered in uncertainty analysis, it is not always easy to assign a realistic uncertainty value, which may be under-estimated, to the measured parameter.

A description of the procedure for evaluating and expressing uncertainty presented in § 8 of the ISO-GUM, may be expressed in a more detailed form developing analysis in parallel to the measurement process of the parameter.

In particular, the first section of § 8 is perhaps expressed in too compact a form: "1   Express mathematically the relationship between the measurand Y and the input quantities X_i, on which Y depends: $Y=f(X_1, X_2, \ldots X_n)$. The function f should contain every quantity, including all corrections and

correction factors, that can contribute a significant component of uncertainty to the result of the measurement (see 4.1.1 and 4.1.2)".

The determination and mathematical formulation of "every quantity, including all corrections and correction factors" is not often easy to perform; to simplify these operations, a procedure for practical evaluation of the uncertainty of a parameter, carried out in parallel with the measurement process during five measurement stages is presented below, in which the distinction is made between the direct measurand Y and the indirect measurand $Z=f(Y_1, Y_2, \dots Y_n)$. Although not explicitly shown in the proposed procedure, the covariances associated with each correlated uncertainty must always be evaluated.

The procedure is first applied to each direct measurand Y_i and then to the indirect measurand Z, according to Figure 1.
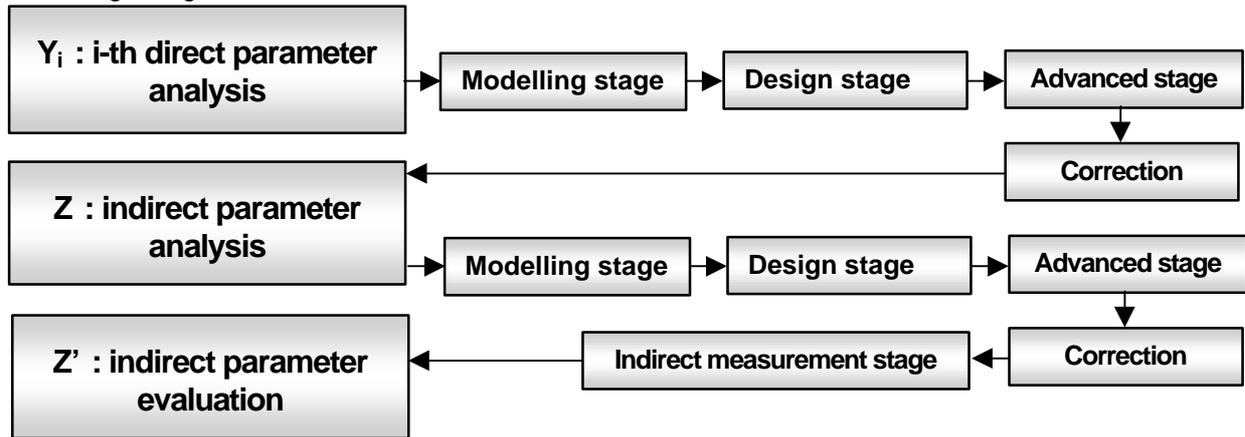

Figure 1. Scheme of the proposed procedure for evaluating uncertainty in parallel with the measurement process; it is first applied to each direct measurand Y_i and then to the indirect measurand Z,.

2.1   Modelling stage uncertainty analysis

Before designing a measurement chain suitable for measuring a parameter, both the modelling of the physical system being considered and the measurement system used to acquire that parameter must be analysed.

a) Modelling of the physical system. The parameter must be defined by referring to a suitable model of the physical system being measured.

If parameter Y_i is directly determined by means of the acquisition of input quantity X_1 and other quantities X_i, such as correction factors, the equation $Y_i = f(X_1, X_2, \dots X_n)$ must be defined. For example, if load F is measured by means of strain gauges, it is a function f of signal conditioning output X_1 and other quantities X_i such as temperature and long wire effects.

If the parameter is indirectly determined by means of the acquisition of other input quantities Y_i, the equation $Z = f_1(Y_1, Y_2, \dots Y_n)$ must be defined in a sufficiently detailed way.

For example, if bending moment M at the mid-point of a beam of length L, simply supported at both ends, is to be measured (see Figure 2), it may be not enough to evaluate uncertainty in the modelling of the physical system of the beam according to the theory of elasticity, which gives the equation: M=FL/4. The uncertainty due to the perpendicularity of applied load F and that of the effective position along the beam which causes a variation in coefficient 4 must also be evaluated.

It may be not enough just to measure the value of load F and length L of the beam in order to measure parameter M; we must also verify to what extent the proposed model corresponds to physical reality.
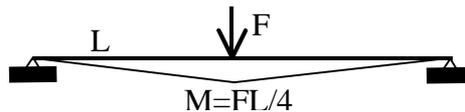

Figure 2. Example of physical system modelling: in some cases equation M=FL/4 may not be sufficiently accurate to evaluate bending moment M because coefficient 4 also depends on effective position of load F.

A more complicated equation may be chosen to define the measurand better, but in any case the representativity of the model used must be evaluated by expressing the modelling-of-the-physical-system-uncertainties (which are frequently only of Type B) $u^a_{mA1}, u^a_{mB1}, u^a_{mA2}, u^a_{mB2}, \dots u^a_{mAn}, u^a_{mBn}$.

According to § 3.4.3 of the ISO-GUM, "in order to predict the standard deviation obtained by combining various uncertainty components", u^a_{mAi}, u^a_{mBi}, may be combined according to § 5 of the ISO-GUM obtaining u^a_{mA}, u^a_{mB} modelling-of-the-physical-system-uncertainties.

b) Modelling of the measurement system. The response of the measurement system used for acquiring the parameter must be modelled, and the effect of inserting this whole measurement chain on the physical system must be also evaluated.

If, in the example of Figure 2, a caliper is used to measure beam length L, the contact surfaces of the caliper must be modelled as smooth parallel planes, and the force applied by the touching sensor must not deform the beam.

Once a representative model of the measurement system has been defined, the representativity of the measured parameter is studied by evaluating the modelling-of-the-measurement-system-uncertainty u^b_{mA}, u^b_{mB}, as discussed in 2.1.a, replacing index a by b.

Lastly, the modelling uncertainties of both physical and measurement systems are combined, to give Type A and Type B modelling uncertainty values u_{mA}, u_{mB} according to § 5 of the ISO-GUM (Figure 3). For example, in the case of a zero correlation coefficient, Type B modelling uncertainty u_{mB} may be evaluated as:

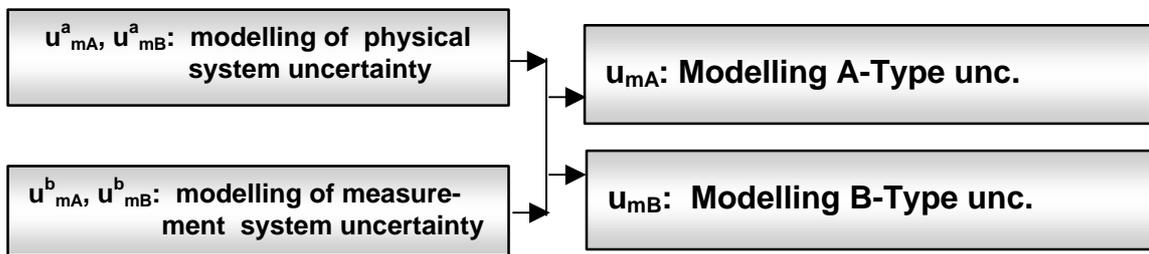$$u_{mB} = \sqrt{(u^a_{mB})^2 + (u^b_{mB})^2} \tag{1}$$

Figure 3. Uncertainty during modelling stage.

2.2   Design stage uncertainty analysis

The metrological characteristics of each component of the measurement chain are analysed during design stage uncertainty analysis. In this phase, uncertainty analysis must also be used to select the most suitable measuring instrument and reference sample.

For example, among the uncertainty components, for each instrument the resolution, repeatability, linearity and drift uncertainties may be evidenced; if the information is incomplete, resolution uncertainty and the manufacturer's declaration regarding instrumental uncertainty could be examined.
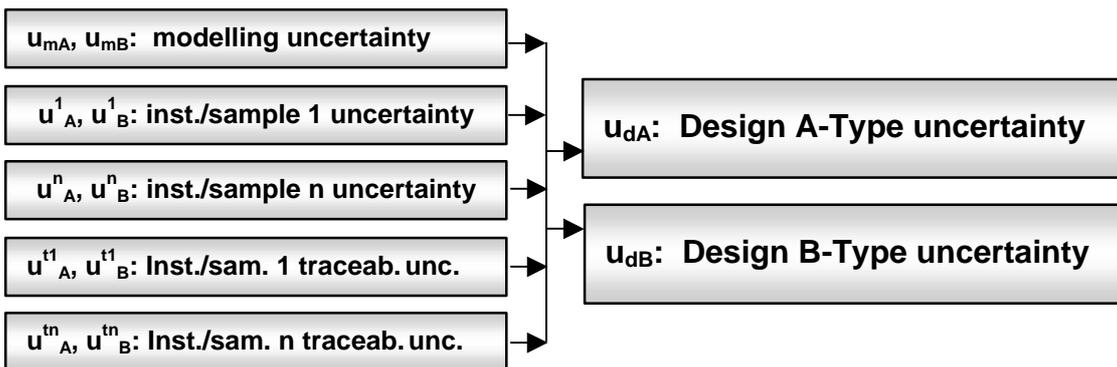

Figure 4. Design stage uncertainty.

The metrological features of each measuring instrument or reference sample are examined and their traceability is considered; there must be no any break in the traceability path. If the i-th measuring instrument or reference sample cannot be maintained in the traceability path for any reason (e.g. time or economy), further traceability uncertainty values u^{ti}_A, u^{ti}_B must be assigned to it.

Estimation of Type A and Type B design stage uncertainty values u_{dA}, u_{dB} is carried out according to § 5 of the ISO-GUM in which modelling uncertainty is included (Figure 4). In the case of a zero correlation coefficient, Type A and Type B design stage uncertainty u_{dA}, u_{dB} may be evaluated as:

$$u_{dA} = \sqrt{(u_{mA})^2 + \sum_{i=1}^{k}\left(u_A^i\right)^2 + \sum_{i=1}^{k}\left(u_A^{ti}\right)^2} \;\; ; \; u_{dB} = \sqrt{(u_{mB})^2 + \sum_{i=1}^{k}\left(u_B^i\right)^2 + \sum_{i=1}^{k}\left(u_B^{ti}\right)^2} \quad (2a,b)$$

2.3 Advanced stage uncertainty analysis

During advanced stage uncertainty analysis, design stage uncertainty values (modelling included) are combined with the components from test operating conditions, e.g., the effects of the measurement procedure in each operating phase, and both interfering and modifying disturbances as extraneous variables. It is also necessary at this point to identify clearly which procedures are to be used.

For example, in the case of repeated dimensional measurements, if several reference samples, of equal nominal length but of relatively slight different real length, are used to compare a specimen having unknown length, the uncertainty of the result is less than that obtainable from repeated comparisons with a single reference sample [4].

Following § 3.2.3 of the ISO-GUM also evaluated here are systematic effects which, according § 2.5.2 of reference [5], must number more than four (alternatively, more detailed uncertainty analysis is necessary).

The logical procedure for calculating uncertainty in the advanced stage is shown in Figure 5.
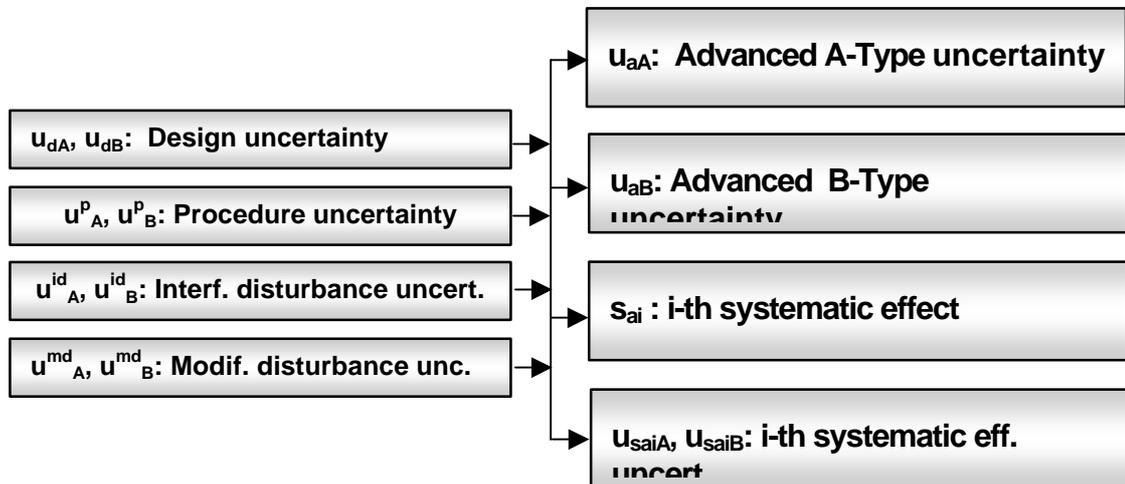

Figure 5. Logical procedure for calculation of advanced stage uncertainty.

Following § 5 of the ISO-GUM, uncertainty estimation of each parameter in the advanced stage is obtained after distinguishing the components, Type A, Type B and systematic effects coming from uncertainty evaluation during the design and procedure stages and including both interfering and modifying disturbances.

Depending on the accuracy of the evaluation of non-negligible systematic effects, i-th systematic effect uncertainty values u_{saiA}, u_{saiB} must also be evaluated. If *all* systematic effects are not deeply studied, a corresponding relatively low uncertainty value could be assigned, and estimation of the parameter may apparently be extremely accurate when in fact it is not.
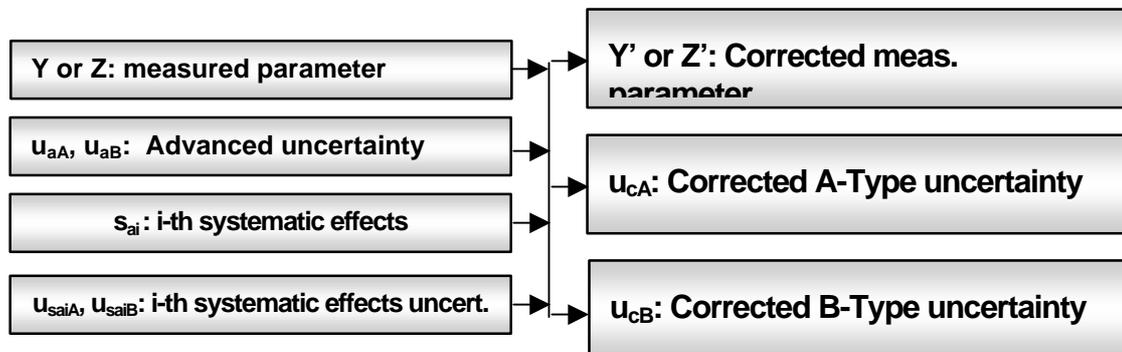

Figure 6. Logical procedure for calculation of correction stage uncertainty.

2.4 Correction stage uncertainty analysis

Systematic effects, with their corresponding uncertainty values detected in the previous stage (Figure 5) are here employed to correct both measured parameter Y or Z and their uncertainty.

Estimation of Type A and Type B correction stage uncertainty values u_{cA}, u_{cB} is carried out according to § 5 of the ISO-GUM (Figure 6).

Following Figure 1, each input quantity Y_i is determined before evaluating indirect parameter Z.

2.5 Indirect measurement stage uncertainty analysis

Uncertainty propagated to the result is obtained by means of sensitivity indexes, according to § 5 of the ISO-GUM. This phase includes all operations of data processing and reduction, e.g., in the case of measuring time-variant parameters. If, for example, one must pass from the time domain to the frequency domain by means of Fast Fourier Transform, the propagation of uncertainty which occurs throughout the analytical transformations inherent in the algorithm used, must be evaluated.
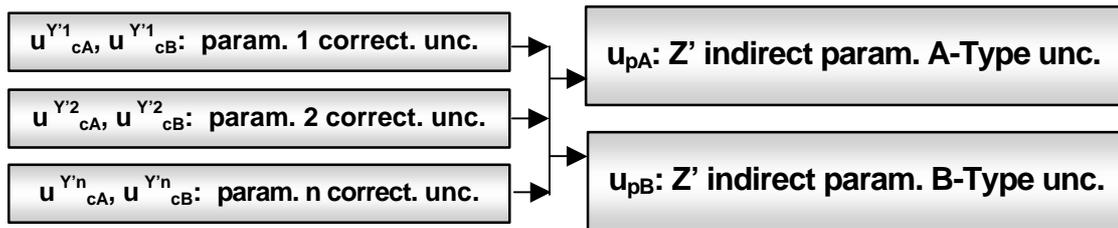

Figure 7. Logical procedure for calculation of indirect measurement stage uncertainty.

2.6 Final uncertainty analysis

Once the indirect parameter symbol Y is substituted with Z, according to § 8 of the ISO-GUM (Points 6, 7, 8), *combined standard uncertainty value $u_c(z)$* of the measurement result is evaluated; then *expanded uncertainty value* $U = k\ u_c(z)$ is determined, if necessary, and finally is compiled a detailed report explaining how uncertainties are obtained.

3 OTHER ASPECTS OF ISO-GUM UNDER POSSIBLE CONSIDERATION

The following points may also be taken into consideration for possible improvement to the ISO-GUM:

-1) Ratio R_{BA} between Type B and Type A uncertainties, named **subjectivity index**, may be evaluated to show which part of uncertainty is evaluated from an assumed probability function (based on the degree of belief that an event will occur) with respect to that part of uncertainty evaluated from a probability density function.

-2) § 3.2.3 of the ISO-GUM states: "If a systematic error arises from a recognised effect … a correction … can be done …", but, in the same section, the possibility that the systematic effect is not recognised is not considered.

For example, in the 1988 radiodating of the Turin Shroud, various systematic effects such as those due to sample contamination which could lead to an uncertainty of the order of a thousand of years, were not taken into account.

Perhaps a new type of uncertainty, **Type C,** could be introduced into the ISO-GUM in order to take into account cases in which uncertainty is assigned on the basis of a completely subjective assumption, because there is a lack of information about the parameter in question.

-3) The same § 3.2.3 of the ISO-GUM states: " It is assumed that, after correction, the **expectation or expected value of the error arising from systematic effect is zero**"

The hypothesis in assuming an expectation value of null is questionable if *all* systematic effects are not completely corrected. The hypothesis that the expectation or expected value is zero appears to be over-restrictive.

For example, in common practice, the linearity error of a caliper may be not so large as to be examined separately and is incorporated with the other uncertainty components. However, if several repeated measurements are carried out near a certain fixed value, while the random component of uncertainty tends towards zero, the correspondent systematic effect remains constant and thus becomes the prevailing component of uncertainty [4].

As it is often not easy to assign a suitable correlation coefficient to the parameter measured, as a practical result, often at a normative level [3], for the sake of simplicity a correlation coefficient of zero is assigned, producing an under-estimation of the uncertainty measurement.

Uncertainty evaluated from ISO-GUM, § 5, eq. 10:

XVI IMEKO World Congress

Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future

Vienna, AUSTRIA, 2000, September 25-28

$$u_{c1}(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)} \tag{3}$$

is thus the minimum possible. It is therefore useful to also evaluate uncertainty in the hypothesis that correlation coefficient r is 1 (or -1 if the correspondent sensitivity coefficient is negative) and use equation eq. 13 of ISO-GUM, § 5, obtaining:

$$u_{c2}(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)u(x_i) \tag{4}$$

Uncertainty is then given by $u_{c1}(y) \leq u_c(y) \leq u_{c2}(y)$ which tends towards value u_{c1} when all the correlation coefficients tend towards zero.

It should be remembered that, in special cases of uncorrelated measurements, non-linear equation (3) may be used, but the following condition must be observed: "The formula can only be applied to equations in which the measured parameters appear directly" [4].

4   CONCLUSIONS

Due to the multiple aspects which must be considered in uncertainty analysis, it is not always easy to assign a realistic uncertainty value to the measured parameter which may be underestimated. On the basis of ISO-GUM, analysis carried out in parallel with the measurement process is proposed to reduce this problem, because the determination and mathematical formulation of "every quantity, including all corrections and correction factors" is not always easy to perform. The proposed procedure is first applied to each direct measurand and then to the indirect measurand.

The procedure is subdivided into: a) *modelling stage:* before designing a measurement chain suitable for measuring a certain parameter, the modelling of the physical system being analysed and of the measurement system used to acquire that parameter is examined; b) *instrument design stage*: the metrological characteristics of each component of the measurement chain are analysed, also considering the traceability of the reference samples; c) *advanced stage:* all the previously detected components are combined with those from test operating conditions, e.g., effects of time, interfering and modifying disturbances; d) *correction stage:* systematic effects are accounted for; e) *indirect measurement stage:* uncertainty propagated to the result of an indirect measurement is obtained by means of sensitivity indexes.

Lastly three other points are taken into consideration for a possible improvement of the ISO-GUM; they are: - the introduction of a subjectivity index, ratio between Type B and Type A uncertainties; - the introduction of a new type of uncertainty, Type C, in order to take into account cases in which uncertainty is assigned on the basis of a completely subjective assumption; - the proposal to evaluate uncertainty values both in the case of correlation coefficients of zero and of one in the frequent cases in which the correlation among the input quantities is not easy to evaluate.

REFERENCES

[1] ISO / TAG 4 / WG 3-GUM, Guide to the expression of uncertainty in measurement, October 1993.

[2] G. Fanti: A procedure to evaluate uncertainty measurement. (Poster session) Proc. 9° Congrès International: Métrologie '99, Bordeaux, France, Oct. 1999.

[3] UNI-EN-ISO 5167-1 Sept. 1997: Misurazione della portata dei fluidi per mezzo di dispositivi a pressione differenziale. Diaframmi, boccagli, venturimetri inseriti in condotti chiusi a sezione circolare. UNI, Milano, Italy 25 Aug. 1997.

[4] G. Fanti, Una certezza nell'analisi dell'incertezza, IV Congresso nazionale di Misure Meccaniche e Termiche, L'Aquila, Italy, July 1999.

[5] ANSI/ASME PTC 19.1.1985: ASME Performance Test Codes Supplement to "Instrument and Apparatus, Part 1: Measurement Uncertainty".

[6] Ronald H. Dieck: "Measurement Uncertainty: Methods and Applications", I.S.A. (Instrument Society of America), North Carolina, U.S.A., January 1995.

[7] B.N. Taylor, C.E. Kuyatt: Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, NIST TN 1297 1994 ed. U.S.A. (http://physics.nist.gov/Pubs/guidelines/TN1297/tn1297s.pdf).

AUTHOR: Assoc. Prof. Giulio Fanti, Dipartimento di Ingegneria Meccanica, University of Padova, Via Venezia 1, 35131 Padova, Italy, Phone +39-49-827-6804, fax +39-49-827-6785,
e-mail: fanti@dim.unipd.it.