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OPTIMIZATION OF A REAL IDENTIFICATION SYSTEM

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Abstract: The results of optimization of a measuring system applied to the identification of a human respiratory tract by a method of forcing non-periodic pressure changes are presented in the paper. The aim of the optimization is minimizing errors of determination of the parameters of a model describing a system being identified in the multidimensional technological space of the applied measuring tools and the parameters of the measurement experiment and identification algorithm. The results obtained in the simulation process showed that the optimum properties of the identification system can be obtained without using the equipment with extreme parameters, i.e. the expensive equipment. This result is contradictory to the approach commonly employed to the design of measuring systems applied in the parametric identification.

Keywords: parameter estimation, identification system optimization, respiratory system identification

1 INTRODUCTION

The design of the human airways parameter estimation system is the aim of the optimization research described in the paper. The results of such measurements are used in diagnosing the respiratory diseases. As one of the simpler human airways models the three-element Mead model [1] was adopted which electric analogue is presented in Fig. 1. The elements of the model represent the compliance C_d of the upper airways, and the resistance R_p and compliance C_p of the lower airways. The additional resistance R_f represents an additional pneumatic resistance of the breathing air afferents within the used measuring system.

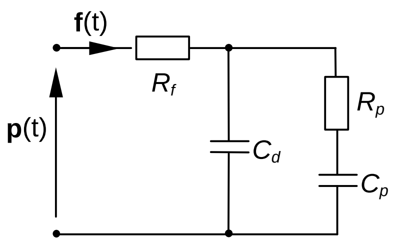

Fig. 1. The Mead model of human airways

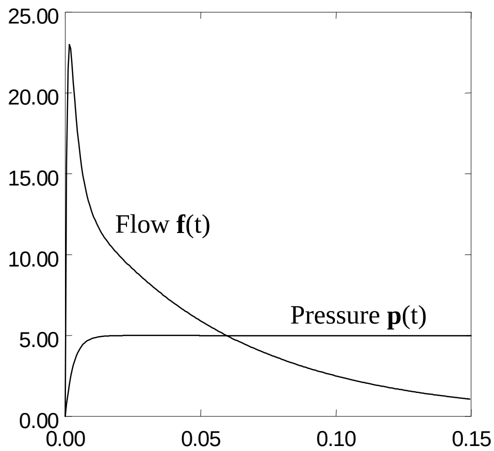

Fig. 2. Inertial forcing signal **p**(t) [cm H_20] and the Mead model response **f**(t) [L/sec] vs. time

The transmittance of the presented model is (1).

$$G(s) = \frac{f(s)}{p(s)} = \frac{1}{R_f} \frac{s\left(sT_pT_{fd} + T_{fd} + T_{fp}\right)}{s^2T_pT_{fd} + s(T_p + T_{fp} + T_{fd}) + 1}, \quad T_p = R_pC_p, \quad T_{fp} = R_fC_p, \quad T_{fd} = R_fC_d \tag{1}$$

The optimized measuring method consists in exciting the patient's airways with a pressure jump applied to the outlet of the airways. The time course of the forced pressure change and the corresponding airflow change at the outlet of the patient's airways, these variables are being measured against the free breath background of the patient under test. The finite rising time of the forcing signal and the dynamic and static properties of the pressure sensors and the pneumotachometer are the factors significantly limiting the identification accuracy. Step-function inputs are used. However, in a real system, the forcing signal **p**(t) changes, which results from the structure of the forcing element, inertially with the time constant T_w and can be seen as the response of a system with the transmittance $G_w(s) = (1 + sT_w)^{-1}$ to the step function A*$\mathbf{1}$(t). The rise time of the forcing signal is limited by the parameters of the used electrovalve and by the properties of the identification system which becomes nonlinear at the forcing signal frequency exceeding 80÷100 Hz. The forcing signal amplitude must be limited for the same reason. These requirements are contradictory to the postulate of using forcing signals with possible wide frequency band, formulated from the identification accuracy point of view.

The following coefficients of the model (1) were adopted in the simulation experiments that were carried out.

R_p=0.35[cm$H_2$0·sec/L], C_p=0.15[L/cm$H_2$0], (T_p=52.5*10^{-3}[sec]), C_d=0.02[L/cm$H_2$0]
R_f=0.1*R_p=0.035[cm$H_2$0·sec/L], (T_{fp}=5.25*10^{-3}[sec], T_{fd}=0.7*10^{-3}[sec])
A=5[cm$H_2$0], T_w=3*10^{-3}[sec]

The Mead model flow **f**(t), determined by simulation, forced inertially by changing the forcing pressure **p**(t) is presented in Fig. 2. The total air volume which flowed as the result of the forcing is near 0.9 L which is about 20% of the adult's lung capacity.

2 SAMPLING PERIOD SELECTION

Based on the model (1) coefficient variance criterion employing the information matrix M, the initial selection will be carried out of the sampling period for the pressure and flow signals. The inverse information matrix is the lower limit of the covariance matrix of these estimates at unbiased estimation. Because of the complex form of the transmittance, analytical calculation of the information matrix is difficult. For the response to the forcing **p**(t), determined by simulation, and assuming uncorrelated and normally $N(0, s)$ distributed disturbances to the quantity **f**(t) measurement results, we can determine with a numerical method an approximation of the information matrix M and the value of the quality criterion defined as the trace of the matrix M^{-1} of the set of N samples taken at the determined time instants t_i. If the assumptions adopted are met, the complete information matrix is the product (2) of the inverse of the measurement disturbance variances and the factor resulting from the properties of the system and the identification process. We will determine only the second factor of the product, the sum of the matrices M_i independent of the measurement disturbance level.

$$M = s^{-2} \sum_{i=1}^{N} M_i = s^{-2} \sum_{i=1}^{N} \frac{\partial f(t)}{\partial q} \frac{\partial f(t)}{\partial q}^T \Big|_{t=t_i} \tag{2}$$

At each time point t_i where the signal is sampled, the value of matrix M_i is determined numerically (3). The symbol q =[R_p C_p C_d]^T denotes the model coefficient vector.

$$M_i = \left[\frac{f(t, q + \Delta q) - f(t, q - \Delta q)}{2\Delta q} \right]_{t=t_i} \left[\frac{f(t, q + \Delta q) - f(t, q - \Delta q)}{2\Delta q} \right]_{t=t_i}^T \tag{3}$$

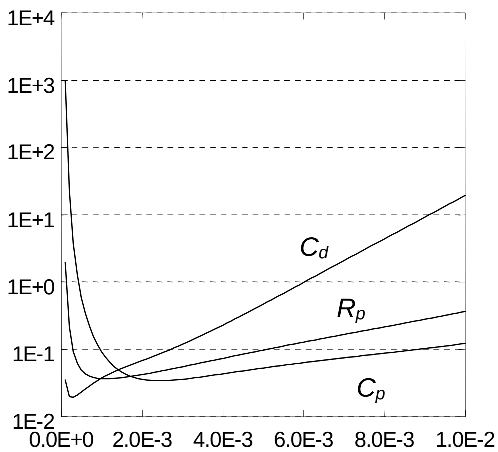

Fig. 3. Relative normalized variance of the estimates of coefficients R_p, C_p, C_d vs. sampling period

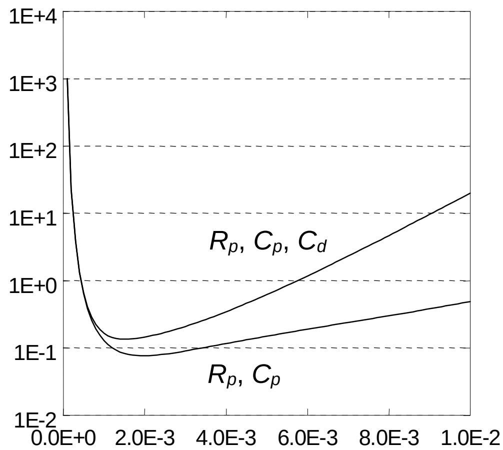

Fig. 4. The trace of the relative normalized covariance matrix S_w of the estimates vs. sampling period for the full vector of coefficients and limited to the coefficients R_p, C_p

The covariance matrix of the estimates is (4) with the matrix S independent of the disturbance variance. To obtain a relative shape S_w of this matrix it is necessary to divide its elements by the corresponding elements of the matrix qq^T.

$$Cov(q,q) = s^2 S = s^2 \left[\sum_{i=1}^{N} M_i(t_i) \right]^{-1} \tag{4}$$

A relative value Δq of coefficient deviation equal to 0.1 % of the coefficient and the size N = 50 of the set of samples of the signal **f**(t) were taken to calculations. A uniform sampling was assumed which began at the moment of the input step. The relationship between diagonal elements of the matrix S_w, corresponding to individual coefficients, and the period dt of sampling of the signal **f**(t) is presented in Fig. 3, and the relationship between the trace of this matrix and the dt parameter is presented in Fig. 4.

Taking into account results described in the further part of the paper, we consider two cases of designing an identification system: to achieve the maximum accuracy of estimation for all coefficients of the Mead model, and for the coefficients R_p and C_p only which bear information on the illness status of the lower airways. Based on the presented results, an optimum sampling period was determined as 1.5 msec for the full vector of the estimated parameters and 2.0 msec for the limited parameter vector.

3 INFLUENCE OF LIMITED DYNAMICS OF THE MEASUREMENT SENSORS

The analysis carried out by the information matrix method assumes the possibility of constructing an unbiased and efficient estimator for which the covariance matrix could be built. As we will see in this section, the limited dynamic properties of the measurement equipment make it impossible to assume that the nonlinear least squares algorithm, used for identification, gives unbiased and efficient results.

The applied identification system optimization method consists in modeling the whole measuring system including forcing and disturbing signals, the identification system, measuring equipment and identification algorithms. Such an approach enables the influence of all modeled elements on the identification accuracy to be taken into account. The identification inaccuracy expressed with the joint rms error (5) is determined by a statistical method with a set of results obtained from simulation of the identification process repeated many times. The joint rms error is equal to the square of the trace of the measurability matrix [5] and, at the same time, is equal to the square of the sum of trace of the parameter estimate covariance matrix and the square norm of the bias vector [6]. This way of describing estimation inaccuracy produces easy to interpret information on the influence of the system identification parameters on the model coefficient estimation error.

$$s = \sqrt{\sum_{i=1}^{n} s_i^2 + \sum_{i=1}^{n} b_i^2} \tag{5}$$

where: s_i^2, b_i^2 - relative variance and squared bias of *i*-th parameter estimator,

n – number of estimated parameters.

The system identification optimization (i.e. the minimization of the criterion (5)) can be carried out not only – as common – in the domain of forcing and sampling frequency [3, 4, 2], but also in the domain of measuring equipment static and dynamic parameters. To this effect, the constructed models of the equipment are used. The main objective of the investigations made was to show that the optimum properties of the identification system can be obtained without the necessity of using the expensive equipment of extreme parameters.

To simulation research, a model of respiratory identification system, presented in Fig. 5, was adopted where P_w is the source of forcing pressure **p**(t) that varies inertially as described in the introduction, F is a model of the Fleich pneumotachometer, R_f is the pneumatic resistance of the forcing pressure connections including the pneumotachometer resistance, P_f is a Validyne pressure sensor model matching to the pneumotachometer, P_p is a similar pressure sensor that measures the forcing pressure. Boxes *Noise* represent the measuring disturbances and the *Human airways* (HA) box describes the Mead model presented in the introduction (Fig. 1). The identification procedure is an implementation of the Levenberg-Marquardt non-linear least squares algorithm available in Matlab.

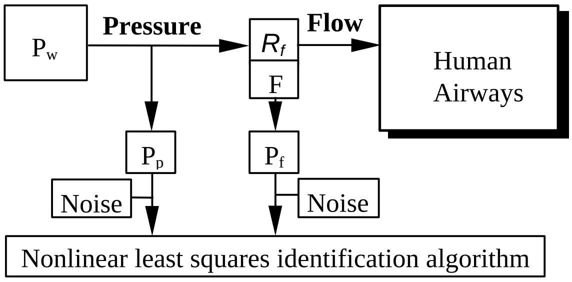

Fig. 5. The identification system model

The model of Validyne pressure sensors P_p, P_f includes only their dominating pneumatic part with transmittance (6), where x is the damping factor, w_n is the natural frequency depending on the length of the pneumatic connections.

$$G_P(s) = \frac{w_n^2}{s^2 + 2xw_n s + w_n^2} \qquad (6)$$

The Fleich pneumotachometer matched to the described pressure sensor is modeled by a first-order inertia of transmittance (7), where T_f is the time constant limiting the transducer dynamics.

$$G_F(s) = \frac{1}{1 + sT_f} \qquad (7)$$

Measuring equipment parameter values were assumed the same which are imposed by the construction, i.e. the pneumotachometer pneumatic resistance $R_f = 0.1*R_p = 0.035$ cmH20·sec/L and the pneumotachometer time constant $T_f = 2$ ms. The pressure sensors' P_p and P_f damping factors were assumed to be $x_p = 0.2$, $x_f = 0.2$, respectively. The length N of the measurement data buffer for the identification procedure is N=50. The pressure and flow disturbances are normally distributed: respectively $N_p(0, 0.01\ cmH_20)$ and $N_f(0, 0.02$ L/sec).

Simulation study was carried out on how the dynamic parameters of measurement quantity transducers influence the identification accuracy. Parameters that can be selected to achieve the highest estimation accuracy of the adopted HA model coefficients are the pressure sensors natural frequencies w_p and w_f changed by selecting the length of the pneumatic connections and the sampling period dt whose initial selection was carried out in Chapter 2 for an idealized case. Based on the initial study and the constructional requirements, the optimum searching area was limited to the intervals of the individual system parameters: dt=0.5÷10 msec, w_p=100÷5000 rad/sec, w_f=100÷5000 rad/sec.

The results of the simulation research on the estimation accuracy of the parameters R_p, C_p, C_d for the equipment parameter values w_p = 5000 rad/sec, w_f = 5000 rad/sec, selected according to the rule of the best possible transducer dynamics, and the sampling period dt = 1.5 msec and dt = 2.0 msec resulting from the estimation in Chapter 2, are presented in Table 1 as the relative (percentage) estimation rms error of individual coefficients. Compared to the results presented in Fig. 3, the estimation rms error of C_d increased substantially more than the errors of the other coefficients.

Table 1. The relative rms error of the coefficients for the initial project of the identification system.

Coefficient	R_p	C_p	C_d
dt =1.5 msec	20.6	1.1	56.8

Coefficient	R_p	C_p	C_d
dt = 2.0 msec	18.3	2.7	57.1

To show a qualitative change in the identification process after the limiting properties of the measuring equipment are taken into consideration, we present the difference between the system response at the output of the transducer P_f and model output for forcing measured with transducer P_p. As it can be seen in Fig. 6 (dotted line), the initial, dynamically changing part of the response is poorly approximated. We conclude from this that the identification accuracy of the coefficient C_d will substantially deteriorate.

Because the rms error of the coefficient C_d is the greatest in the joint estimation rms error and, at the same time, this coefficient is of little use in diagnosis of HA diseases, we decide to limit our description of inaccuracy to the joint estimation rms error of the coefficients R_p and C_p only. The expression containing this error will be an estimation quality criterion for the identification system optimization process.

4 JOINT OPTIMIZATION OF THE SAMPLING PERIOD AND THE PARAMETERS OF THE MEASURING SENSORS

We will carry out optimization with a quality criterion defined as the joint estimation rms error of the coefficients R_p and C_p with predetermined variability ranges of the optimized measuring system parameters, i.e. the sampling period and the pressure sensor natural frequencies. Because of random character of the quality criterion estimate calculated by statistical methods, numerical minimization algorithms using differential information (of the *steepest descent*, *quasi-Newton* type) cannot be used in the optimization procedure. The random component disturbs the performance of these algorithms causing the inadmissible deterioration of the convergence. An application of a Simplex algorithm at an assumed low level of the simulated disturbances of the measuring variables produced good results. Trials made at higher disturbance levels required using some simple algorithms, of the *Mesh* and *Monte Carlo* type, of searching for the global minimum. The best value of the designed measuring system parameters, found with the mentioned methods, are presented in Table 2, and the estimation rms errors at these values of the identification system parameters are presented in Table 3.

Table 2. Designed values of the identification system parameters

Parameter	w_p [rad/sec]	w_f [rad/sec]	*dt* [msec]
Value	390.0	5000.0	5.0

Table 3. Relative estimation rms errors at the designed parameter values

Parameter	R_p	C_p
Criterion **s** [%]	0.6	4.0

We stress the large difference in the optimum sampling period related to the value selected by the information matrix method. The optimum parameter value w_p confirms the proposition that the highest estimation accuracy can be attained for non-extreme measuring equipment parameters. Optimization in the domain of the identification system parameters resulted in the essential increase of estimation accuracy of the parameter R_p with some deterioration of estimation accuracy of the parameter C_p, which is connected with the averaging character of the adopted quality criterion. At the assumed disturbance level, the estimation rms errors are pre-dominated by the parameter bias errors due to the quality of the system response approximation in identification model. The quality of approximation is presented in Fig. 6 as the fitting error. From the identification system design point of view it is essential that a different disturbance level can change the location of the optimum point in the measuring system parameter space. This is a general feature of the projects performed with the criteria linking the random and bias errors [6]. The second effect described in the literature, is the relationship between the solution to the optimization problem and unknown estimated parameter values. The argument supporting the usability of the solution presented in this paper is that the optimization of an identification system can be made only for a particular assumed model of an identification system. Lack of *a priori* knowledge about the system means no possibility to optimize. No doubt that mentioned effects are inconvenience. However, they are an unavoidable feature of the solutions to the stated optimization task.

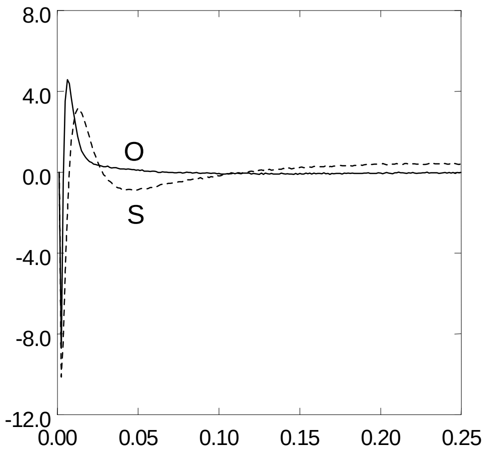

Fig. 6. The error of model response fit to the system response measured with transducers with optimum (O) and starting (S) parameters vs. time

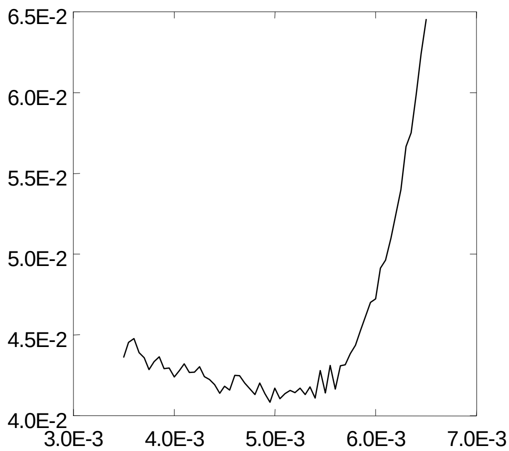

Fig. 7. Variability of the rms error vs. parameter *dt* at the designed values of other parameters

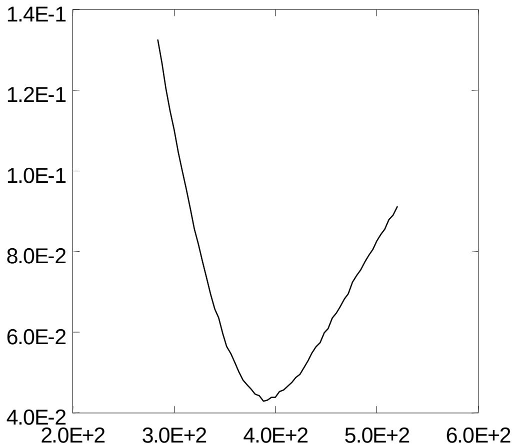

Fig. 8. Variability of the rms error vs. parameter w_p at the designed values of other parameters

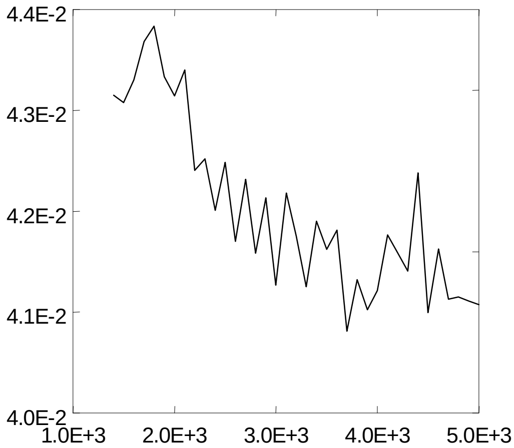

Fig. 9. Variability of the rms error vs. parameter w_f at the designed values of other parameters

In Figs. 7, 8 and 9, where the quality criterion vs. the designed identification system parameters about the designed parameters values is shown, the strong dependence of the quality criterion on the parameter w_p should be stressed together with weaker sensitivity to the parameter *dt* and little to w_f.

5 CONCLUSIONS

Solutions to the problem of the quality criterion minimization in the set of real parameters of measuring tools used in an identification experiment were sought in the paper. Suboptimal values of these parameters were found and they do not lie on the borders of the assumed areas of parameter variability. This means that the problem of such an optimization is not trivial.

The method presented can be applied to designing an identification process with the essential limitations to selecting the static and dynamic parameters of the selected measuring tools while the other tools, forcing signals and algorithms should be selected in such a way (with regard also to technical conditions) that they would possibly compensate for the adverse influence of these limitations.

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