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CONTROLLABILITY OF OBJECTS MEASURED FOR THE NEED OF EXPERIMENT DESIGN

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Abstract − Methods of experiment design require performing measurements in selected input space points of a model of the object measured. To make it possible, an object must have a property called controllability. In the paper, controllability criteria have been formulated based on the fundamental principles of metrology. In addition, restrictions of controllability of certain objects of measurement have been described. Finally, consequences of disregarding the criterion for controllability occurring at a stage of the verification of the model have been described.

Keywords: measurement, design of experiment

1. INTRODUCTION

Measurement is the identification of a mathematical model of an object performed experimentally. Identification is of a structural character in creative measurements, and of a parametric one in routine measurements [1], [2]. Each measurement is an experiment, and each experiment – in empirical sciences – requires making a measurement. A designed experiment, which can be called the identification of a model of a multidimensional object, is a special kind of experiment based on the rules of experiment theory. It has the character of creative measurement since identification also refers to the structure of the mathematical model. Making such a design experiment requires following certain rules concerning the selection of measuring points, which result from the mathematical criterion for the design. The satisfaction of this criterion ensures definite features of the formal relationship searched for, referred to as the function of an object [3], between the output quantity Y and input quantities X_k in a qualitative mathematical model of an object:

$$Y = f(X_k, b_q), \text{ with } C_j = const. \qquad (1)$$

In this formula b_q are the relationship parameters, while C_j are constant quantities determining conditions in which measurements are performed.

For example, satisfying the orthogonality criterion [4] guarantees stochastic independence and ease of calculations of the coefficients b_q. Satisfying the criterion for rotatability ensures minimization of so called normalized uncertainty of the output quantity Y [5] and the relative independence of the coefficients b_q from the values of input quantities X_k [6].

There are many similar criteria determining experiment designs [7], [8]. Each of them requires that measurements of the values of X_k be made at the n points imposed by the design P_m i – the dimensional space of the input quantities $(k = 1,...,i)$. The number of measuring points $(m = 1,...,n)$ depends on the level of normalization of the values of X_k and is related to the non-linearity of the model. In the paper it is assumed that all the input quantities are normalized in the same manner. These points, whose coordinates are the reference values of the design, are placed inside a hypersolid of determinability of the model limited by closed intervals of variation of the values of quantities X_k: $[x_{kmin}, x_{kmax}]$. The function of an object found as a result of the experiment 'is valid' only in this i – dimensional hypercube.

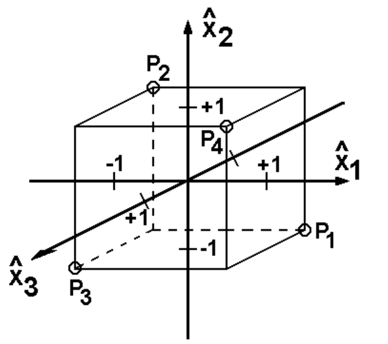

Fig.1 Geometric interpretation of a fractional design

In Fig. 1 a space of three input quantities, normalized in two levels way, is presented. The ranges of variability of each of the input quantities were limited to the interval [-1; +1]. The dependence between the normalized values and the real values is shown in the simple relationship

$$\hat{x}_k = \frac{2(x_k - x_{kav})}{x_{k\max} - x_{k\min}} \qquad (2)$$

where x_{kav} - the central value of the variability interval. In the cube corners, of the length of sides of 2, are placed the reference points of a complete design. As a result of the application of generating relation,

$$\hat{x}_3 = \hat{x}_1 \hat{x}_2 \qquad (3)$$

we can create a saturated fractional design described by Finney [12]. Points P_m are a graphical image of the system of this plan. The normalized values of the input quantities are coordinates of these points (Fig. 2):

$P_1(+1,-1,-1)$; $P_2(-1,+1,+1)$; $P_3(-1,-1,+1)$ i $P_4(+1,+1,+1)$

The fractional design realizes the criterion for orthogonality and rotatability on condition that the same distance is preserved between the points P_m, which for any pair of points is a diagonal in the square and it is $2\sqrt{2}$.

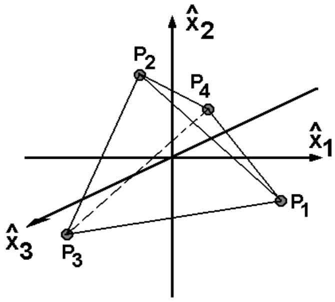

Fig. 2 Graphic image of the system of fractional design

Thus, we can say that the corners of a regular tetrahedron are a graphic image of the fractional design system.

2. GENERAL CRITERION FOR CONTROLLABILITY

When starting making experiments, an experimenter must be able to set any values of the input quantities according to the reference values of the design. It is said that [9] a qualitative mathematical model of an object must be controllable. For example, a design requires that $x_{1m} = 154{,}46$ orifices of a conical angle $x_{2m} = 102{,}43^o$ be drilled in an aluminium plate, while technological considerations allow the total number of $x^f_{1m} = 154$ orifices of an angle $x^f_{2m} = 102^o$ to be drilled. Thus, a question arises whether the difference between the reference values, x_{km}, and the set ones, x^f_{km}, is sufficiently small for the object to have the properties resulting from the design criterion.

The situation described can be illustrated geometrically referring to a fractional plan. Fig. 3 shows two sets of points: indicated by the design of reference points (P_m) and set by the experimenter (P^f_m). As can be seen, the geometrical figure of fig. 2 was deformed and the distances between the points P^f_m are no more equal to $2\sqrt{2}$. A question arises whether this deformation is sufficiently small for the plan to preserve its properties, i.e. orthogonality and rotatability.

The answer to this question must be given before one starts to measure the output quantity, i.e. prior to the further part of the experiment. This answer should have the form of a unique criterion which determines the greatest possible difference between the above-mentioned values of each input quantity. It is sufficient to state that a model is controllable when for each input quantity X_k, at each measuring point P_m, the inequality holds true

$$\left| x_{km} - x^f_{km} \right| \le \Delta_{km} \qquad (4)$$

where Δ_{km} – the values determined by measurement inaccuracy and design susceptibility. The latter should be understood as a set of the maximum permissible values of the differences (4), with which the plan criterion is still satisfied. For the normalized values, this dependence should be replaced with the relationship:

$$\left| \hat{x}_{km} - \hat{x}^f_{km} \right| \le \hat{\Delta}_{km} \qquad (5)$$

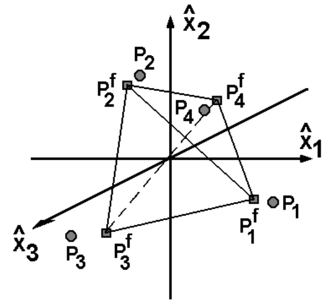

Fig. 3 Measuring points of fractional design: reference ones (P_m) and set ones (P^f_m)

The relationship (4) can be called a general criterion for controllability of a mathematical model of the object measured. The values Δ_{km} form a matrix of controllability, which has a shape of the transposed design matrix:

$$\mathbf{\Delta} = \left[\Delta_{in} \right] \qquad (6)$$

Elements of the matrix Δ are functions of two variables: -the sensitivity of a design to a change in the reference values and -the inaccuracy of the instruments used for the measurement of the input values. In the further part of the paper, criteria taking into consideration only the second variable are formulated.

3. METROLOGICAL CRITERION FOR CONTROLLABILITY

In the space R^i of the input quantities there are n reference measuring points indicated by the experiment design P_m $(x_{1m}, x_{2m}, \ldots, x_{im})$ and n measuring points set by the experimenter P^f_m $(x^f_{1m}, x^f_{2m}, \ldots, x^f_{im})$ as a result of the practical accomplishment of the design systems. In Fig. 3 the deformation is presented of a three-dimensional space of input quantities, caused by the inaccuracy of setting and measurement of values at the set measuring points.

The coordinates of the measuring points, i.e. the values, x^f_{km}, are verified by the measurement. Each of them has its own estimate, $\bar{x}^f_{km}$, and a measure of accuracy of its determination. Since in the designed experiment the measurement results are real numbers, an extended uncertainty, U^f_{km} [10], can be taken as a measure of inaccuracy. Thus, for each co-ordinate of the set point P^f_m one can write

$$x^f_{km} \in [\bar{x}^f_{km} - U^f_{km};\ \bar{x}^f_{km} + U^f_{km}] \qquad (7)$$

In this formulation the point P^f_m becomes a hypercube of dimensions given by extended uncertainties of the measurement n of the set values of each input quantity X_k. They form a solid of uncertainty around this point, whose centre is determined by estimates of the set values.

In a normalized space of input quantities, extended uncertainties should be replaced by normalized extended uncertainties according to the relationship

$$\hat{U}^{f}_{km} = \frac{2U^{f}_{km}}{x_{k\max} - x_{k\min}} \qquad (8)$$

Metrological criterion for controllability can be formulated as follows [9]:

-the object is controllable if its mathematical model can be constructed in such a way that all the measuring reference points indicated by the design lie inside the uncertainty solids of the measuring points set as a result of the practical accomplishment of the model. It results from the above that for each $k \in [1; i]$ and for each $m \in [1; n]$ the following relationship must be fulfilled:

$$x_{km} \in [\bar{x}^{f}_{km} - U^{f}_{km}; \bar{x}^{f}_{km} + U^{f}_{km}], \qquad (9)$$

By comparing criteria (9) and (4) it can be stated that in this case the extended uncertainties U^{f}_{km} are elements of the controllability matrix:

$$\Delta = [\, U^{f}_{in} \,] \qquad (10)$$

In Fig. 4 a graphical form of a situation is shown in which the object is controllable despite clear differences between the coordinates of both groups of points. The figure was made on an assumption that the extended uncertainties do not depend on the values of the input quantities and that at all the points of the model definiteness the values of extended uncertainties have the same value.

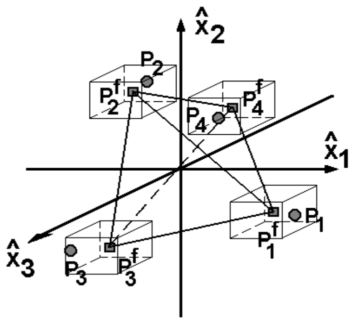

Fig. 4 An example of fulfilment of metrological criterion for controllability in a normalized space

The metrological criterion is of the general character and can be used for a preliminary assessment of the feasibility of a continuation of the experiment.

4. CRITERION OF CONTROLLABILITY FOR A FRACTIONAL DESIGN

The fractional design described above was formed on the basis of a complete design as a subset of points realizing relationship (3). This relationship will also be valid for the set points. Thus, at each point

$$\hat{x}^{f}_{3m} = \hat{x}^{f}_{2m}\hat{x}^{f}_{1m} \qquad (11)$$

As mentioned earlier, this relationship causes the normalized distance between any pair of points P_m is equal to $\hat{d} = 2\sqrt{2}$.

The criterion for controllability for this case can be formulated as follows:

- if in a space of three input quantities, the distance d_r between any pair of points set as a result of the practical realization of the model does not differ from the values of $\hat{d}$ by more than it would result from the inaccuracy of measurement of the input quantities at these points, then the object under study can be considered controllable.

In Fig. 5 two reference points of a fractional design P_3 and P_4 are shown. Shaded areas are geometrical places of points in which set points can put for the criterion for controllability to be satisfied.

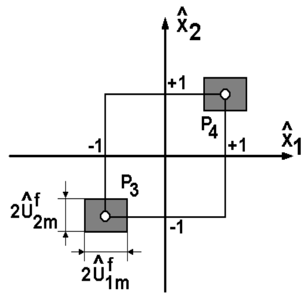

Fig. 5 Two reference points of a fractional design

Assuming that extended uncertainties do not depend on the values of input quantities and at all the points of the definiteness of a model have the same values, it can be seen tha n the boundary values of the normalized distance $\hat{d}_r$ between them are:

$$\hat{d}_{r\max} = 2\sqrt{(1+\hat{U}^{f}_{1m})^{2} + (1+\hat{U}^{f}_{2m})^{2}}$$

$$\hat{d}_{r\min} = 2\sqrt{(1-\hat{U}^{f}_{1m})^{2} + (1-\hat{U}^{f}_{2m})^{2}}$$

Since $\hat{U}^{f\,2}_{km} \langle\langle\, 1$, then, having disregarded small terms and made an obvious assumption that $\hat{U}^{f}_{1m} + \hat{U}^{f}_{2m} \langle\, 1$, we obtain:

$$\hat{d}_{r\max} = \hat{d}\sqrt{1 + \hat{U}^{f}_{1m} + \hat{U}^{f}_{2m}}$$
$$\hat{d}_{r\min} = \hat{d}\sqrt{1 - \hat{U}^{f}_{1m} - \hat{U}^{f}_{2m}} \qquad (12)$$

Changing over to non-normalized values leads to unnecessary complication of these relationships. Therefore, it appears that the criterion of the form below would better be used:

$$\hat{d}_{r\min} \le \hat{d}_r \le \hat{d}_{r\max} \qquad (13)$$

bearing in mind that in a normalized space, relationships (2) and (8) are valid.

5. EXAMPLES OF UNCONTROLLABLE OBJECTS

Textile raw materials and products are the typical objects of limited controllability. Their features like: rheological properties, the essential influence of measuring processes on the parameter's value, high sensitivity on climate influences and great dispersion of surface and space parameters, make the investigation very difficult. They require special efforts for the formation of a representative test specimen collection, as well as for the progress of the measuring process itself. These properties also cause great difficulties in using traditional methods of experiment designing.

The quantities which values are not continuous variables are the first reason of uncontrollability of the objects, e.g. such as raw material and a kind of the threads and weave of the fabric cannot exist in the object's function as continuous real variables. The other reasons of uncontrollability are the properties of technological processes, in this case spinning and weaving. They are the elements of the practical realisation of the experiment model. Experience shows [11], that the relative differences between the nominal values of thread and woven fabric parameters and the estimators of measured true values reach the values of a few to ten or more per cent for both processes.

Let us assume that the design requires to measure the properties of the fabric of the following parameters: -the warp and weft threads made of cotton yarn of the 36 tex (linear mass) and 281 twists/m, -linen wave, -density of 290 threds/10cm. The properties of the process cause that, at the nominal machine value of twist which equals 280 twist/m, the following value can be obtained: $\bar{x}_{km}^{f} = 266 twists/m$. This value, at about 3 % of related expanded uncertainty, gives the range:

$$[\bar{x}_{km}^{f} - U_{km}^{f}; \bar{x}_{km}^{f} + U_{km}^{f}] = [258;\ 274]\ twists/m.$$

In this way we have a situation where the reference value x_{1m} = 281 twists /m, and is no longer included in the range of the uncertainty of the estimation value of the thread twist's true value, and the dependency (9) is not fulfilled. A similar consideration, with the same result, can be conducted for the linear density. This signifies that the controllability criterion is not fulfilled for this object.

In the case of many other textile quantities, we have to deal with similar situations. Therefore a conclusion can be drawn, that most of the textile products are not controllable objects. It means that their investigations require another methods like e.g. steady-system design and non-linear estimation [9].

6. CONSEQUENCES OF NOT SATISFYING THE CRITERION FOR CONTROLLABILITY

The controllability of the model is of prime importance on the stage of verification. The verification of the model consists in assessing possibilities of using the object function found for predicting properties of objects in the area of the definiteness of the object. This assessment is always based on the analysis of the significance of differences between the values of the output quantity calculated on the basis of the object function and that obtained as a result of measurements of a new object called a verifying object. If the model is not controllable, this means that the criterion for the experiment design used is not fulfilled. In the case of the orthogonal design, there is a danger that the coefficients of the function of an object depend on one another, which is manifested by an absence of the diagonality of the design matrix. In the case of a rotatable-uniformal design, this means that there is, for example, a significant, although unknown relationship between the inaccuracy of identification of the value of the coefficients of the object function and the values of the particular input quantities. This may lead to a danger of positive verification in one area of the definiteness of the model, and negative in the other. Thus, in the case of difficult-to-control objects, verification should be made at many points of the space of the input quantities, while its results should be assessed unambiguously.

7. CONCLUSIONS

1. Controllability is one of the most important features of measured objects while performing standard designed experiments; this feature can be expressed by means of a quantitative measure formulated in the form of a criterion for controllability.

2. During an experiment, before measuring the output quantity, the metrological analysis of the results of the set value measurements of the input quantities in a model of an object should be made; otherwise, the function of the object can have properties different from the expected ones, which can, in turn, cause problems at a stage of the verification of the model.

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