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A STUDY OF THE MOISTURE EFFECTS ON THE ACOUSTIC WAVES

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Abstract – This paper focuses on the study of a mathematical model to measure the moisture content of agricultural soils. After the proposal and analysis of a measurement method based on the propagation of elastic waves in a granular unconsolidated medium, a mathematical relationship between the propagation velocity of an acoustic wave and the moisture content of the soil is obtained. The authors carefully study and verify the conditions that guarantee the applicability of the model; the analysis shows that a right choice of the frequency of the transmitted signal allows the applicability of the model to a wide range of agricultural soils.

Keywords: Acoustic waves, soil measurements, water content

1. INTRODUCTION

This article focuses on the feasibility of a sensor to be used on-site for the measurement of the moisture content of agricultural soils; the requirements for this kind of sensor are quite stringent: it should be simple to use, accurate and as much inexpensive as possible.
This research is part of a wider project focused on the microclimatic analysis of agricultural soils and aiming at an ecocompatible utilization of them thanks to a complete monitoring of their properties. Knowing that the velocity of propagation of an acoustic wave in a given medium is a direct function of its density, its porosity and its moisture content, the authors examine the possibility to use acoustic waves to stimulate the soil and to use the signal received at a known distance to measure the degree of moistness.
The analysis aims to the definition of a mathematical relationship between the moisture content of the soil and the velocity of sound; the final model would be as accurate and simple as possible.
The theory of propagation of acoustic waves in a granular, unconsolidated and unsaturated medium (as is an agricultural soil) has been already studied with remarkable success by Brutsaert [1]; he, starting from the general theory of propagation of elastic waves in a porous medium, proposed a complete mathematical model for this kind of phenomenon. In this paper the authors examine the possibility to measure the moisture content of a soil measuring the velocity of propagation of acoustic waves; all of the possible simplifications of the basic mathematical model are carefully analysed aiming to determine a relationship as simple as possible. The final model depends on a limited number of independent parameters and is applicable to the most part of agricultural soils. In detail, once obtained the functional relationship between moisture content and velocity of propagation of the acoustic wave, the range of applicability of the model is analysed and the range of usable frequency for the transmitted stimulus wave is determined.

2. THE PROPOSED MODEL

Many researchers have already and deeply studied the problem of elastic wave propagation in liquid-saturated granular and porous mediums; generally the model of soil assumes a frame of unconsolidated solid particle (modelled as spheres) and two mixed fluids: a wetting one (the water) and an unwetting one (the air).
There are two important parameters to consider in soil moisture measurement: the *porosity*, which represents the ratio between volume of voids and total volume, and the *degree of saturation*, which is the ratio of volume of water and volume of voids.
Defining the following symbols: V_{tot} = total volume of soil specimen = volume of soil + volume of air content + volume of water content = $V_s + V_a + V_w$, V_v = volume of voids = $V_a + V_w$, then the soil *porosity* f is expressed as:

$$f = \frac{V_v}{V_{tot}} \tag{1}$$

The *moisture u* is given by:

$$u = \frac{V_w}{V_{tot}} \tag{2}$$

while the *degree of saturation with liquid* S is given by:

$$S = \frac{V_w}{V_v} \tag{3}$$

Then the condition $S = 1$ identifies a soil fully saturated of water, while the condition $S = 0$ identifies a completely dry soil. Using the previous definitions and after some simple mathematical manipulations we obtain the relationship between the porosity of the soil and its degree of moistness:

$$S = \frac{u}{f} \tag{4}$$

The rigorous solution of equations associated with the propagation of sound waves in porous mediums produces *three compressional waves* and *one share wave*. The theory introduced by Brutsaert [1] assures that every of this waves propagates with a different velocity and is affected by a different attenuation. In [2] it is demonstrated that, under the suitable hypotheses, only one of the compressional waves dominates over the others which are instead deeply attenuated; the *surviving* wave is known as *wave of the first kind* or as *perfectly coupled compressional wave*. For this kind of wave the velocity of propagation is given by the following expression:

$$v = \sqrt{\frac{0.306 \cdot a \cdot p_e^{1/3} \cdot Z}{\rho_{tot} \cdot f \cdot b^{2/3}}} \qquad (5)$$

where a and b are parameters tied to the granular properties of material, ρ_{tot} and p_e are the total density and the effective pressure of soil, whereas Z is a parameter which describes the effects of the interstitial fluids.

Relationship (5) by itself expresses a useful model for the realization of a soil moisture sensor because, as we will see in more detail in the following sections, the variables ρ_{tot}, p_e and Z can be related to *S*, the degree of saturation with liquid of the soil.

3. THE MODEL APPLICABILITY

Relationship (5) in the previous section gives the velocity of sound in an unsaturated and unconsolidated soil; it has been obtained in the hypotheses of a continuous medium and negligible attenuation only for a wave of the first kind.

To guarantee a good behaviour of the model, a *suitable frequency of the transmitted signal* must be used and the following two conditions must be verified:

1. *the inertia-viscosity factor must be less than one;*
2. *the distant field constraint must be satisfied;*

The first limit gives the following mathematical relationship:

$$\beta = \left(\frac{16 \cdot \pi \cdot f_s \cdot k_i}{f \cdot g \cdot S} \right)^{\frac{1}{2}} < 1 \qquad (6)$$

which ensures that the propagation of an acoustic signal in the soil is approximately due only to an elastic compressional wave; in (6) g is the acceleration of gravity. The soil properties influence β by means of the hydraulic conductivity k_i and of the porosity f ; β depends from the moisture content of the soil by means of S, the degree of saturation with liquid previously defined. Eqn. (6) takes into account also the dependence of the model on the frequency of the transmitted signal f_s ; consequently only the use of a definite range of frequencies will guarantee that the propagation of an acoustic wave will be purely elastic and without dissipation. An in-depth analysis of (6) requires expressing the electrical conductivity as a function of the degree of saturation S. Using the Van Genuchten's model [5] we can write:

$$k_i = k_{is} \cdot S^{\frac{1}{2}} \left[1 - \left(1 - S^{\frac{1}{m}} \right)^m \right]^2 \qquad (7)$$

where k_{is} is the hydraulic conductivity at complete saturation ($S = 1$) and $m = 1 - 1/n$ represents a parameter which depends both on the textural class of the soil (fig. 1) and on the degree of the saturation with liquid S.

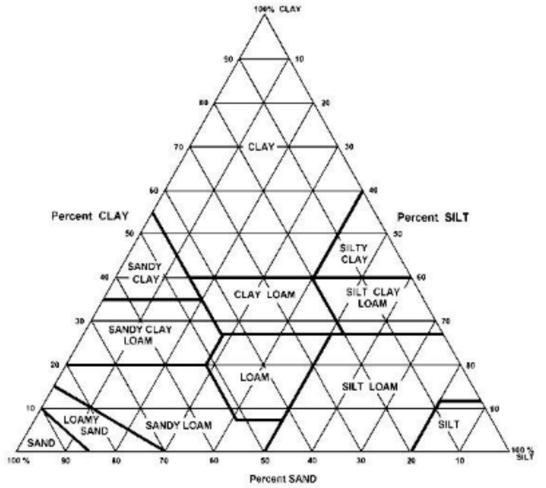

Fig. 1. Drawls triangle: representation of twelve principal textural classes.

To apply the Van Genuchten's model we must first determine the constants that appear in (7) which can be different also for soils belonging to the same textural class. Considering that our aim is the study of the variability of β and the fulfillment of (6), in this analysis it will be sufficient to use the medium values of the various coefficients corresponding to the twelve possible textural classes. These coefficients have been calculated by Carsel e Parrish [3] averaging the values measured for the soils of 42 Countries (Tab. 1).

Substituing (7) in (6) we obtain:

$$\beta = \left(\frac{5.13 \cdot f_s \cdot k_{is}}{f} \left[\frac{1 - \left(1 - S^{1/m}\right)^m}{S^{1/4}} \right]^2 \right)^{\frac{1}{2}} \qquad (8)$$

The previous relationship says us that β grows with the degree of saturation of the soil and with the frequency of the acoustic wave, whatever is the textural class (Fig. 2). β is also a monotonically growing function of *f*; then it assumes its greater value when *f* has its minimum value (that is when *f = 0.3*); however, as we can see from the results of Carsel and Parrish (Tab. 1), that value of *f* is cautionary with respect to the most frequent ones.

An in-depth analysis of (8) shows that the factor between square brackets is always less than one even if we consider various types of soil (that is if we change the coefficients *n* and *m*); then β will be always less than the threshold value β_t:

$$\beta_t = \left(\frac{5.13 \cdot f_s \cdot k_{is}}{f} \right)^{\frac{1}{2}} \qquad (9)$$

Tab. 1. Values of k_{is}, f, α and n for some different soil texture.

Soil	k_{is} $[ms^{-1}]$	f	α	n
Sand	8.25*10^{-5}	0.43	0.145	2.68
Loamy Sand	4.05*10^{-5}	0.41	0.124	2.28
Sandy Loam	1.23*10^{-5}	0.41	0.075	1.89
Sandy Clay Loam	3.64*10^{-6}	0.39	0.059	1.48
Loam	2.89*10^{-6}	0.43	0.036	1.56
Silt Loam	1.25*10^{-6}	0.45	0.020	1.41
Sandy Clay	3.33*10^{-7}	0.38	0.027	1.23
Clay Loam	7.22*10^{-7}	0.41	0.019	1.31
Clay	5.56*10^{-7}	0.38	0.008	1.09
Silty Clay Loam	1.94*10^{-7}	0.43	0.010	1.23
Silt	6.94*10^{-7}	0.46	0.016	1.37
Silty Clay	5.56*10^{-8}	0.36	0.005	1.09

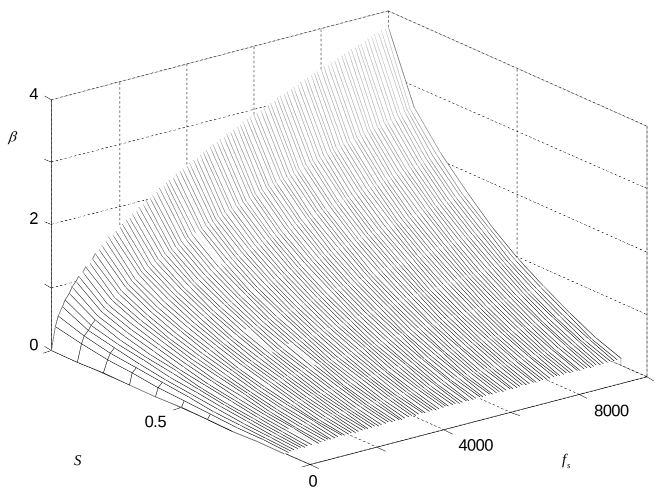

Fig. 2. Representation of theoretical values of β to different values of S and f_s when $f = 0.3$

Then to guarantee the validity of (5) it is sufficient to verifies that

$$\beta_t < 1 \qquad (10)$$

and this requires an in-depth knowledge of properties of three parameters: the *porosity of soil* f, the *hydraulic conductivity at complete saturation* k_{is}, and the *frequency of transmitted signal* f_s. Given that $f \in [0.30, 0.85]$, it is simple to verify that β_t is a decreasing function of f and that the limit condition is verified when $f = 0.30$. However a greater f permits the use of an extended range of frequencies; as an example for a soil with a value of hydraulic conductivity at complete saturation $k_{is} = 10^{-5}\ ms^{-1}$ the maximum usable frequency is $f_s \cong 6\ kHz$, when $f = 0.30$; this value rises to $f_s \cong 9\ kHz$ when $f = 0.45$, and to $f_s \cong 11\ kHz$ when $f = 0.60$ (Fig.3). It is important to note that these values of limit frequency change greatly for soil with the same value of porosity but with a different value of k_{is}.

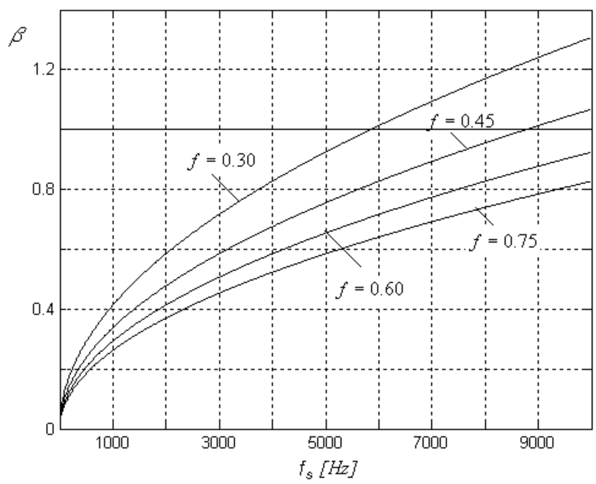

Fig. 3. Representation of theoretical values of β_t vs frequency for different typical values of porosity ($k_{is} = 10^{-5}$)

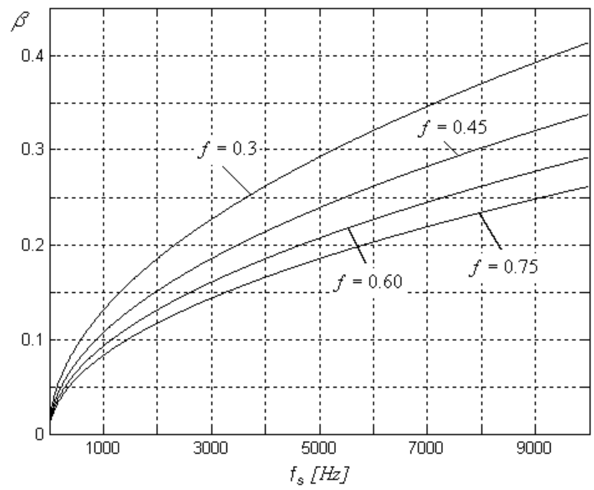

Fig. 4. Representation of theoretical values of β_t vs. frequency for different typical values of porosity ($k_{is} = 10^{-6}$)

As an example for a soil with $k_{is} = 10^{-6}\ ms^{-1}$ the maximum usable frequency is $f_s \cong 55\ kHz$ when $f = 0.30$, $f_s \cong 85\ kHz$ when $f = 0.45$ and it is about $f_s \cong 110\ kHz$, when $f = 0.60$ (Fig.4). An in-depth analysis of (9) shows that the threshold value β_t decreases when k_{is} decreases; this means that we could have some problems to satisfy the condition (10) for soils with great values of k_{is}. However we can verify that, if $k_{is} \in \left[10^{-7} \div 10^{-9}\right]$, β_t is always less than one whatever is the frequency of the acoustic wave (Fig. 5), even in the worst case (value of porosity $f = 0.3$).

However it is better to highlight that these results are valid only if we assume valid the results reported by Carsel and Parrish.

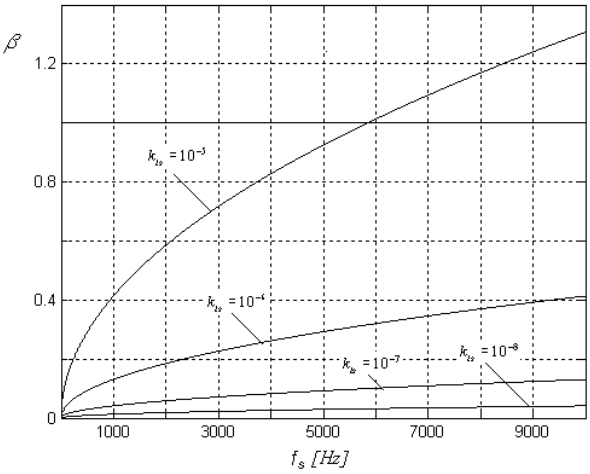

Fig. 5. Representation of theoretical values of β_t vs. frequency for different values of hydraulic conductivity at saturation.

To verify those results and to guarantee the applicability of (5) to a large range of agricultural soils it is better to consider the general expression proposed by Saxton [4] which gives the hydraulic conductivity at complete saturation k_{is} as a function of the percentages of clay and sand present in the soil:

$$k_{is} = k_{is}\left(sand_{\%}, clay_{\%}, u_s\right) \tag{11}$$

This model gives us the possibility to cover almost all of the textural classes of agricultural interest (Fig. 6); the results obtained by means of Eqn. (11) when applied to soils with varying percentages of sand and clay content confirm the values published by Carsel and Parrish and reported in Tab. 1; as a consequence using a value of $k_{is} \cong 10^{-5}\ ms^{-1}$ in (9) corresponds to the worst case.

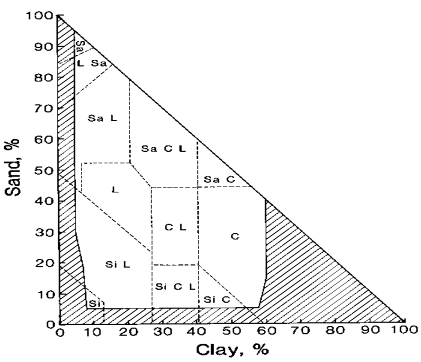

Fig. 6. Validity region of Saxton model

In Fig. 7 is reported a graph of β vs. f_s, with k_{is} as a parameter and with a fixed value of 0.3 for the porosity f; from that figure we can see that the worst case is that of a soil of type *Sand* with k_{is} = $8.25 \cdot 10^{-5}\ ms^{-1}$ and that condition (10) is verified for fs $\cong$ *900 Hz*. It is also better to highlight that this range of frequencies is extremely precautionary because, for soils of agricultural interest as are that classified as *Loam*, the value of k_{is} grows by about one order of magnitude and it can assume values of about $10^{-7}\ ms^{-1}$ and also taking into account that for $k_{is} = 1 \cdot 10^{-5}\ ms^{-1}$ the minimum value for f_s is about $6\ kHz$ (Fig. 7).
The constriction imposed to β is only the former condition that must be satisfied to guarantee the applicability of (5).

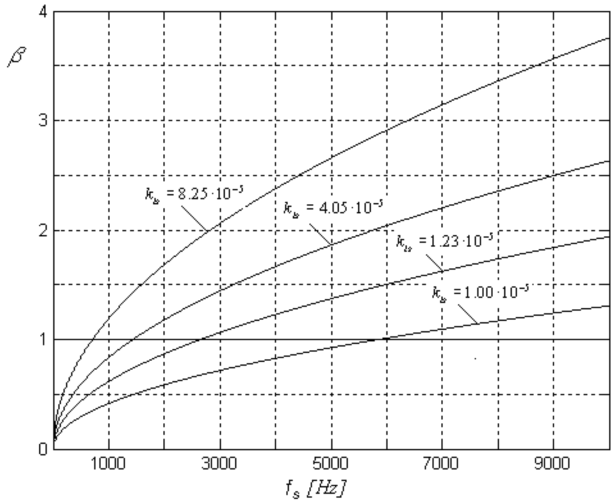

Fig. 7. Representation of theoretical values of β_t vs. frequency for different values of hydraulic conductivity at saturation.

The latter must be applied to the distance d between transmitter and receiver; it must be:

$$d \geq d_{min} = \frac{v}{2 \cdot \pi \cdot f_s} \tag{12}$$

If (12) is verified it can be demonstrated that the propagating wave can be considered a *plane wave*; in this case the velocity is a linear function of the distance from the transmitter and it is in phase with the acoustic pressure. Eqn. (12) says that the minimum usable distance is a direct linear function of the velocity of propagation v and an inverse function of f_s. It is evident that, being v a uncontrollable parameter, the frequency of the signal must be properly choose to guarantee the usability of a sufficiently short distance between transmitter and receiver; actually an excessively long distance could generate a deep attenuation of the received signal. The only parameter we can control in (12) to avoid this problem is the frequency of the transmitted signal. The velocity of sound in the soil is approximately included in the range [300, 1500] *m/s*; the lower value being the velocity of sound in the air and the higher the velocity of sound in the water.
Using $d = 50\ cm$, we obtain that the minimum usable frequency is about $f_{s\min} = 400\ Hz$ if $v = 1500\ m/s$ (Fig. 8). f_{smin} decreases to about 300 Hz if $v = 1100\ m/s$ and to about 90 Hz if $v = 300\ m/s$. However, being the typical value of

v in soils $\cong$ 600 m/s, the minimum usable frequency will be $f_s \cong 150\ Hz$.

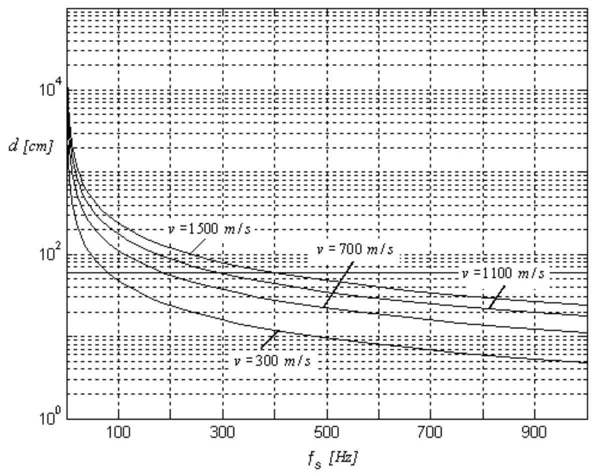

Fig. 8. Minimum theoretical distance between transmitter and receiver vs. frequency for different values of v.

To summarize: to use the model of propagation of acoustic signals in soil given by (5) it is necessary to satisfy both (6) and (12). The first equation requires that the frequency of the transmitted signal must be less than $900\ Hz$; this value is extremely prudential because it was calculated considering a *Sand* soil characterized by a value of the hydraulic conductivity at full saturation $k_{is} = 8.25 \cdot 10^{-5}\ ms^{-1}$. The second equation fixes the minimum value of f_{smin} to about $150\ Hz$. Also this value is prudential because it is obtained in the hypothesis of a distance between receiver and transmitter of about $50\ cm$ and f_{smin} decreases with that distance. Now an in-depth examination of the expression of velocity of sound in the soil is required to investigate if there is the possibility to simplify it without prejudice its generality.

4. THE SOUND VELOCITY

In the previous paragraph the conditions which permit the applicability of (5) were examined; it has been shown that, if the frequency of the transmitted signal becomes to a definite range, the velocity of propagation of the sound in the soil is tied as well as to the parameter Z (which takes into account the effects of the fluids present in the soil), and also to the *total density* ρ_{tot} and to the *effective pressure* p_e. In the sequel we will show that Eqn. (5) may be simplified and expressed as a function of the water content of the soil.

If ρ_b e ρ_w are, respectively, the *dry bulk density* and the *liquid bulk density*, it is possible to express the product $\rho_{tot} \cdot f$ in Eqn. (5) as a function of the degree of saturation with liquid S:

$$\rho_{tot} \cdot f = 0.1424\Big[\left(\rho_w \cdot S - 2.65\right)\rho_b^2 + \\ + \left(7.0225 - 5.3 \cdot \rho_w \cdot S\right)\rho_b + 7.0225 \cdot \rho_w \cdot S\Big] \tag{13}$$

Also the effective pressure p_e may be expressed as a function of S; if the sensor is placed at a depth h from the surface of the soil and expressing the total pressure p_t as:

$$p_t = \rho_m \cdot g \cdot h \tag{14}$$

the effective pressure is given by:

$$p_e = p_t - S \cdot p_c \tag{15}$$

where p_c is the *capillary pressure*, which is tied to S by means of the Van Genuchten's model parameters (Tab. 1) [5]:

$$p_c = -\frac{\rho_w \cdot g}{\alpha}\left[S^{-\frac{1}{m}} - 1\right]^{\frac{1}{n}} \tag{16}$$

The parameter Z, which describes the influence of the air and of the water content present in the granular medium on the velocity of the sound, may be expressed as [6]:

$$Z = \frac{\left[1 + \dfrac{30.75 \cdot k_e^{3/2} \cdot b}{p_e^{1/2}}\right]^{5/3}}{\left[1 + \dfrac{46.12 \cdot k_e^{3/2} \cdot b}{p_e^{1/2}}\right]} \tag{17}$$

where the effective modulus k_e is defined by the following relationship, corrected according to [7]:

$$k_e = \frac{k_a \cdot k_w}{k_a\left(1 - S\right) + k_w \cdot S} \tag{18}$$

being k_a and k_w the bulk modulus of air and water, respectively [6]. Substituting (18) in (17) and taking into account that $k_a = 1.4 \cdot 10^5 Pa$ and $k_w = 2 \cdot 10^9 Pa$ it is possible to show that Z is not so much sensitive to S and that it is close to one, even if the parameter b spans over its entire range of variability $\left[10^{-10} \div 10^{-12}\right] Pa^{-1}$ (Fig. 9).

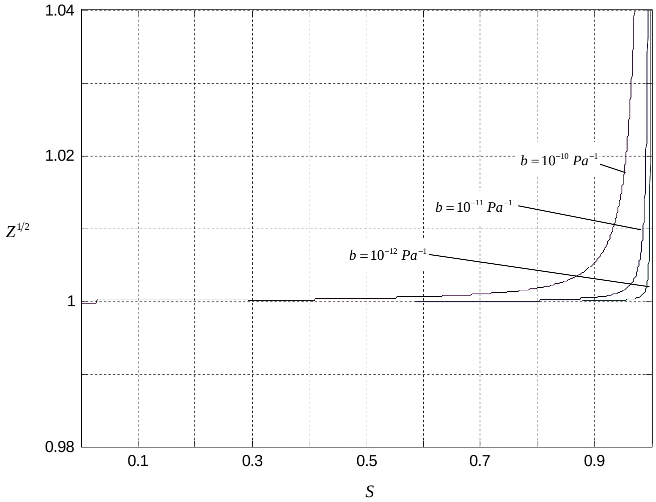

Fig. 9. Representation of $Z^{1/2}$ vs. saturation for different typical values of b in a loimy sand soil.

In Fig. 10 we can see that Z is almost constant and close to one for all types of soils and for varying porosity *f* and depth *h*.

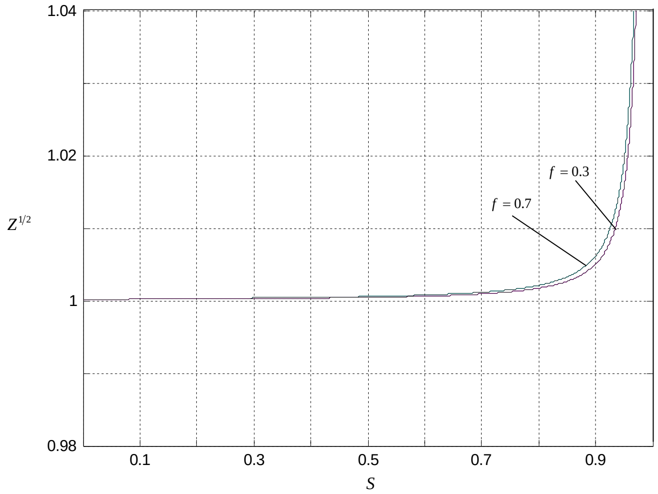

Fig. 10. Representation of $Z^{1/2}$ vs. saturation for different typical values of b in a loimy sand soil.

In further analyses has been verified that if $S \in [0.2,\ 0.8]$, using a fixed value of one instead of Z causes a maximum relative error on the velocity less than 0.02%.

Defining $\varsigma = \left(a / b^{2/3}\right)^{1/2}$, the velocity of propagation *v* may be expressed as:

$$v = \zeta \sqrt{\frac{0.306 \cdot p_e^{1/3}}{\rho_{tot} \cdot f}} \qquad (19)$$

then ζ appears as an "amplification factor" of *v*.

An in-depth analysis of (19), also taking into account (13) and (16), shows that the velocity of sound in the soil is greatly influenced by the water content of the soil and that its decay with rising *S* depends on the type of soil. Generally only three types of velocity trend can be obtained depending on the kind of soil [8].

5. CONCLUSIONS

In this work the functional relationship between the water content of agricultural soils and the velocity of propagation of sound waves in them have been analysed. A detailed analysis has been made aiming to an in-depth knowledge of the model to verify its applicability to a large selection of agricultural soils.

The analysis showed that, using a range of stimulus frequencies of $150 \div 900\ Hz$, the model can be used for almost all of the soils even in the worst case, that is when the hydraulic conductivity at saturation is $k_{is} = 8.25 \cdot 10^{-5}\ ms^{-1}$.

The analysis also showed that the expression of velocity may be simplified and can be related to a limited number of parameters all of which are tied to the degree of saturation *S*; the final relationship is applicable to almost all of the soils of agricultural interest.

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