XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

ESTIMATION OF UNCERTAINTIES IN THE DISTANCE SENSOR CALIBRATION

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Abstract: Calibration of the ultrasonic distance sensor is described in the paper. Measuring range of the sensor is 100 - 600 mm. Design scheme of the calibrating device is presented. The calibrating device utilises gauge blocks as a standard. Non-linear mathematical model of the calibration is presented. Matrix method for the unknown parameters estimation is described. Analysis of the type A and type B sources of the uncertainties is given as well as their evaluation.

Keywords: ultrasonic sensor, calibration, uncertainties determination

1. Introduction

Ultrasonic measurement systems offer the non-contact distance measurement also in dusty or noisy environment. Their properties do not depend on the target material and its color.

2. Operating Principle of the Ultrasonic Distance Sensor

Ultrasonic distance sensors utilise an ultrasonic transducer that is used for transmitting and receiving as well. The transducer transmits 300 to 5000 specially coded ultrasonic impulses per second. Their amount is affected by the distance of the target. Ultrasonic impulses are reflected by the target. Sensing and decoding by the receiving part of the sensor follows. Distance of the target a is calculated from the time between transmitting of the signal and its receiving. The distance is calculated by the formula

$$a = (c \cdot t)/2$$

where

a is the unknown distance of the measured object,
c is the velocity of the sound propagation in the environment,
T is the time between impulse transmitting and receiving.

Ultrasonic signal is not propagated from the source in the single direction but it creates a lobe. The apex angle of the lobe represents the level of 3 dB. If the target is close enough, its distance can be measured even it is outside the lobe. If the target distance fits to the upper part of the measurement range, the target must be situated in the sensor's axis. The closer is the target to the axis the better results are obtained.

Different ultrasonic distance sensors can be found on the market. Their measurement range varies from 100 ÷ 200 mm to 500 ÷ 6000 mm. Apex angle of the ultrasonic lobe varies from 6 to 10 degrees (Fig. 1). Output signal linearity is better than 0,2% for smaller measurement ranges and better than 1% for bigger measurement ranges [4, 5].

3. Basic Influences to the Resulting Measurement Uncertainty

Application possibilities of the ultrasonic distance sensors are affected by many influences. They include shape of the target, surface of the target, declination of the ultrasonic lobe axis. Properties of the environment between the sensor and the target are important as well.

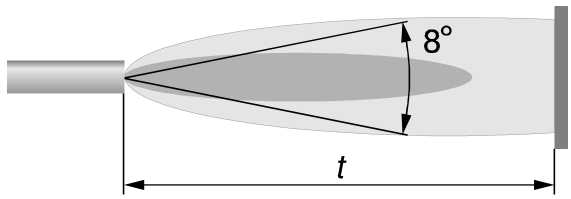

Fig. 1 Radiation of the ultrasonic distance sensor

Sensor's behavior is affected mainly by the reflective properties of the target. Almost each material reflects the sound. Exceptions are represented by the cotton, rubber etc. The shape of the target is also important.

Declination of the lobe axis restricts the application possibilities. In the case of the smooth flat target the declination of the lobe axis must be less than half of its apex angle. Otherwise no reflected signal is detected. The target with rough surface reflects the signal more dissipate. It enables to decline the target from the lobe axis more than 50°, the measurement range decreases accordingly.

Velocity of the sound propagation is in the close chain with the environment temperature. When the environment temperature increases for 1°, the velocity increases for 0,18%. That is why lot of ultrasonic distance sensors include the temperature compensating facilities. On the other hand the humidity and the atmospheric pressure do not affect performed measurements significantly.

4. Calibration of the Ultrasonic Distance Sensor

The Ultrasonic Distance Sensor is sensor with continuous scale that will be transformed by the p degree polynomial. Regress experiment model has been selected for the evaluation. It is important to include also uncertainties of the measurement standard, calibration model is non-linear. That is why it will be linearised by developing to the Taylor's series, neglecting higher order members. Regress model is set directly for the calibration function, not for the transformation one (see [1], [2], [3]).

Mathematical model of the measuring instrument to be calibrated is based on a polynomial function (consider 4th order polynomial)

$$E = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 \tag{1}$$

where

E is the output voltage;
a_0, a_1, a_2, a_3, a_4 are unknown parameters;
t is the value of the standard.

Calibration of the measuring instrument is held in more points of the standards t_j, whereby j = 1, 2, ..., n; $n \geq 7$ and related voltage values E_j are:

$$E_j = a_0 + a_1 t_j + a_2 t_j^2 + a_3 t_j^3 + a_4 t_j^4 \tag{2}$$

The calibration task is to determine the unknown parameters estimations and corresponding uncertainties and covariances as well.

Because the standard values t_j are not fixed points, model (1) is linearised by its developing to the Taylor series. Members of the higher orders are neglected so that the model is in the form:

$$E = \left(a_0^{(0)} + Da_0\right) + \left(a_1^{(0)} + Da_1\right)t^{(0)} + \left(a_2^{(0)} + Da_2\right)t^{(0)^2} + \left(a_3^{(0)} + Da_3\right)t^{(0)^3} + \left(a_4^{(0)} + Da_4\right)t^{(0)^4} +$$

$$\left(a_1^{(0)} + 2a_2^{(0)}\,t^{(0)} + 3a_3^{(0)}\,t^{(0)^2} + 4a_4^{(0)}\,t^{(0)^3}\right)\Delta t \tag{3}$$

where

$a_0^{(0)}$, $a_1^{(0)}$, $a_2^{(0)}$, $a_3^{(0)}$, $a_4^{(0)}$ are zero estimations of the unknown parameters, it means those are fixed values;
$t_j^{(0)}$ is the zero estimation of the quantity that is measured by the standard.
After modification (3) will be

$$\boldsymbol{E} - bDt = a_0 + a_1 t^{(0)} + a_2 t^{(0)^2} + a_3 t^{(0)^3} + a_4 t^{(0)^4} \qquad (4)$$

where

$$b = a_1^{(0)} + 2a_2^{(0)}\, t^{(0)} + 3a_3^{(0)}\, t^{(0)^2} + 4a_4^{(0)}\, t^{(0)^3}\ .$$

If we set

$$W = E\ \ - cDt \qquad (5)$$

then the measurement model (4) changes to the form (if $t^{(0)} = t$)

$$W = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 \qquad (6)$$

where

t is the fixed value.

Calibration experiment for the model [6] is organised in n points t_1, t_2, …, t_n. Then calibration model in the matrix form can be expressed as

$$\boldsymbol{W} = \boldsymbol{A}\boldsymbol{y} \qquad (7)$$

where

$\boldsymbol{W}$ is *n-dimensional vector defined by* (5) for t_1, t_2, …, t_n, b_1, b_2, …, b_n, and E_1, E_2, …, E_n,
$\boldsymbol{A}$ is the design matrix of the $n\times 4$ type,
$\boldsymbol{y} = (a_0, a_1, a_2, a_3, a_4)^{\mathrm{T}}$ is the vector of the unknown parameters estimations.

Based on relation (5), vector $\boldsymbol{W}$ has the form

$$\boldsymbol{W} = \boldsymbol{E} + \boldsymbol{B}\, D\boldsymbol{t} \qquad (8)$$

where

$\boldsymbol{E} = (E_1, E_2, \ldots, E_n)^{\mathrm{T}}$,
$\boldsymbol{B} = \mathrm{diag}(b_1, b_2, \ldots, b_n)$,
$\Delta\boldsymbol{t} =(Dt_1, Dt_2, \ldots, Dt_n)$.

Considering $\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{C}_1\boldsymbol{E}_2$ and $\boldsymbol{t} = \boldsymbol{t}_1 + \boldsymbol{C}_2\boldsymbol{t}_2$ (thus also $D\boldsymbol{t} = D\boldsymbol{t}_1 + \boldsymbol{C}_2 D\boldsymbol{t}_2$) where$E_1$ is vector of the measured values of the output voltage, E_2 is vector of quantities affecting measurement of the output voltage, matrix $\boldsymbol{C}_1$ expresses structure of individual affecting quantities, $\boldsymbol{t}_1$ is vector of values measured by measuring standard (values from the measurement standard’s certificate in this case), $\boldsymbol{t}_2$ is vector of the measurement standard’s deviations (including affecting quantities acting on value reproduced by the measurement standards), $\boldsymbol{C}_2$ is matrix reflecting structure of the influences of measurement standard’s deviations for individual measurements, then measurement model (8) (using the same way as for the single measurement) can be expressed as:

$$\boldsymbol{W} = \boldsymbol{E}_1 + \boldsymbol{C}_1\boldsymbol{E}_2 + \boldsymbol{B}\boldsymbol{C}_2 D\boldsymbol{t} \qquad (9)$$

Corresponding covariance matrix $\boldsymbol{U}_{\boldsymbol{w}}$ of the $\boldsymbol{W}$ vector is expressed in the form (see [4])

$$\boldsymbol{U}_{\boldsymbol{W}} = \boldsymbol{U}_{\boldsymbol{E}1} + \boldsymbol{C}_1\boldsymbol{U}_{\boldsymbol{E}2}\boldsymbol{C}_1^{\mathrm{T}} + \boldsymbol{B}\boldsymbol{C}_2\boldsymbol{U}_{\boldsymbol{t}}\boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} \qquad (10)$$

Estimation $\hat{\boldsymbol{y}}$ of the unknown parameters vector $\boldsymbol{y}$ are calculated by the formula

$$\hat{\boldsymbol{y}} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{U}_{\boldsymbol{w}}^{-1}\boldsymbol{A}\right)^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{U}_{\boldsymbol{w}}^{-1}\boldsymbol{w} \qquad (11)$$

and corresponding covariance matrix is determined as

$$\boldsymbol{U}_{\hat{\boldsymbol{y}}} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{U}_{\boldsymbol{w}}^{-1}\boldsymbol{A}\right)^{-1} \tag{12}$$

Necessary condition to determine estimation (11) and covariance matrix (12) represents the knowledge of the diagonal matrix $\boldsymbol{B}$. Each of its elements b_i is equal to $b_i = a_1^{(0)} + 2\, a_2^{(0)} t_i^{(0)} + 3\, a_3^{(0)} t_i^{(0)2} + 4\, a_4^{(0)} t_i^{(0)3}$. Thus zero estimations of the calibration function's unknown parameters must be known. They can be calculated using the equation (11). In this case $\boldsymbol{U}_{\boldsymbol{w}}$ is equal to $\boldsymbol{U}_{\boldsymbol{w}} = \boldsymbol{U}_{\boldsymbol{E}1} + \boldsymbol{C}_1 \boldsymbol{U}_{\boldsymbol{E}2} \boldsymbol{C}_1^{\mathrm{T}}$. Uncertainties of the measurement standard are not included in the first step.

Presented procedure respects fully also appointments stated in [6, 7].

5. Sources of uncertainties

Covariance matrix $\boldsymbol{U}_{\boldsymbol{W}}$ defined by the equation (10) is required for calculation of estimations of the parameter vector, defined by (11) as well as for calculation of their covariance matrix, defined by (12).

a) **Covariance matrix $\boldsymbol{U}_{\boldsymbol{E}1}$**

Covariance matrix $\boldsymbol{U}_{\boldsymbol{E}1}$is filled by uncertainties of the output voltage measurements E_1, E_2, …, E_n as well as by covariances among them, evaluated by the type A method. Quantity E_i has been measured for n times in each point. Arithmetic mean of those measurements has been considered as the measurement result. Elements of the covariance matrix $\boldsymbol{U}_{\boldsymbol{E}1}$ are calculated as experimental standard deviations of arithmetic means and experimental covariances among them.

b) **Covariance matrix $\boldsymbol{U}_{\boldsymbol{E}2}$**

Covariance matrix $\boldsymbol{U}_{\boldsymbol{E}2}$ is filled by uncertainties of quantities affecting measurement of the output voltage values E_1, E_2, …, E_n as well as by covariances among them evaluated by the type B method. The only considered source of uncertainty in presented case was the error of used PC acquisition card PCL 812 that obtains values $\pm D$ where D is given by error of the 12 bit A/D converter (its resolution ability).

c) **Covariance matrix $\boldsymbol{U}_{\boldsymbol{t}}$**

Covariance matrix $\boldsymbol{U}_{\boldsymbol{t}}$ is filled by uncertainties of measured standards used for individual measurements. They are obtained by merging of uncertainties stated in measurement standard's calibration certificates and uncertainties caused by the fact that measurement standard is used in measurement conditions that differs from those during its calibration.

Non-standard measurement gauges have been used as a measurement standard. Individual measurement gauges have been calibrated with uncertainty u_{ti}^2. Another uncertainty source represents measurement temperature differing from those during calibration.

6. Calibration results

Ultrasonic distance sensor 945-L4Y-2D-1C0 of the HONEYWELL Inc was calibrated. Calibration was performed in seven points, the measurement range was set to 100 ÷ 600 mm, step 50 mm. Measurement was repeated for ten times in each point.

Calculation of the unknown parameters estimation, corresponding uncertainties and covariances was performed using a software MATLAB®.

Satisfactory results of the calibrating curve have been obtained after the third iteration (difference among new estimations and estimations from the previous step were smaller then corresponding uncertainties of those estimations).

Obtained results are used for design of calibration table, calibration curve as well as for design of curve of residuals. Calibration curves and curves of residual, obtained for the 1st and 4th order polynomial, are presented on Fig. 2 and Fig. 3. Table 1 represents calibration table.

7. Conclusion

The paper describes briefly one of the possible approaches to the single component sensor calibration. After the estimation vector of the unknown parameters and corresponding uncertainties and covariances a good agreement among data obtained and data given by the producer can be presented.

This paper was prepared under the financial support of the VEGA grant agency, grant No. 1/4157/97.

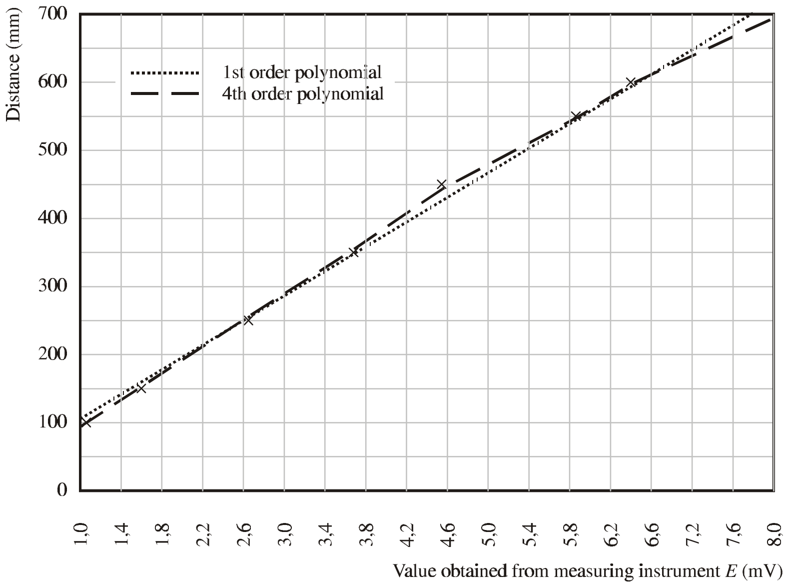

Fig. 2 Calibration curves

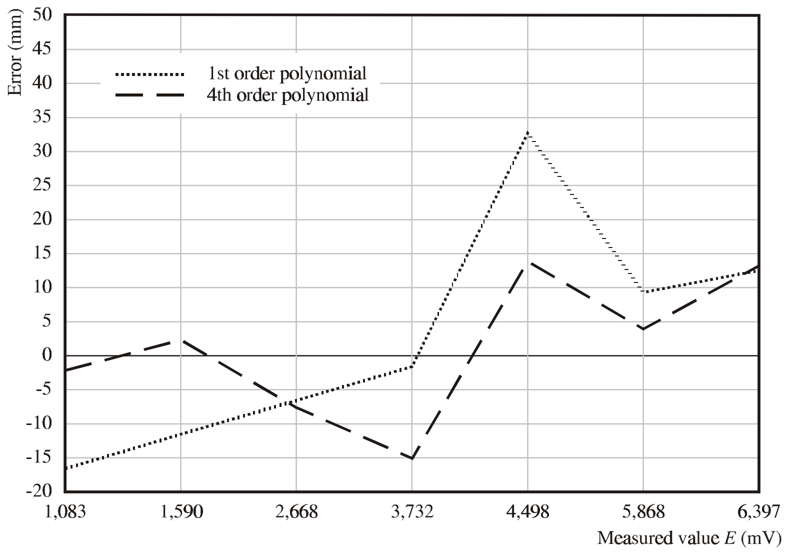

Fig. 3 Curves of residual

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