

CALIBRATION OF MATRIX SENSOR ARRAYS

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Abstract: The contribution deals with evaluation of uncertainties in the process of calibration of multicomponent sensors with special aim to force matrix sensor arrays. Matrix sensors are designed for simultaneous measurement of acting forces. It is used in applications in which sensing of varying forces acting in an area is needed (e.g. in robotics, telemanipulating devices, medical applications). Model for calibration of matrix sensor arrays and the methodology for uncertainties and covariances calculation is submitted. The base is regress polynomial model of calibration. The result is estimation of model parameters and covariance matrix of those estimators. Considered are all uncertainties and covariances, resulting from measuring standard device, transmission of values from measuring standard to sensor, repeating of values by calibrating sensor. The most important characteristics of current state-of-art in this field is absence of unique procedures used for objective matrix sensor arrays evaluation and testing.

Keywords: calibration, matrix sensor arrays, uncertainty, covariance

1 INTRODUCTION

One of the main problems in successful use of matrix sensor arrays is interference influence of different loads (Fig. 1). This effect acts very importantly not only in the measurement process but also in the calibration process. This common influence of output signals obtained from different forces is particularly important for the sensors that are used for determination of actual force values, not only for obtaining of load's pattern.

In the matter of fact calibration of matrix sensor arrays requires determining of signal's sensitivity of different elements to the influence of different components of the load.

As the force matrix sensor array can be manufactured in different sizes up to thousands elements, let us consider calibration problem of the basic sensor with 4x3 elements (Fig. 1). Procedure described in following text is fully applicable for any size sensor, differing only in model difficulty.

$i-1$ $j-2$	$i-1$ $j-1$	$i-1$ j	$i-1$ $j+1$
i $j-2$	i $j-1$	i j	i $j+1$
$i+1$ $j-2$	$i+1$ $j-1$	$i+1$ j	$i+1$ $j+1$

Fig. 1 Scheme of the force matrix sensor array with 4x3 elements

Loading components of the sensor during measurement can be written as

$$F_{(i-1,j-2),h} = a_{0(i-1,j-2)} + a_{1(i-1,j-2)} \cdot E_{(i-1,j-2),h} + a_{2(i-1,j-2)} \cdot E_{(i-1,j-2),h}^2 + b_{(i-1,j-1)(i-1,j-2)} \cdot E_{(i-1,j-1),h} + b_{(i,j-2)(i-1,j-2)} \cdot E_{(i,j-2),h} + b_{(i,j-1)(i-1,j-2)} \cdot E_{(i,j-1),h}$$

$$F_{(i-1,j-1),h} = a_{0(i-1,j-1)} + a_{1(i-1,j-1)} \cdot E_{(i-1,j-1),h} + a_{2(i-1,j-1)} \cdot E_{(i-1,j-1),h}^2 + b_{(i-1,j-2)(i-1,j-1)} \cdot E_{(i-1,j-2),h} + b_{(i-1,j)(i-1,j-1)} \cdot E_{(i-1,j),h} + b_{(i,j-2)(i-1,j-1)} \cdot E_{(i,j-2),h} + b_{(i,j-1)(i-1,j-1)} \cdot E_{(i,j-1),h} + b_{(i,j)(i-1,j-1)} \cdot E_{(i,j),h}$$

⋮

(1)

$$F_{(i,j),h} = a_{0(i,j)} + a_{1(i,j)} \cdot E_{(i,j),h} + a_{2(i,j)} \cdot E_{(i,j),h}^2 + b_{(i-1,j-1)(i,j)} \cdot E_{(i-1,j-1),h} + b_{(i-1,j)(i,j)} \cdot E_{(i-1,j),h} + b_{(i-1,j+1)(i,j)} \cdot E_{(i-1,j+1),h} + b_{(i,j-1)(i,j)} \cdot E_{(i,j-1),h} + b_{(i,j+1)(i,j)} \cdot E_{(i,j+1),h} + b_{(i+1,j-1)(i,j)} \cdot E_{(i+1,j-1),h} + b_{(i+1,j)(i,j)} \cdot E_{(i+1,j),h} + b_{(i+1,j+1)(i,j)} \cdot E_{(i+1,j+1),h}$$

⋮

$$F_{(i+1,j+1),h} = a_{0(i+1,j+1)} + a_{1(i+1,j+1)} \cdot E_{(i+1,j+1),h} + a_{2(i+1,j+1)} \cdot E_{(i+1,j+1),h}^2 + b_{(i)(i+1,j+1)} \cdot E_{(i),h} + b_{(i+1)(i+1,j+1)} \cdot E_{(i+1),h} + b_{(i+1,j)(i+1,j+1)} \cdot E_{(i+1,j),h} + b_{(i+1,j+1)(i+1,j+1)} \cdot E_{(i+1,j+1),h}$$

where

$F_{(i-1,j-2),h}, F_{(i-1,j-1),h}, \dots, F_{(i+1,j+1),h}$ is load of individual sensor's elements,
 $E_{(i-1,j-2),h}, E_{(i-1,j-1),h}, \dots, E_{(i+1,j+1),h}$ is voltage of the sensor's output signal, measured at individual elements,

$h = 1, 2, \dots, n$ is number of load,

$a_{0(i-1,j-2)}, a_{1(i-1,j-2)}, a_{2(i-1,j-2)}, \dots, b_{(i+1,j)(i+1,j+1)}$ are unknown parameters of the model.

or in the matrix notation

$$F = CE \quad (2)$$

where

F is the vector of sensor's load,

C is the calibration matrix of the sensor,

E is the vector of the sensor's output signals.

More research works deal with calibration in last years. Evaluation of calibration uncertainties is based mainly on one-component model. In this paper multicomponent approach is submitted.

During a calibration process unknown parameters of calibration matrix are determined. Experimental methods for identification of the model (1) or of the following model are used

$$E = AF \quad (3)$$

Majority of contemporary approaches is dedicated to finding the transformation matrix A by solution of the model (3) following by determining of the calibration matrix. Determining of uncertainties of calibration matrix elements and their covariances can be difficult. It seems to be more advantageous directly to find elements of the calibration matrix during the calibration process.

2 COMPUTATIONAL MODEL OF THE CALIBRATION

Theoretical model of the calibration for n loads in the matrix form

$$F = Tc \quad (5)$$

where

$F = (F_1^T, F_2^T, \dots, F_n^T)^T$ is a $12n$ dimensional vector of acting loads,

$F_h^T = (F_{(i-1,j-2)h}, F_{(i-1,j-1)h}, F_{(i-1,j)h}, \dots, F_{(i+1,j+1)h})$, $h = 1, 2, \dots, n$,

c is a 94-dimensional vector of elements of the calibration matrix C ,

T is a $12n \times 94$ design matrix.

Aim of the calibration is experimental determination of estimation of the calibration matrix C elements as well as uncertainties and covariances of those estimations. It is possible to do it for the model in the form (5) (see [6]), if covariance matrix U_F is known or the matrix H_F is known, where $U_F = S^2 H_F$ or if the model is in the special form (see [4]) and design matrix T is the matrix of fixed numbers at the same time.

Because elements $E_{(i-1,j-2)}, E_{(i-1,j-1)}, \dots, E_{(i+1,j+2)}$ of the matrix T are measured and their estimations are affected by errors then matrix T is random and the model is non-linear. Such model will be linearised by development to the Taylor series and by neglecting of members of higher orders. Model (5) then changes to the form

$$F = T^{(0)}c^{(0)} + T^{(0)}dc + DdE \quad (6)$$

where

$T^{(0)}$ is the zero estimation of the $12n \times 94$ dimensional design matrix,
 $c^{(0)}$ is the zero estimation of the 94 dimensional vector of the calibration matrix coefficients,
 dc is the 94 dimensional vector of increased values of the calibration matrix elements,
 dE is the $12n$ dimensional vector of errors of the sensor's output signals estimations,
 D is the $12n \times 12n$ dimensional diagonal matrix, whose i -th element is a partial derivative of the i -th element of the vector F by the i -th element of the vector E , if zero estimations $E^{(0)}$ are substituted as values of the E vector.

If $T^{(0)}c^{(0)} + T^{(0)}dc = T^{(0)}(c^{(0)} + dc) = T^{(0)}c$ is indicated, model (6) changes to the form

$$F - DdE = T^{(0)}c \quad (7)$$

If following indication is introduced

$$W = F - DdE \quad (8)$$

then

$$W = T^{(0)}c \quad (9)$$

where

$T^{(0)}$ is the design matrix of fixed (non-random) numbers,
 c is the vector of unknown parameters,
 W is the random vector of input quantities.

3 COVARIANCE MATRIX OF THE VECTOR W

For vector W in the form (8) it is supposed that:

a) vector of loads F can be written as

$$F = F_1 + C_1F_2$$

where

F_1 is the vector of measured values of the load,
 F_2 is vector of quantities affecting the load estimation,
 C_1 is known matrix that represents structure of those influences,
 C_1F_2 is correction vector,

b) vector of errors dE can be written as

$$dE = dE_1 + C_2dE_2$$

where

dE_1 is vector of random errors,
 dE_2 is vector of quantities that influence measurement result,
 C_2 is known matrix representing structure of those influences,
 C_2dE_2 is vector of systematic errors that influence measurement of output signals from the sensor.

So that vector W can be expressed in the form

$$W = F_1 + C_1F_2 - D(dE_1 + C_2dE_2) \quad (10)$$

If independence between vectors F_1 and C_1F_2 as well as between vectors dE_1 and C_2dE_2 is supposed, then covariance matrix of the vector (11) can be written as

$$U_w = U_{F_1} + C_1U_{F_2}C_1^T + D(U_{dE_1} + C_2U_{dE_2}C_2^T)D^T + U_{F_1E_1} + U_{F_2E_2} \quad (11)$$

where

$\mathbf{U}_{F_1 E_1}$ is a covariance matrix expressing covariances among elements of vectors \mathbf{F}_1 and \mathbf{E}_1 ,
 $\mathbf{U}_{F_2 E_2}$ is a covariance matrix expressing covariances among elements of vectors \mathbf{F}_2 and \mathbf{E}_2 .

If it is considered that \mathbf{F}_1 and \mathbf{E}_1 are vectors of input quantities measured during calibration experiment, one can write

$$\begin{aligned}\mathbf{U}_{F_1} &= \mathbf{S}_F^2 \mathbf{H}_{F_1}, \\ \mathbf{U}_{E_1} &= \mathbf{S}_E^2 \mathbf{H}_{E_1}, \\ \mathbf{U}_{F_1, E_1} &= \mathbf{S}_{FE} \mathbf{H}_{F_1, E_1}\end{aligned}\quad (12)$$

and covariance matrix \mathbf{U}_W of the input quantities vector \mathbf{W} can be written as

$$\mathbf{U}_W = \mathbf{S}_F^2 \mathbf{H}_{F_1} + \mathbf{S}_E^2 \mathbf{H}_{E_1} + \mathbf{S}_{FE} \mathbf{H}_{F_1, E_1} + \mathbf{D} \mathbf{C}_2 \mathbf{U}_{dE_2} \mathbf{C}_2^T \mathbf{D}^T + \mathbf{U}_{F_2 E_2} \quad (13)$$

where

$\mathbf{U}_{F_1} = \mathbf{S}_F^2 \mathbf{H}_{F_1}$ is a covariance matrix of directly measured sensor's loads determined by a type A method, matrix \mathbf{H}_{F_1} is known, parameter \mathbf{S}_F has not always to be known,
 $\mathbf{U}_{F_1, E_1} = \mathbf{S}_{FE} \mathbf{H}_{F_1, E_1}$ is a covariance matrix among measurements (estimations) of the sensor's loads and measurement (estimations) of the output signals, determined by a type A method, matrix \mathbf{H}_{F_1, E_1} is known, parameter $\mathbf{S}_{F, E}$ has not to be known,
 \mathbf{U}_{F_2} is a covariance matrix of measurements of sensor's loads determined by a type B method (see [1], [2], [3]),
 \mathbf{U}_{dE_2} is a covariance matrix of measurement of output signals, determined by type B method,
 \mathbf{U}_{FE} is a covariance matrix among measurements (estimations) of sensor's loads and measurements (estimations) of sensor's output signals determined by a type B method,
 $\mathbf{C}_1, \mathbf{C}_2, \mathbf{D}$ are known matrixes.

Parameters $\mathbf{S}_F, \mathbf{S}_E, \mathbf{S}_{FE}$ are unknown in general and they are determined in calibration experiment. To enable finding of unknown parameter's estimation, calibration experiment must be properly designed and thus calibration model must have proper structure.

4 ESTIMATION OF THE UNKNOWN PARAMETERS VECTOR AND ITS COVARIANCE MATRIX

For calibration model (9) and covariance matrix in the form (11) (if the covariance matrix is known) will be estimation $\hat{\mathbf{c}}$ of the unknown parameters vector \mathbf{c} in the form (see [6])

$$\hat{\mathbf{c}} = (\mathbf{T}^T \mathbf{U}_W^{-1} \mathbf{T})^{-1} \mathbf{T}^T \mathbf{U}_W^{-1} \hat{\mathbf{E}} \quad (14)$$

and covariance matrix $\mathbf{U}_{\hat{\mathbf{c}}}$ of the vector $\hat{\mathbf{c}}$

$$\mathbf{U}_{\hat{\mathbf{c}}} = (\mathbf{T}^T \mathbf{U}_W^{-1} \mathbf{T})^{-1} \quad (15)$$

where

$$\mathbf{w} = \hat{\mathbf{F}}_1 + \mathbf{C}_1 \hat{\mathbf{F}}_2 - \mathbf{D} (\hat{\mathbf{a}} \hat{\mathbf{E}}_1 + \mathbf{C}_2 \hat{\mathbf{a}} \hat{\mathbf{E}}_2) \quad (16)$$

$\hat{\mathbf{F}}_1, \hat{\mathbf{F}}_2, \hat{\mathbf{a}} \hat{\mathbf{E}}_1, \hat{\mathbf{a}} \hat{\mathbf{E}}_2$ are estimators of relevant quantities.

\mathbf{T}_0 is design matrix for zero estimators of the output voltage values \mathbf{E}_0 ,

\mathbf{w} is estimation of the \mathbf{W} vector,

\mathbf{U}_w is covariance matrix of the estimators vector \mathbf{w} .

Knowledge of the covariance matrix (13) has been supposed. Besides knowledge of individual covariance matrixes in (13) one must know also matrix \mathbf{D} . That assumes knowledge of zero estimators of sought parameters vector \mathbf{C}_0 as well as zero estimators of the measured input values vector \mathbf{E}_0 . Measured values vector $\hat{\mathbf{E}}$ is considered to be a zero estimator \mathbf{E}_0 of the vector \mathbf{E} . Similarly, matrix (6) is

considered as a zero estimator T_0 of the design matrix T , where measured values \widehat{E}_{ij} are substituted instead of E_{ij} .

Zero estimators vector c_0 (element of the calibration matrix C) can be derived for example from (14), if non-random design matrix T is considered, so that following equation is substituted for matrix U_w

$$U_w = U_{F_1} + C_1 U_{F_2} C_1^T + U_{F_1 E_1} \quad (17)$$

New estimators of the calibration matrix elements as well as their uncertainties and covariances among them are determined from (14) and (15) for such zero estimators. If new estimators are not "good enough", new estimators are considered to be zero ones and the procedure repeats. As "good enough" can be considered estimators with values differing from the zero estimators less than its uncertainty determined from (17).

Knowledge of covariance matrix U_w requires to know $U_{F_1}, U_{E_1}, U_{F_1, E_1}, U_{F_2}, U_{\Delta E_2}, U_{FE}$ and matrixes C_1, C_2, D . Only covariance matrixes $U_{F_2}, U_{\Delta E_2}, U_{FE}$ (determined by the type B evaluation) are known more often and matrixes $U_{F_1}, U_{\Delta E_1}, U_{F_1, E_1}$ are unknown.

Two general cases can be noted in here:

- 1) more repeating measurements (of the load as well as of the sensor's output signal) are performed for each sensor's load,
- 2) the only measurement is performed for each sensor's load.

In the case of repeating measurements in each point (let us assume the same number of repeating in each point) m pairs of values F_1, E_1 to F_n, E_n is obtained. They can be used for calculation of standard deviations and standard covariances that are substituted into covariance matrixes $U_{F_1}, U_{E_1}, U_{F_1, E_1}$.

Procedure follows as in the case of known covariance matrix. If calibration model is not adequate, correction factor σ^2 is introduced for multiplying the matrix (13). Estimator \widehat{c} of the parameter's vector does not depend on value σ^2 (see [6]) and covariance matrix (15) is then multiplied by σ^2 factor those value can be determined (see [6]).

Single measurement in each point does not allow determination of covariance matrixes $U_{F_1}, U_{E_1}, U_{F_1, E_1}$ using type A evaluation as in previous case. Mentioned covariance matrixes can be determined under certain conditions by procedure stated in [4].

CONCLUSION

Contribution defines calibration model of the tactile sensors as well as method of the calibration determination. Calibration model is multicomponent and very complicated. Model can be simpler for most practical situations. Matrix notation simplifies calculation of the calibration model very much. Evaluation method is based on least-squares method preceded by linearization. Method enables incorporation of all uncertainty sources.

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