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EXACTNESS OF LIFE-CYCLE EXAMINATIONS

K. Meissner
Laboratory of Quality Assurance
Jena University of Applied Sciences, D-07745 Jena, Germany

Abstract: The analysis of the simulation results showed that the Herd-Johnson method is suitable to establish examination plans. In order to receive more accurate results, sampling plans have to be varied according to the simulation results. During the time of early failures only few machine actually fail. Therefore the confidence interval of the parameter ß is big enough to consider conclusions concerning sampling sizes. Besides one can make safe statements for the guaranteed time of the use (one year), so the economical consequences on behalf of extending this period are now known.
Keywords: life –cycle examinations, simulation results, confidence interval

1 INTRODUCTION

A machine's life-cycle or thus of a functional module or a component depends on its kind of use, its surrounding conditions and the tolerance of each element. Hence, an individual life-span can't be determined in advance. However, by applying statistical methods it is possible to evaluate a machine's life, if a probability is assumed beforehand.
The reliability of measuring machines which are to be used for ten years in a three shift working zone has been investigated. In order to recognise early failures that still run on the manufacturer's one-year-guarantee, a shortened testing series has been recorded. This test is a multiple censored one as machines with diverging faultless periods are removed from the test respectively as machines at the time of the analysis are still to be tested. The tests' analysis was evaluated by the Herd-Johnson method. With the help of a computered simulation the procedure's applicability was examined. This was necessary in order to evaluate the accuracy of statements and to be able to suggest effective dimensions of sampling.

2 EVALUATION OF A MACHINE'S LIFE-CYCLE

A machine's life-span is described as the time between its starting and its failure. The distribution function F(t) is characterised by a random variable X. (1)

$$F(t) = P(X \le t) \tag{1}$$

The surviving probability R(t) of a machine can be calculated by (2).

$$R(t) = 1 - F(t) = P(X > t) \tag{2}$$

If F(t) is a continuous function, the density function f(t) can be specified as (3).

$$f(t) = F'(t) = \frac{dF(t)}{dt} \tag{3}$$

The present failure rate r(t) then equals (4).

$$r(t) = \lim_{\Delta x \to 0} = \frac{P(X \le t + \Delta x / X > t)}{\Delta t} = \frac{f(t)}{R(t)} \tag{4}$$

With the help of these equations an average life-cycle $\overline{X}$ and the variance VAR(X) can be determined by (5) and (6).

$$\overline{X} = m = \int_0^\infty t * f(t)dt = \int_0^\infty R(t)dt \tag{5}$$

$$VAR(X) = d^2 = \int (t-m)^2 * f(t)dt \tag{6}$$

Often used life-cycle distributions are:

- the exponential distribution (7)

$$F(t) = 1 - \exp(-l * t) \tag{7}$$

with the loss percentage r

$$r(t) = \frac{l * \exp(-l * t)}{\exp(-l * t)} \tag{8}$$

and the mean of the life cycle

$$\overline{X} = \frac{1}{l} \tag{9}$$

- the Weilbull-distribution (10), while with φ(t) (11) a two-parametric Weilbull-distribution arises.

$$F(t) = 1 - \exp(-j(t)) \tag{10}$$

$$j(t) = \left(-\frac{t}{a}\right)^2 \tag{11}$$

β now equals the *forming parameter* and α is a *scaling parameter*. When β=1, the Weilbull-distribution equals the exponential distribution, while β<1 indicates early failures and β>1 emphasises altering failures. Weilbull-distributions compared to other distributions are advantageous because they describe different failure rates clearly and their *forming parameter*s α and β can be graphically determined in an easy and short manner („life-cycle net„).
IDB-distribution (Increasing, Degreasing, Bathtub-shaped) is given by (12) and are suitable for descriptions concerning bathtub-shaped rates. ε, τ, γ are parameters of this distribution.

$$F(t) = 1 - \frac{\exp^{-t\frac{t^2}{2}}}{(1 + e * t)^{\frac{g}{e}}} \tag{12}$$

with the loss percentage r

$$r(t) = \frac{ß}{a} * \left(\frac{t}{a}\right)^{b-1} \tag{13}$$

2.1 Application of life-cycle distributions

Machines' life-cycle distributions are determined by the observed life-span of each object. For this, a distribution type must be accepted, the *estimation values* of all parameters for the distribution and the confidence intervals of all parameters must be calculated. Usually a two-parametric Weilbull-distribution is used for the explained steps. With this paper, the accuracy of estimated parameters is to be examined, so that the Maximum-Likelihood method can be used.

2.2 Sampling

The examination of machines can be done by random sampling, independent sampling, any or simulated sampling. The examination's course can be stated in three main steps (figure 1).

Step 1:	**Preparation** - determination of sample-size - type of sample - definition of errors - stop criterion
Step 2:	**Implementation of samples**
Step 3:	**Utilisation** **-** estimation of parameters - calculation of variance

Figure 1. Course of a life-cycle examination

The examination course needs a stop criterion, such as a maximum examination period T or a quantity of failures r is given. These examination plans are single censored tests. If the examination takes place until the failure of all machines, the multiple censored test applies. In order to examine life-cycles cheaper and faster, reduced samples can be used. The examining time is limited by a stop criterion.

There are two types:

Type I: censored sampling with/without replacement with examination end T
Type II: censored sampling with/without replacement with examination end r

Figure 2 and 3 is show the scheme of a single censored examination plan (type I and type II).

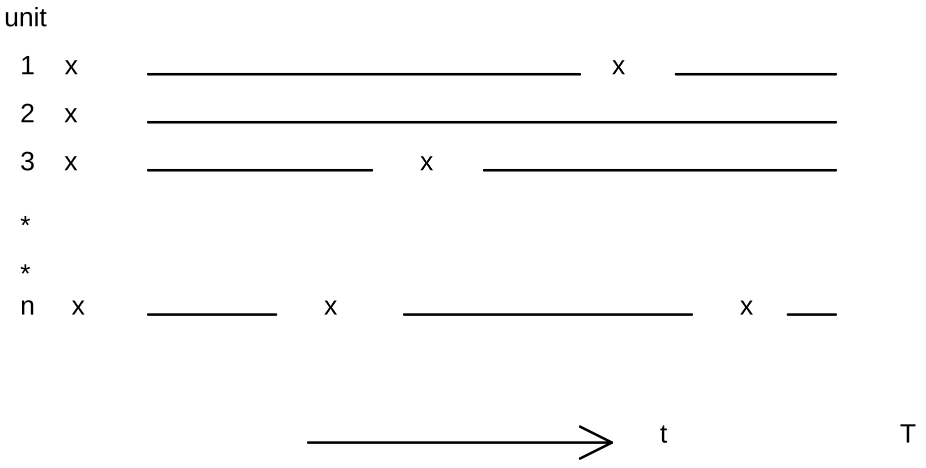

Figure 2: Examination plan type I: Censored sampling with replacement with examination end T

The examination is effective and economic if failed examines are replaced, because this way all examination places are used.

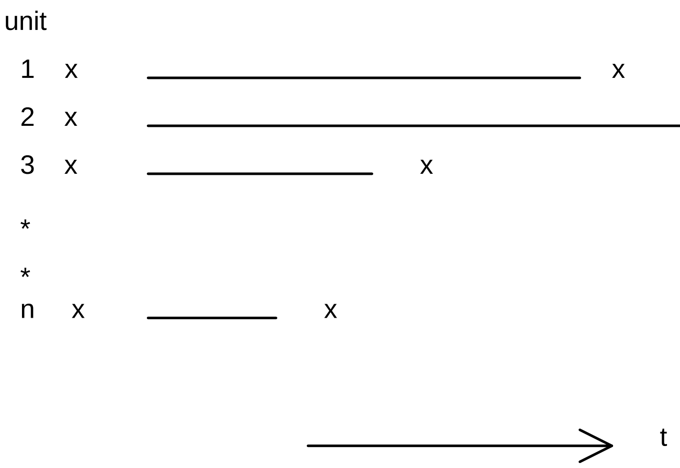

Figure 3: Examination plan type II (Single censored sampling without replacement with examination end r)

2.3 Evaluation

The data exactness, the test's economic aspect and the limited time span are significant for the evaluation. In most cases hardly any test's, in consideration of the economic aspect, realised completely (all objects are tested until each object failed). That's why uncomplete tests with stop criterion T and quantity of failures r are preferred.
The Herd-Johnson procedure can be used when evaluating multiple censored tests. The moments of failures of the multiple censored tests with n examines (maximum n runs) are used as the input dates of the Herd-Johnson method. The established periods of times are recorded. Periods of times of the still working examines are marked with a +. In the next step the opposite row is examined. The next step are the calculation of dependent, previous and present reliability and the probability of errors. After that, the connected probability of errors are registered on life-cycle paper so that function of distribution can be determined.

3 SIMULATION

To simulate failure reactions (loss percentage), three typical situations which occur at a measuring machine's failure have been considered.

- early failures at 1 up to 100 applications (forming parameter ß = 0.7)
- one year usage with 250 applications (forming parameter ß = 0.7; 1; 1.7)
- random failures at 101 up to 1000 applications (forming parameter ß = 1)
- late failures at 1001 up to 2500 applications (forming parameter ß = 1.7)

The cumulative failure rates have been evaluated with the examination plan of the reliability tests. For each of the three failure reactions a certain amount of machines was tested. The Herd-Johnson method is used because a composed function and a multiple censored test are present.
A further point of view is the evaluation of the forming parameter's ß accuracy depending on the sampling size and error rate. Particular problems arise at early failures because only few machines fail which make it hard to estimate ß exactly.

4 RESULTS OF SIMULATION

A distribution's characteristics are given by its forming parameter ß which therefore is more important than the scaling parameter α. From the calculated sum frequencies in dependence on ß, we determined the confidence interval for ß from two confidence levels of life cycles P = 0.9 and P = 0.6. Figure 4 and 5 sketch two examples for a sum frequency as a function of ß in dependence on the percentage of failed machines p and for P = 0.9.

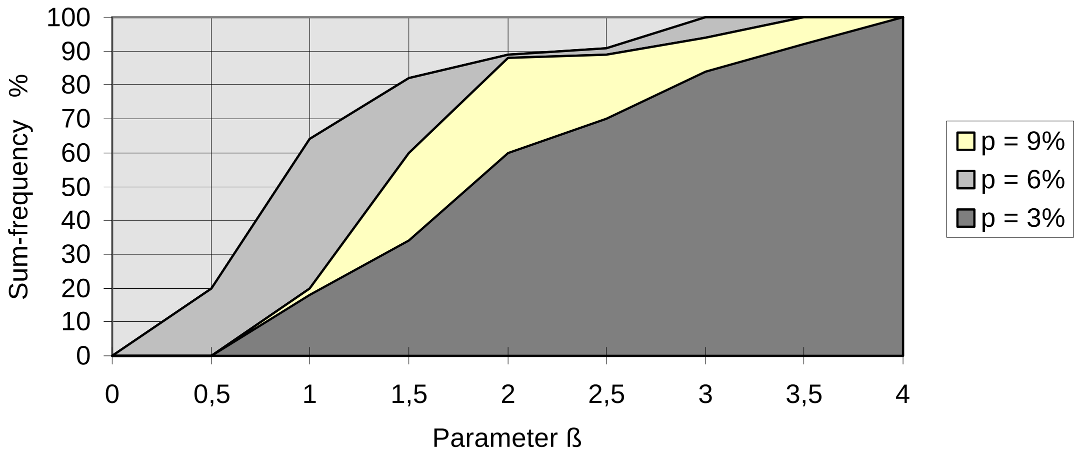

Figure 4: Sum frequency of forming parameter ß for p = 3% (one year usage)

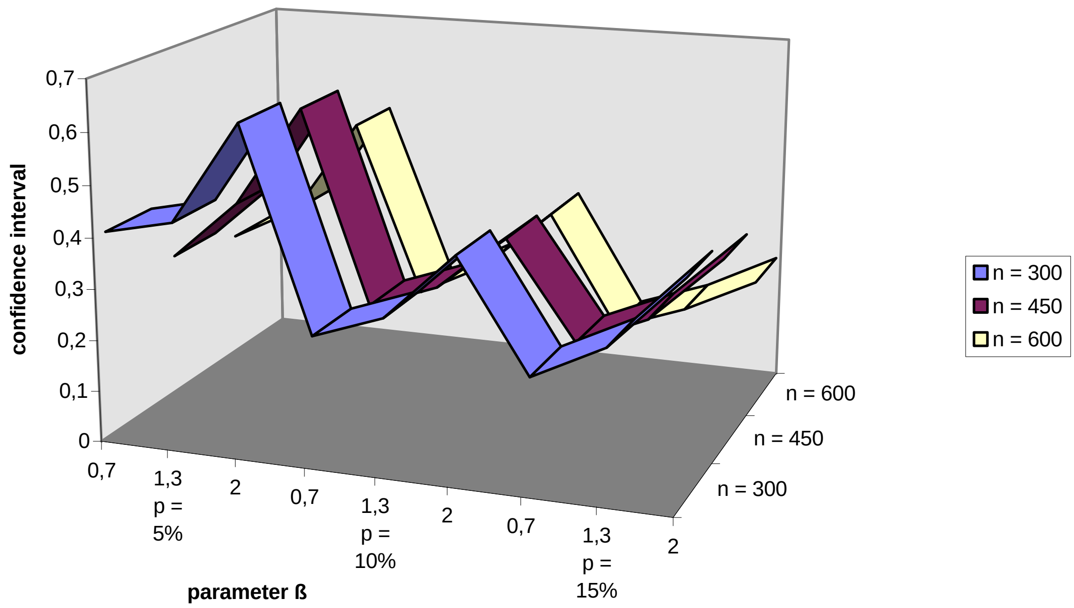

Figure 5: Confidence interval of ß in dependence at P = 0,9 (one year usage)

A summary of the results of confidence intervals for P = 0,9 is shown in table 1.

Parameter ß	0,7			1,3			2		
	upper limit	lower limit	confidence interval	upper limit	lower limit	confidence interval	upper limit	lower limit	confidence interval
percentage p									
5	0,87	0,59	0,25	1,41	1,09	0,51	2,15	1,64	0,51
10	0,83	0,66	0,17	1,40	1,14	0,26	2,11	1,75	0,36
15	0,81	0,67	0,14	1,39	1,20	0,19	2,11	1,85	0,26

Table 1: Confidence interval of ß with P = 0,9

A further interesting aspect is the evaluation of ß's confidence interval when dependant on the life-cycle examinations sampling size. Table 2 presents an example.

percentage of one year p	5%			10%			15%		
form parameter ß	0,7	1,3	2	0,7	1,3	2	0,7	1,3	2
sample size n									
300	0,40	0,43	0,63	0,24	0,29	0,42	0,21	0,28	0,42
450	0,28	0,40	0,60	0,22	0,27	0,38	0,19	0,25	0,38
600	0,25	0,32	0,51	0,17	0,24	0,36	0,14	0,19	0,26
confidence interval									

Table 2: Confidence interval of ß for P =0.9 (one year usage)

In figures 6 and 7 you can see two graphics (sum frequency and confidence interval) which belong to the both tables.

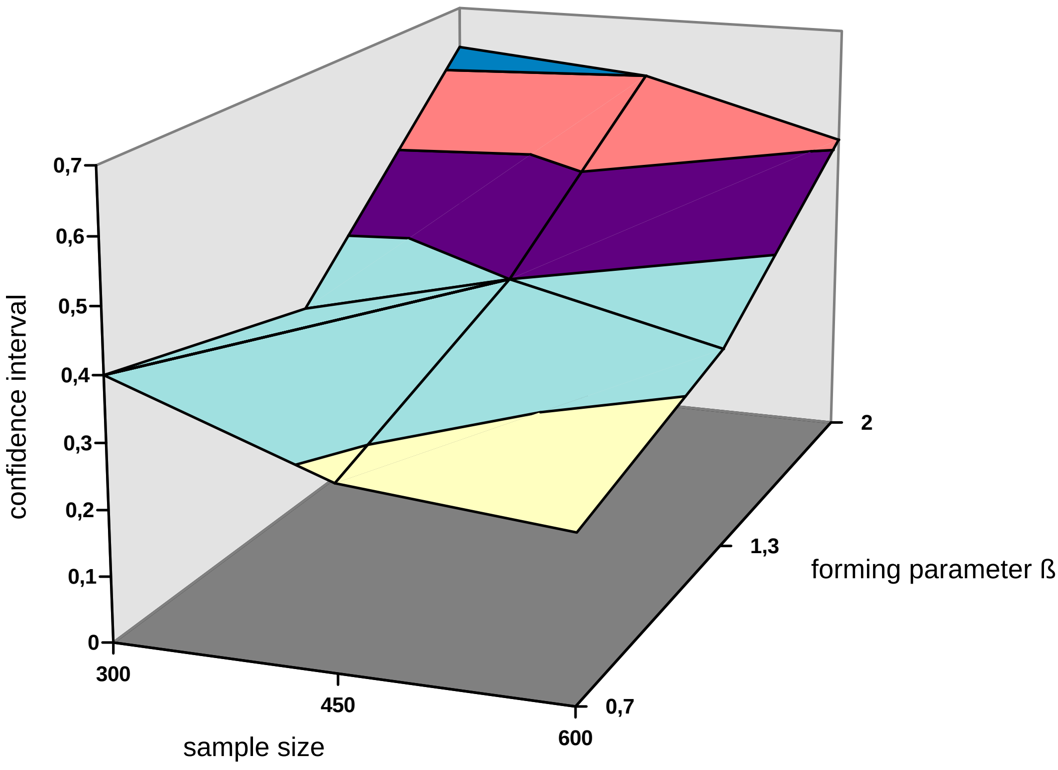

Figure 6: Confidence interval of ß at P =0.9 and ß = 5% (one year usage)

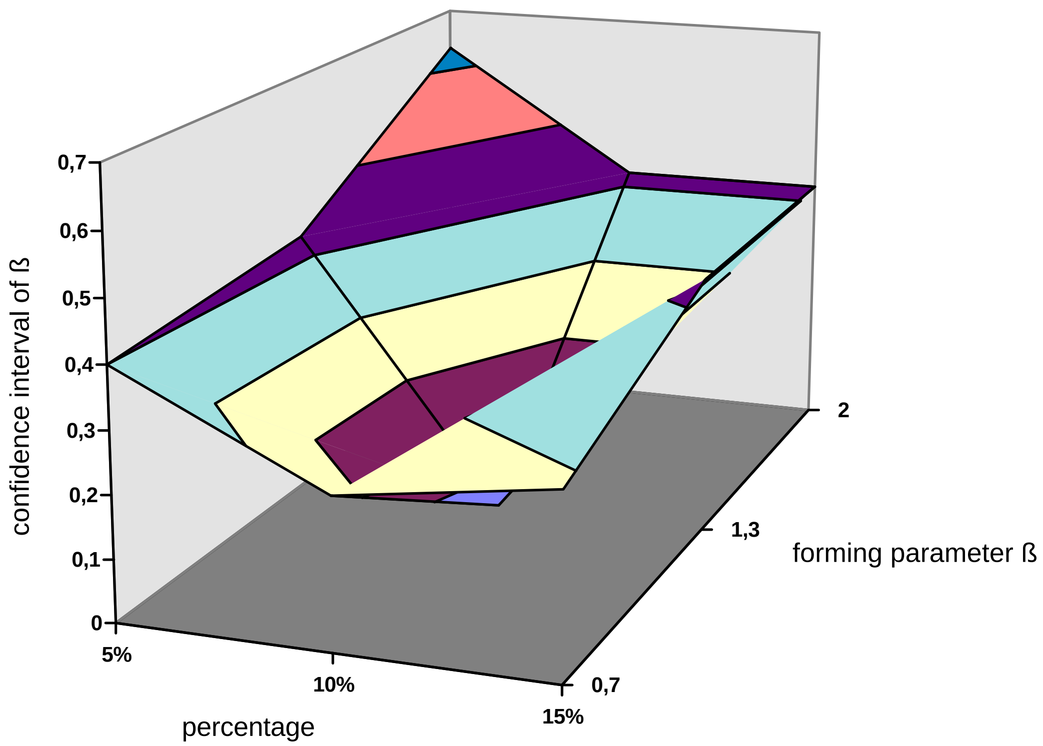

Figure 7: Confidence interval of ß for a sample size n = 300

5 CONCLUSION

The analysis of the simulation results showed that the Herd-Johnson method is suitable to establish examination plans. In order to receive more accurate results, sampling plans have to be varied according to the simulation's results. During the time of early failures only few machines actually fail. Therefore the confidence interval ß is big enough to consider conclusions concerning sampling sizes. Besides one can make safe statements for the guaranteed time of use (one year) so that economical consequences on behalf of extending this period are now known.

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AUTHOR: Dean of Faculty Precision Engineering. Jena University of applied sciences, PB 100314, D 07703 Jena, Germany, Phone ++49 3641 205400, Fax ++49 3641 205401,
E-mail: klaus.meissner@fh-jena.de