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KNOWLEDGE - BASED EVALUATION OF THE SIGNAL PARAMETER

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Abstract: The main problems related with taking into account the a priori knowledge about the measured signal have been outlined. Then two examples have been presented, showing how the knowledge about one signal parameter narrows down the uncertainty about the other one. Firstly, bounding curves for the mean-square value when the mean value is known have been examined and some practical conclusions concerning single and multiple signal rectification have been presented. Then the mean value uncertainty for a given signal bandwidth has been studied.
Keywords: knowledge-based measurement, RMS measurement, mean value uncertainty;

1 INTRODUCTION

For quantitative determination of the signal characteristics, specialised signal parameters are used rather than its shape description. The signal parameter, the value of which is to be estimated with a given uncertainty, will be referred to as estimated parameter.

To make the measurement process as effective as possible, the whole available knowledge concerning the investigated signal should be taken into account not only during calculations of the estimated parameter value and its uncertainty but also for the sake of measurement process simplification.

The a priori knowledge may contain, for instance, information about: the signal probability distribution [1] or some properties of it (e.g. the signal maximum and minimum values), signal bandwidth, signal properties of symmetry, signal mathematical model (see e.g. [2]-[6]), or -sometimes- some values of the signal parameters other than those estimated. The information about signal parameters is often valuable, since they are usually known with a relatively low uncertainty.

The completion of the ultimate task: to narrow down the estimated parameter uncertainty as much as possible or necessary, does not depend on the fact which information entities are known a priori and which are obtained from the currently made measurement. Two facts are important however:

– how large is the uncertainty of the estimated parameter for a given value of some other known parameter or for a given fact known about the investigated signal, assuming that our knowledge is strictly precise. This problem addresses to the notion of „meaning” from semiotics [7] , as it concerns the correspondence between the known information and the estimated parameter value or the structural similarity between the known and estimated parameters;

– how large is the actual uncertainty of the known parameter and what is its influence on the estimated parameter uncertainty.

This paper deals with the first problem. Two examples are presented in it, showing how the knowledge about one periodic signal parameter allows to narrow the uncertainty of another parameter. This problem can be analysed in different ways (see e.g. [1]). Here, in both examples the method of the bounding curves was utilised. In this method, for all possible values of the known parameter, limit values are found within which the estimated parameter value must lie.

2 THE MEAN-SQUARE VALUE UNCERTAINTY FOR A SIGNAL OF A GIVEN MEAN VALUE

As a first example, the uncertainty of the mean-square value:

$$n_2 = \frac{1}{T}\int_0^T x^2(t)dt \qquad (1)$$

of the periodic signal will be analysed, for which only minimum m, maximum M and the mean value n_1 EINBETTENare known. The relation between the estimated parameter n_2 and the known parameter n_1 is in this case as follows:

$$n_2 = n_1^2 + n_{20} \qquad (2)$$

where n_{20} denotes the mean-square value of the signal $x_0(t)$ which remains after removing, from $x(t)$, its mean value: $x_0(t) = x(t) - n_1$. Because n_1 is assumed to be known, the uncertainty of the n_2 in (2) is determined by the range of the n_{20} variability. The minimum possible value of the n_{20} is zero and is taken by the signals $x(t)$ which are constant and equal n_1 almost everywhere, while the maximum possible value of n_{20} is obtained for the rectangular waves jumping between levels M and m once per period.

As a final result, the shadowed area in Fig. 1a indicates all possible values the parameter n_2 may have for different values of n_1. The height of the shadowed area in Fig. 1a changes parabolically: $-n_1^{\,2} + (M+m)n_1 - mM$ taking the greatest value: $(M-m)^2/4$ for $n_1 = (M+m)/2$.

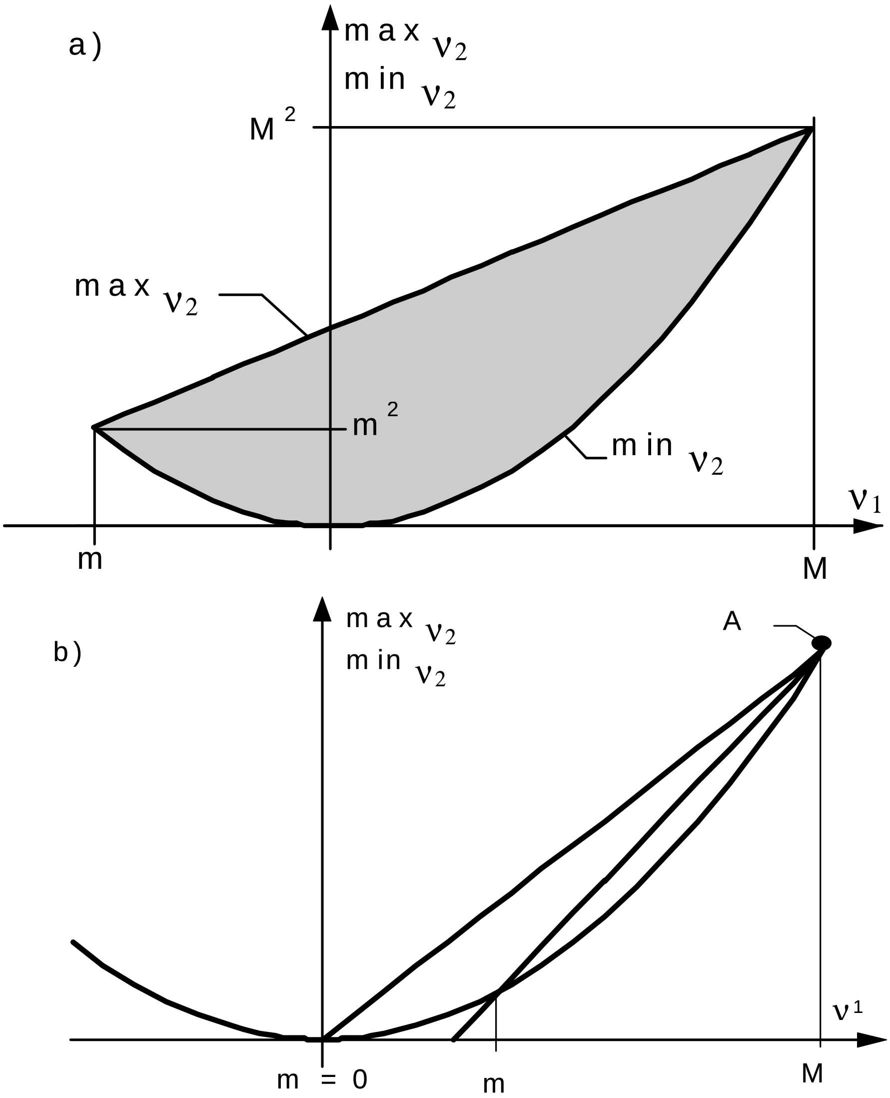

Figure 1. Bounding curves for n_2 for different values of n_1: a) - general case,
b) - the evolution of the upper limit as m tends towards M.

As it may be deduced from Fig. 1a, the worst case takes place when $m = -M$, for which, when additionally $n_1 = 0$, the measurement of the mean doesn't narrow down at all the initial n_2 uncertainty equal to M^2. On the other hand, Fig.1b shows an interesting case, when the signal is full wave rectified, which corresponds to m=0. As it can be seen, the „bowstring" of the n_2 upper limit rotates round the point A, reducing the maximum width of the n_2 uncertainty. It is also shown, in Fig. 1b, that the final n_2 uncertainty, after n_1 measurement, may be very little, especially for the signals of the small span $M - m$, for which both M and m have the same sign.

As no assumptions according to the signal $x(t)$ shape have been made so far, the uncertainties obtained in Fig. 1b, may also be treated as a upper bounding of the so called signal shape error, important for the instruments where rectified average is used instead of true n_2 measurement. The obtained uncertainties are valid irrespectively of the signal shape the instrument has been calibrated for, and the specific actual shape it is used for.

From Eq. (2) and Fig. 1, the method is visible for the n_2 uncertainty reduction to any acceptably small level. It consists in n_{20} measurement from Eq. (2) by the full-wave rectification of the signal $x_0(t)$ (rectification doesn't change n_{20} but reduces the span of the signal), then by the mean value $n_{1,1}$ measurement of the obtained signal and, then, by removal of this mean value. Such the procedure of the signal rectification, the mean value measurement and its removal from the signal, may be repeated as many times as needed. The value of the n_2 should be estimated as :

$$\sum_i n_{1,i}{}^2 , \qquad (3)$$

where $n_{1,i}$ denote the mean values of the signals in consecutive iterations.

The convergence of the above procedure is strongly dependent on the probability distribution of the measured signal, and only for the uniform one (e.g. for the signal of the triangle shape) the signal span $M - m$ decreases by a half per each step. In all other cases, the probability distribution of the signal after consecutive rectifications and $n_{1,i}$ removal, becomes strongly positively skewed, thus slowing down the procedure convergence. Nevertheless, for many signal shapes, the number of iterations needed for n_2 uncertainty reduction to the level of $10^{-2}\%$ was found to be less than ten, similarly as in the method described in [8].

The main advantage of the presented method is that it doesn't need any fast quadrator. Squaring is performed here over constant values $n_{1,i}$ (3) and may be carried out slowly, e.g. numerically. An important disadvantage is , however, that the frequency range of the signal after folding it up by rectification increases drastically because even simple initial signals - like sine ones - acquire a shape with many sharp spikes.

3 THE MEAN VALUE UNCERTAINTY FOR A GIVEN SIGNAL BANDWIDTH

If the mean value n_1 of a continuous signal is to be very close its minimum m or maximum M value, the signal must have the shape of a sharp pulse for which a significant bandwidth is required. Since, in practice, the bandwidth of the periodic signal is often known or may be estimated, the problem has been investigated what is the most possible asymmetry of the n_1 location in the interval [m,M] when the number n of its highest non-zero harmonic is known. First of all, the appropriate parameters were introduced, defining quantitatively the asymmetry of the signal mean value location between m and M.

Then, the values of the consecutive harmonics' phases j_i were found numerically, for which n_1 can lie in the closest neighbourhood of the values m or M. It was stated that assuming $j_1 = 0$, for $i = 2,3,..$ should be: $j_i = (i-1)p/2$. Next, for a given bandwidth n, the signal harmonics content a_i / a_1 (amplitudes' quotient) has been found analytically, for which the mean value greatest asymmetry is obtained: $a_i / a_1 = (n - i + 1)/n$, $i = 1,..,n$, and the corresponding measures of the asymmetry have been calculated. The exemplary shapes of such signals for n = 2, 5 and 10 are presented in Fig. 2. Then, finally, the actual mean value uncertainties were found for a given m, M and the signal bandwidth n, as presented in Fig. 3.

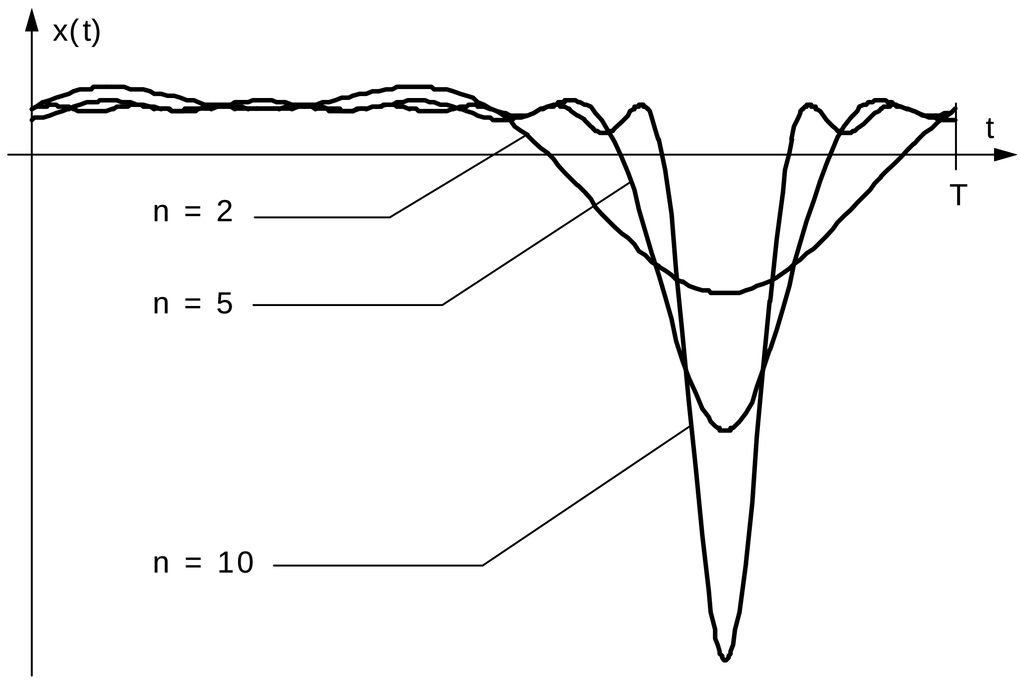

Figure 2. Signals of the bandwidth limited to the n-th harmonic for which the mean value is the closest to the maximum value

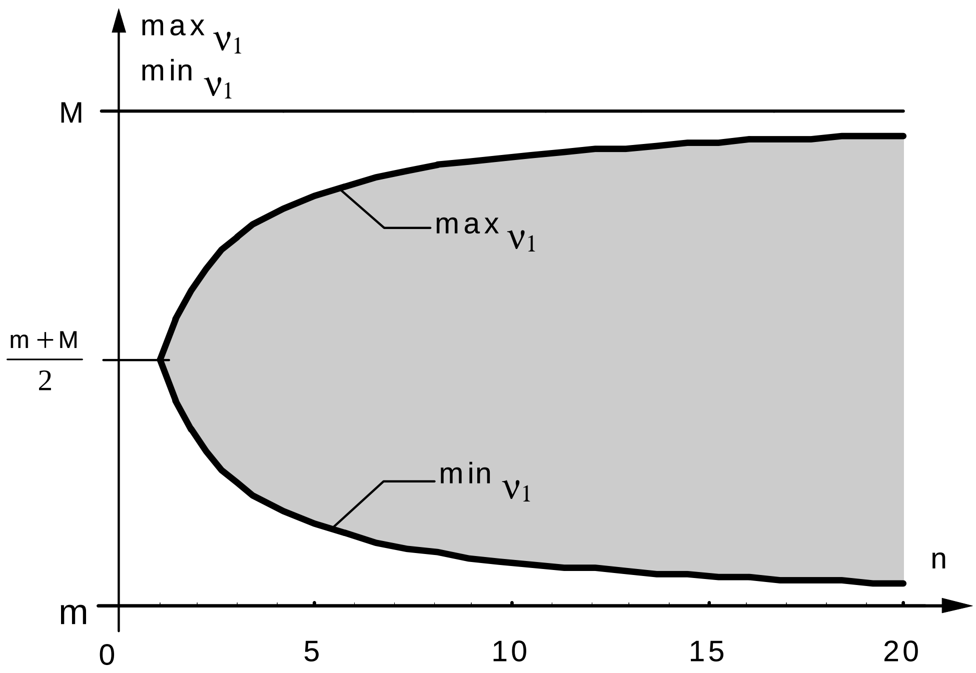

Figure 3. Bounding curves for n_1 vs number n of harmonics present in the signal

As it is seen from the plot, Fig.3, the message: n = 1 completely eliminates uncertainty according to n_1, as it actually means that the signal is sinusoidal with a previously known extremes. For n=2,3,..., the initial uncertainty $M - m$ is reduced by the factor (n+1)/(n-1), that is: threefold for n=2, by a half for n=3, by about 40% for n=4, and only by about 18% for n=10. As it can be seen from this mathematical model, a significant reduction of the n_1 uncertainty may be expected only for such cases where relatively narrow bandwidth bounding n<10 is possible.

4 CONCLUSIONS

The problem of the estimated parameter uncertainty under the assumption that some other given signal parameter is known, was discussed on two examples. The method of the bounding functions was used in both.

It was stated that the mean value significantly bounds the mean-square value uncertainty when the signal span $M - m$ is relatively narrow according to the value of M.

Knowledge about the signal bandwidth can also be used for the mean value uncertainty limitation, but it is practically valuable only when the number of the highest non-zero harmonic is less than ten.

Bounding functions founding for some estimated parameter is interesting not only from the theoretical point of view, but sometimes gives also some suggestions about how the measuring apparatus or procedure can be simplified utilising the a priori knowledge.

In the future, besides influence analyses of the a priori knowledge of simple structures, the methodology of data fusion should be investigated in order to answer the question how the bounding curves based on single parameter knowledge may be utilised for the resultant bounds finding when many parameters are known.

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