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DITHER IN DIGITAL CORRELATION MEASUREMENTS

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Abstract: An analysis of estimation errors of correlation measurements due to A/D conversion with a dither signal has been carried out. It has been shown that any random signal satisfying appropriate conditions can be used as dither. Gaussian noise, although not satisfying one of these conditions, may influence the improvement in the accuracy of digital measurement. It is also easier to generate than dither with rectangular or triangular envelope probability density function (PDF), and, moreover, it may occur in a system in a form of intrinsic noise described by the term 'self-dither'.

Because the signal mean square value can be expressed by means of an autocorrelation function, the analysis carried out is also valid for the use in this important measurement.

Keywords: correlation measurement methods, A/D conversion with dither, normalized bias error

1 INTRODUCTION

The tremendous technological progress observed recently has not lead to the elimination of non-multilevel correlators from the market. They continue to be made chiefly for specialized measurements in, e.g. radio astronomy, the investigation of ionospheric dissipation, radiolocation. Simple correlation function estimators are still being sought after in order to increase the working rate, simplify measuring procedures or reduce the production cost of the equipment. Hence the already historical one-bit correlator becomes anew an object of interest [1, 2, 3]. Rough quantization in the correlator circuit leads to the deterioration in its accuracy in relation to a multibit system. This may be counteracted by the employment of dither signals in its channels.

The paper is devoted to an analysis of the influence of dither signals on the accuracy of correlation measurements, and the signal mean square value measurement in particular.

2 ESTIMATION ERROR OF CORRELATION FUNCTIONS FOR A/D CONVERSION WITH DITHER

2.1 Crosscorrelation function determination and autocorrelation function determination by crosscorrelation

The crosscorrelation function of ergodic processes {x(t)} and {y(t)} can be expressed as [4]

$$R_{xy}(t) = \lim_{T \to \infty} \frac{1}{T}\int_0^T x(t)y(t+t)dt \qquad (1)$$

where x(t) and y(t) are the process realizations, τ - the time delay, and T is the observation time.

Let us assume that the signals $d_1(t)$ and $d_2(t)$, called dither signals, are added to the signals $x_1(t)$ and $x_2(t)$. The signals $y_1(t)$ and $y_2(t)$ thus obtained are converted into the digital forms $y_{1q}(n)$ and $y_{2q}(n)$, which are delayed by k samples in relation to each other, multiplied, and the result of the multiplication is averaged (Fig. 1). The resulting estimator can be expressed as

$$\tilde{R}^{d}_{xy}(k,N) = \frac{1}{N}\sum_{n=0}^{N-1} x_{1q}(n)y_{1q}(n+k) \qquad (2)$$

If $x_1(t) = x_2(t)$, then the estimator $\tilde{R}^{d}_{x}(k,N)$ of the autocorrelation function (Fig. 2) in the following form is obtained

$$\tilde{R}_x^d(k,N) = \frac{1}{N}\sum_{n=0}^{N-1} x_{1q}(n)\,x_{2q}(n+k) \tag{3}$$

Let us note that the mean square value is the autocorrelation function value for τ=0, and this is why the analysis carried out is also valid for the use in this important measurement.

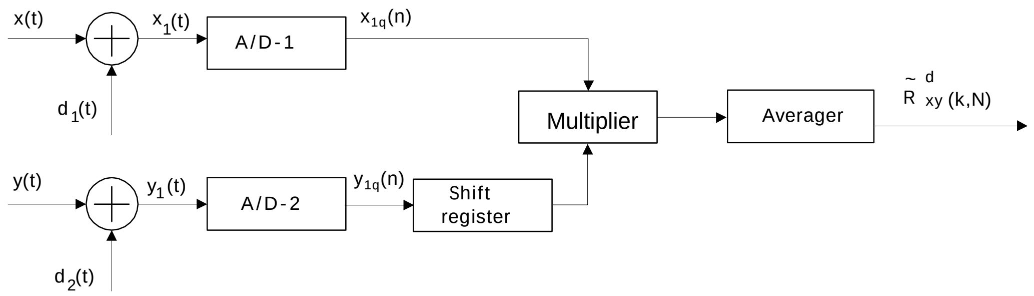


Figure 1. Digital analyzer of the crosscorrelation function with dither signals.

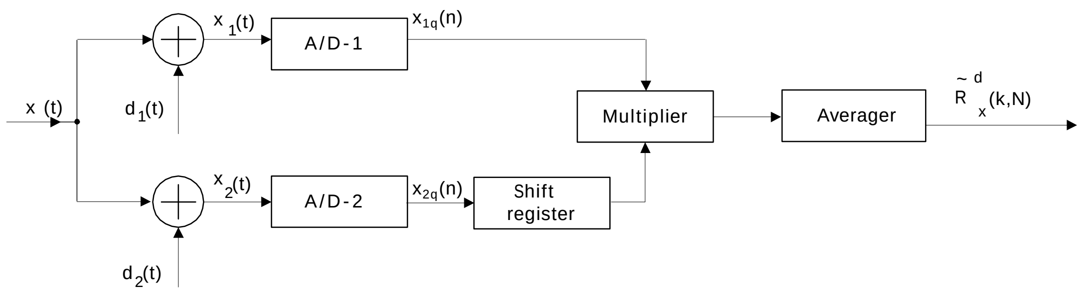


Figure 2. Digital analyzer of the autocorrelation function by crosscorrelation.

It can be shown that the estimator $\tilde{R}_{xy}^d(k,N)$ mean value is equal to [5]

$$E\left[\tilde{R}_{xy}^d(k,N)\right] = E\left[x_{1q}y_{1q}\right] \tag{4}$$

where $x_{1q} = x_{1q}(0)$, $y_{1q} = y_{1q}(k)$. In other words, the mean value of the estimator with dither obtained on the basis of the quantized samples is identical with the estimator obtained from the quantized signals x_{1q} and y_{1q}.

In [6] by using [5] the mean value expression (4) for the roundoff (also called midtread) quantization characteristic is derived, with the assumption that the dither signals satisfy the following conditions:

- they are stochastically independent of the measured signals and of each other,
- they have zero mean values.

It can be expressed as

$$\begin{aligned} E\left[x_{1q}y_{1q}\right] = E[xy] &+ \sum_i \frac{q_1}{2\pi i}(-1)^i \Phi_{d1}\left(-\frac{2\pi i}{q_1}\right)\frac{\partial\Phi_{xy}(-2\pi i/q_1, v_2)}{\partial v_2}\Bigg|_{v_2=0} \\ &+ \sum_j \frac{q_2}{2\pi j}(-1)^j \Phi_{d2}\left(-\frac{2\pi j}{q_2}\right)\frac{\partial\Phi_{xy}(v_1, -2\pi j/q_2)}{\partial v_1}\Bigg|_{v_1=0} \\ &- \sum_i\sum_j \frac{q_1q_2}{4\pi^2 ij}(-1)^{i+j}\Phi_{d1}\left(-\frac{2\pi i}{q_1}\right)\Phi_{d2}\left(-\frac{2\pi j}{q_2}\right)\Phi_{xy}\left(-\frac{2\pi i}{q_1};-\frac{2\pi j}{q_2}\right) \end{aligned} \tag{5}$$

where $\Phi_{xy}(\cdot,\cdot)$, $\Phi_{d1}(\cdot), \Phi_{d2}(\cdot)$ the are the characteristic functions (CF's) of the signals x and y, dither d_1, and dither d_2, respectively; q_1 (q_2) is the quantization step in the first (second) analyzer channel, and the summation limits include the range from $i = -\infty$, $j = -\infty$ to $i = \infty$, $j = \infty$, excluding $i = j = 0$.

It follows from (5) that the crosscorrelation function of the quantized signals x_{1q} and y_{1q} is equal to the real correlation function (of the signals x and y) if and only if the dither signals CF's become zero at the points $2\pi i/q_1$ and $2\pi j/q_2$ ($i = \pm1, \pm2, \pm3, \dots$, $j = \pm1, \pm2, \pm3, \dots$), respectively. This means that the employment of dither signals with the above mentioned properties does not cause the bias - or the systematic component of the estimation error - of the crosscorrelation function estimator (2). This conclusion also concerns the autocorrelation function estimator (3).

If the CF's of the dither signals do not become zero, then the estimator bias error occurs of the level

$$
\begin{aligned}
b\left[\tilde{R}^{d}_{xy}(k,N)\right] = {} & \sum_i \frac{q_1}{2\pi i}(-1)^i \Phi_{d1}\left(-\frac{2\pi i}{q_1}\right)\frac{\partial \Phi_{xy}(-2\pi i/q_1, v_2)}{\partial v_2}\Bigg|_{v_2=0} \\
& + \sum_j \frac{q_2}{2\pi j}(-1)^j \Phi_{d2}\left(-\frac{2\pi j}{q_2}\right)\frac{\partial \Phi_{xy}(v_1, -2\pi j/q_2)}{\partial v_1}\Bigg|_{v_1=0} \\
& - \sum_i \sum_j \frac{q_1 q_2}{4\pi^2 ij}(-1)^{i+j}\Phi_{d1}\left(-\frac{2\pi i}{q_1}\right)\Phi_{d2}\left(-\frac{2\pi j}{q_2}\right)\Phi_{xy}\left(-\frac{2\pi i}{q_1}; -\frac{2\pi j}{q_2}\right)
\end{aligned}
\tag{6}
$$

Gaussian dither belongs among signals which do not satisfy the CF zeroing condition. Its employment in the correlator circuit leads to the estimator bias, and consequently to an error of a systematic nature. However, it is easier to generate than rectangular or triangular envelope PDF dither - satisfying this condition - and, moreover, it may occur in the circuit in a form of intrinsic noise described by the term 'self-dither'. As it turns out, any unintentional analog noise, e.g. due to the electronic circuits of the data acquisition system or embedded in the system input, can often be seen as Gaussian dither. This type of dither is sometimes called a 'self-dither' signal.

2.2 Autocorrelation function determination

Let us assume that an auxiliary signal $d_1(t)$ is added to the signal $x(t)$. The signal $x_1(t)$ thus obtained is converted into the digital form $x_{1q}(n)$. Next, it is multiplied by a copy of itself delayed by k samples, and the multiplication result is averaged. The estimator thus obtained can be expressed as

$$\tilde{R}^{1d}_{x}(k,N) = \frac{1}{N}\sum_{i=0}^{N-1} x_{1q}(n)\, x_{1q}(n+k) \tag{7}$$

Figure 3 presents the basic structure of a system working according to this algorithm.

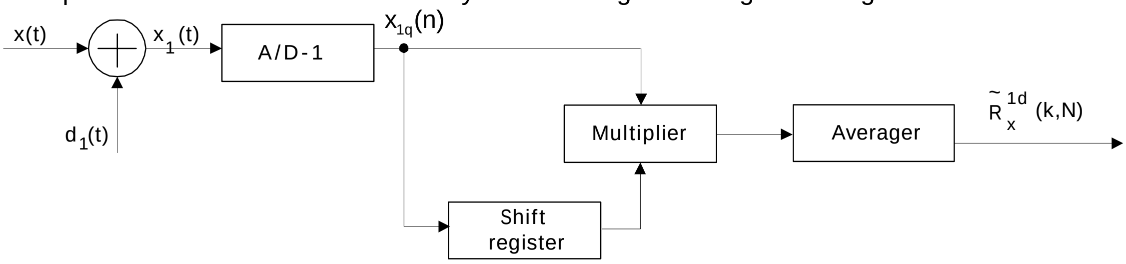


Figure 3. Digital analyzer of the autocorrelation function with dither.

Formulae (4)÷(6) do not apply here, and the evaluation of the estimator bias becomes complicated [6]. As it has been mentioned in Section 2.1, the signal mean square value is the autocorrelation function for $\tau=0$. Therefore, let us consider as an example the metrologically important situation when a sinusoidal signal $x(t)$, whose mean square value $A^2/2$ is to be determined, is accompanied by stochastically independent Gaussian dither $d_1(t)$ with a zero mean value and the dispersion σ_d (equal to rms value).

The signal, dither, and their sum CF's have the form [6]

$$\Phi_x(v) = J_0(Av) \tag{8}$$

$$\Phi_{d1}(v) = \exp(-v^2\sigma_d^2/2) \tag{9}$$

$$\Phi_{x1}(v) = \Phi_x(v)\Phi_{d1}(v) = J_0(Av)\exp(-v^2\sigma_d^2/2) \tag{10}$$

where $J_0(\cdot)$ is a Bessel function of the first kind, order 0.

The estimator $\tilde{R}_x^{1d}(0,N)$ bias of the signal mean square value $A^2/2$ can be expressed as [6]

$$b = \sigma_d^2 + \frac{q^2}{12} + \\ +2\sum_{i=1}^{\infty}(-1)^i\left\{\left[2\sigma_d^2 + \frac{1}{2}\left(\frac{q}{i\pi}\right)^2\right]J_0\left(i\frac{2A\pi}{q}\right) + A\frac{q}{i\pi}J_1\left(k\frac{2A\pi}{q}\right)\right\}\exp\left[-2\left(\frac{i\pi\sigma_d}{q}\right)^2\right] \quad (11)$$

where $J_1(\cdot)$ is a Bessel function of the first kind, order 1.

Let us notice that if $\Phi_{d1}(v) \to 0$ for $v = 2\pi i / q$ $(i = \pm1, \pm2, \pm3, ..)$, then the bias error (11) would become reduced to the sum $\sigma_d^2 + q^2/12$ and could be disposed of by means of adding the correction $-(\sigma_d^2 + q^2/12)$ to the result.

Figure 4a presents the normalized bias error $\delta = b/(0.5 \cdot A^2)$ as a function of the amplitude A for q=1 for different values of σ_d (0; 0.2q; 0.3q; 0.5q, q). The specific values of σ_d correspond to the situation in which, with the probability of 0.997, the dither signal peak-to-peak value does not exceed 0; 1.2; 1.8; 3, and 6 LSB, respectively.

Figure 4b presents the normalized bias $\delta_r = b_r/(0.5 \cdot A^2)$, where $b_r = b - (\sigma_d^2 + q^2/12)$.

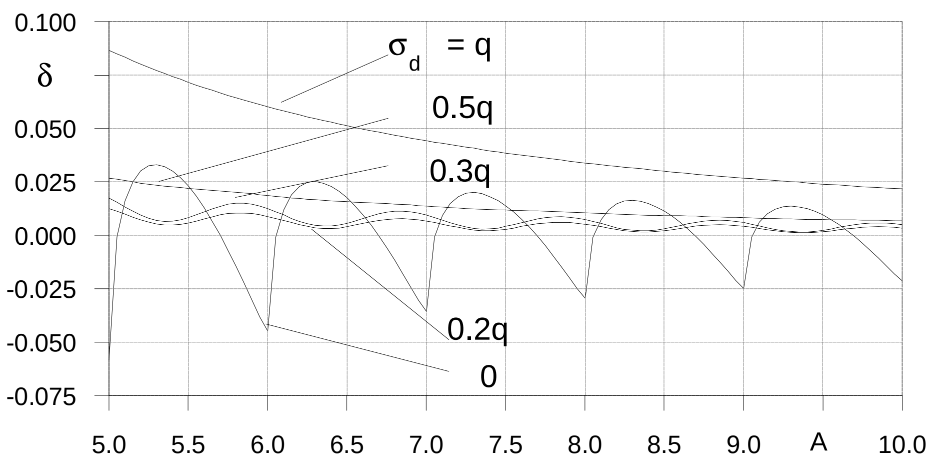
The presented diagrams can be used to estimate the estimator bias of the mean square value of a sinusoidal signal depending on its amplitude (for the sake of simplicity, q=1 is assumed, therefore the number A is at the same time a multiple of the quantization step) for different levels of Gaussian dither signal. It can be noted that it is an oscillatory attenuated function of the argument A.

It turns out that an increase in the dither dispersion from 0.5q to q may cause a 10^6-fold decrease in the error δ_r for A from the range of 1÷20, however the correlator A/D conversion with A ranging from 1000 to 2000 is - for σ_d =0.3q - burdened with the error δ_r of the 10^{-6} order [6]. The employment of dither of the level σ_d =q may cause, for A from the range of 100÷200, an approximately 10^8 -fold decrease in the bias in relation to the estimation without dither (Fig. 5).

The application of the correction $-(\sigma_d^2 + q^2/12)$ to the result is no trivial matter, as it may lead to a multiple decrease in the error (Fig. 4). Such an action, though, requires both a detailed knowledge and control of the Gaussian noise dispersion σ_d, which may pose a difficulty.

The bias component $\sigma_d^2 + q^2/12$ will not occur when two-channel measurement is employed, in which the signal $x_1(t)$ is simultaneously fed to both meter inputs (Fig. 2 - the time delay equals zero). Two-channel measurement requires a further development of the mean square value meter.

(a)



(b)

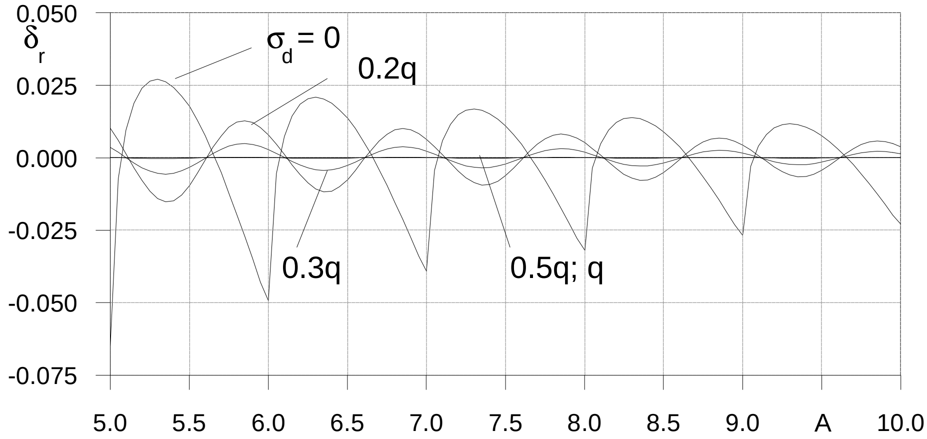
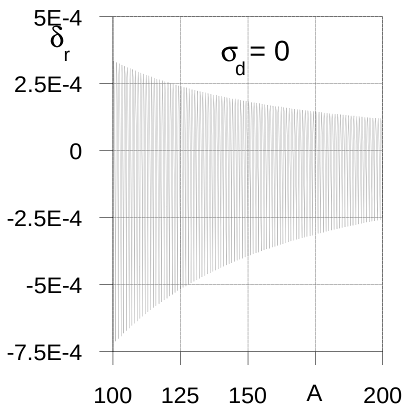


Figure 4. Normalized bias error: (a) - δ, (b) - δ_r of the digital estimator of the sinusoidal signal mean square value as a function of the signal amplitude A (q=1, A from the range of 5÷10) for different values σ_d of Gaussian dither.

(a) (b)



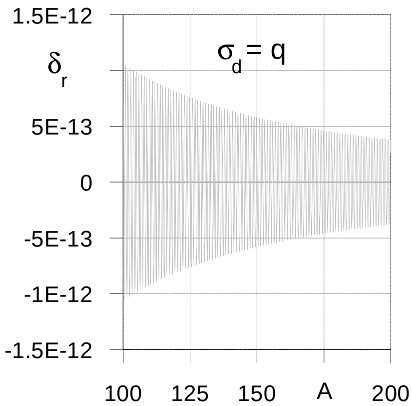


Figure 5. Normalized bias error δ_r ranges of the digital estimator of the sinusoidal signal mean square value as a function of the signal amplitude A (q=1, A from the range of 100 ÷ 200) for: (a) σ_d=0, (b) σ_d=q.

3 SUMMARY

The error level in digital correlation measurement with dither signals depends, among other things, on the A/D conversion resolution, kind and level of dither signals, as well as on the probabilistic characteristics of the investigated signals.

It can be stated that the condition of the non-occurrence of the bias (and thus the systematic component of the estimation error) of the digital crosscorrelation function estimator or digital autocorrelation function estimator determined by means of crosscorrelation, is that the dither signals satisfy the following conditions:

1. they assume zero mean values,
2. they are stochastically independent of the measured signals and of each other,
3. their CF's become zero for the arguments $2\pi i/q_1$ and $2\pi j/q_2$ ($i = \pm1, \pm2, \pm3, \ldots$, $j = \pm1, \pm2, \pm3, \ldots$), respectively.

Normal distribution dither does not satisfy condition 3. However, it is easier to generate than rectangular or triangular PDF dither, and, moreover, it may occur in the circuit in a form of intrinsic noise described by the term 'self-dither'.

It turns out that the application of the correction $-(\sigma_d^2 + q^2/12)$ to the measurement result of the sinusoidal signal square value for A/D conversion with a dither signal may lead to a multiple decrease in the systematic error. In practice, the application of the correction requires both a detailed knowledge and control of the dispersion σ_d , which may pose a problem.

If two-channel autocorrelation measurement is employed, which in correlation technology is termed autocorrelation analysis by crosscorrelation, then the estimator bias will decrease, and in the case of the mean square value measurement, the bias component $\sigma_d^2 + q^2/12$ will not occur. Such measurement requires further development of the analyzer.

The presented analysis concerns ideal A/D conversion. If the converter differential nonlinearity is taken into account, then the condition of the accuracy improvement is a further increase in the dither level growing with an increase in that nonlinearity. As the research has shown, for certain converter types employed in acoustics, the recommended dither rms value may even be in the order of 10 LSB [7].

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