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Pedagogical experience with hydrostatic weighing system

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Abstract: This project investigates the performance of two assemblings of hydrostatic weighing system. Low uncertainty of 0.004% to measure solid volume and 0.007% to measure liquid density was achieved with relatively simple measurements. An error analysis was made and the method could be used in calibration laboratories as well as for education on calibration procedures.

Keywords: hydrostatic weighing system, volume standard, liquid density, and error analysis.

1. INTRODUCTION

Two different assembling of hydrostatic weighing system were investigated, one is simpler to operate and the other has less influence quantities. Both assembling are simple and inexpensive, since most parts of the system were available in the laboratory, such as an analytic balance, volume standards, adjustable table of support, thermometers, barometer, hygrometer, distilled water, etc. The results were compared to a digital density meter and a volume comparator system.

Bidistilled and deinonized water and atmospheric air were used as reference of density. Performing the measurements in a controlled ambient, one can accurately measure the volume and the density of solids as well as the density of liquids by employing an analytical balance and a volume standard.

This experiment would have sufficient accuracy to calibrate and evaluate the performance of several instruments to measure liquid density, such as hydrometer, pyknometer and digital density meter. This technique could also be used for pedagogical purposes and for a calibration laboratory due to its accuracy, simplicity, relative low costs and instructive error analysis involved.

2. METHODS

The proposed method relies on the accurate measure of the weigh and the value of the specific density of two fluids, in our case bidistilled and deionized water and air. The density of air and water can be found by using equation from ISO/ TR20461[(2)].

The first and the second assemblings of the hydrostatic weighing system were mounted as shown respectively on the scheme on fig.1 and fig.2.

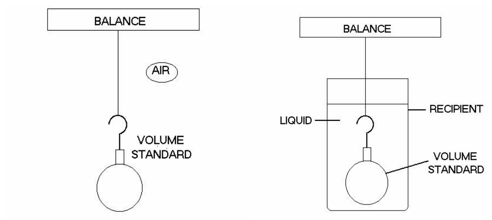

Fig. 1. First assembling of hydrostatic weighing system

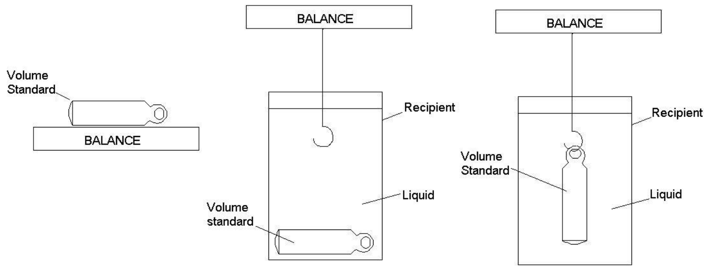

Fig. 2. Second assembling of hydrostatic weighing system

The forces in these two figures are described by the following equations:
Weighing in the air.

$$F_A = P - B_A \quad (1)$$

Weighing in the liquid.

$$F_L = P - B_L - S \quad (2)$$

With "F" the force on the balance, "P" the weigh of the solid volume, "B" the force due to buoyancy on the solid volume and "S" the force due to the surface tension of liquid on the stainless steel wire. The subscripts "A" and "L" correspond respectively to the situations with air and liquid on fig.1 and fig.2.

Using the density of water and atmospheric air as reference standard, one can find the volume standard by solving eq.1 and eq.2.

$$V = \frac{M_A - M_L}{(\rho_L - \rho_A)} \cdot \left(1 - \frac{\rho_a}{\rho_b}\right) + \frac{d\pi\sigma}{(\rho_L - \rho_A) \cdot g} + \delta V \quad (3)$$

$$V_r = V\left(1 + \alpha\left(T_r - T\right)\right) \quad (4)$$

With:

V_r: Measured volume standard at reference temperature "T_r" (m^3)
V: Measured volume standard at measured temperature "T" (m^3)
M_A: Apparent mass of volume standard immersed in air (g)
M_L: Apparent mass on balance for volume standard immersed in liquid (g)
ρ_A: Air density during measurement of M_A (g/m^3)
ρ_L: Liquid density during measurement of M_L at temperature "T" (g/m^3)
α : Volumetric coefficient of thermal expansion ($^oC^{-1}$)
T: Liquid temperature during measurement of volume standard (^oC)
T_r: Temperature of reference of volume standard (^oC)
d: Wire diameter (m)
σ: Surface tension of liquid (N/m)
ρ_a: Air density during balance calibration (g/m^3)
ρ_b: Standard weigh density used for balance calibration (g/m^3)
g: Local acceleration of gravity (m/s^2)
δV: Repeatability/reproducibility error

Once the volume standard is determined, an unknown liquid density "ρ_L"can be measured by using eq.5.

$$\rho_L = \frac{M_A - M_L}{V_r(1+\alpha(T-T_r))}\left(1-\frac{\rho_a}{\rho_b}\right) + \frac{d\pi\sigma}{V_r(1+\alpha(T-T_r))g} + \rho_A + \delta\rho_L \quad (5)$$

With "$\delta\rho_L$" the repeatability/reproducibility error. We tare both assemblings differently, the first with the volume standard immersed in air, as shown in the left side of fig.1, and the second with the hook immersed in water, as shown on the middle of fig.2. This way, we avoid having to calculate the absolute value of the surface tension for the second assembling, giving, instead of eq.3 and eq.5, the following equations (6) and (7). This procedure diminishes the uncertainty for both measurements of the volume standard and of the liquid density.

$$V = \frac{M_A - M_L}{(\rho_L - \rho_A)} \cdot \left(1-\frac{\rho_a}{\rho_b}\right) + \delta W_V + \delta V \quad (6)$$

$$\rho_L = \frac{M_A - M_L}{V_r(1+\alpha(T-T_r))} \cdot \left(1-\frac{\rho_a}{\rho_b}\right) + \rho_A + \delta W_\rho + \delta\rho_L \quad (7)$$

With "δW_V" and "δW_ρ" the variations caused by the surface tension and buoyancy force acting on the wire for the measurements of volume and density. "δV" and "$\delta\rho_L$" are the repeatability errors for each measurement.

The uncertainty contribution due to each influence quantity and measuring instrument is determined for the measurement of the volume standard and of the liquid density. The results are shown on section 4 on table 4 and 5.

3. EXPERIMENTAL SETUP

Both setups were mounted using balance with capacity of 200 g and resolution of 0,0001 g. The volume standards used were different for the assemblings, the first was made out of stainless steel while the second was made out of crystal. The holding structure (wire and hook) was flexible for the first assembling and rigid for the second assembling.

Fig. 3 shows the improved assembling that reached smaller uncertainty. The main differences in order to improve the system were the rigid holding structure, the crystal volume standard and the tare of the analytical balance made with the hook inside the liquid.

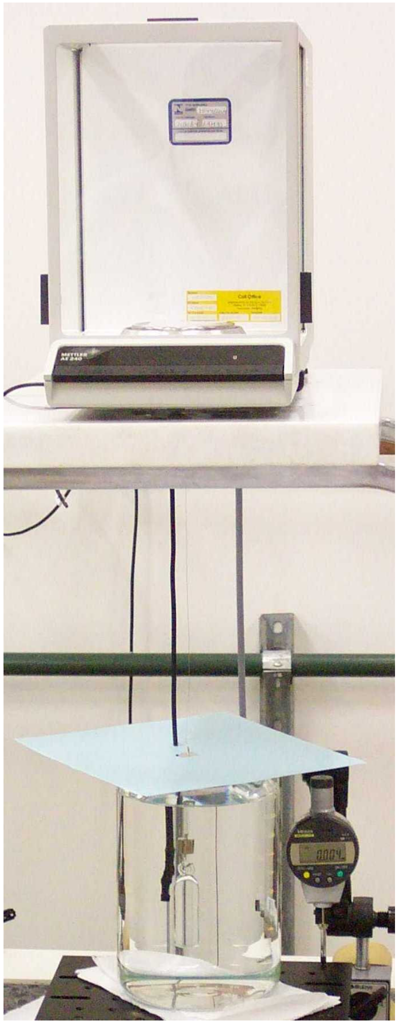

Fig. 3. Second assembling of hydrostatic weighing system

4. RESULTS

The results for the measurements of the volume standard are shown on table 1, while for the measurements of liquid density are shown on table 2 and 3. All values consider coverage factor equal to 2.00, which for a normal distribution corresponds to a coverage probability of approximately 95%.

For the measurements of liquid density, we used a mineral oil for the first assembling and a fluorinert electronic liquid FC-40 for the measurements with the second assembling.

We may calculate the normalized error for the measurements of volume standard, comparing our experiment with the volume comparator system, which is 0,045.

Volume standard at 20ºC	Volume (cm^3)	Expanded uncertainty (cm^3)	Relative uncertainty
First assembling	10,03810	0,00092	0,010%
Second assembling	10,01268	0,00043	0,004%
Volume comparator	10,01270	0,00011	0,001%

Table 1. Measurements of volume standard

From the measurements on table 2 and table 3, we calculate the normalized erros between our experiments and the measurements with the digital density meter and the volume comparator system. We find it to be equal to 0,068 and 0,281 respectively.

Mineral oil density at 20,136ºC	Density (g/cm^3)	Expanded uncertainty (g/cm^3)	Relative uncertainty (%)
First assembling	0,82423	0,00015	0,018%
Digital density meter	0,82425	0,00024	0,029%

Table 2. Measurements of liquid density for first assembling

FC-40 density At 20,645ºC	Density (g/cm^3)	Expanded uncertainty (g/cm^3)	Relative uncertainty (%)
Second assembling	1,88105	0,00026	0,014%
Volume comparator	1,881060	0,000020	0,001%

Table 3. Measurements of liquid density for second assembling

The following tables (4 and 5) show the error contribution due to the various influence quantities and measuring instruments.

Quantity	Relative uncertainty contribution for sphere volume (%) 100% x $u_i(y)/y$	Relative uncertainty contribution or oil density (%) 100% x $u_i(y)/y$
M_A+M_L	21.9	7.7
α	0.3	0.4
T	14.2	3.1
d	10.1	3.2
σ	8.1	1.8
ρ_a	1.3	0.9
ρ_b	1.6	1.2
g	0	0
ρ_{ar}	10.2	8.9
ρ_L	25.4	-
δV	7.0	-
$\delta\rho_L$	-	36.4
V	-	36.5

Table 4. Uncertainty contributions for first assembling

Quantity	Relative uncertainty contribution for volume (%) 100% x $u_i(y)/y$	Relative uncertainty contribution or density (%) 100% x $u_i(y)/y$
M_A	10,2	2,2
M_L	10,2	2,2
T	13,0	0,1
ρ_a	6,1	2,5
ρ_b	4,0	1,6
α	0,6	0,0
$\delta\rho_{ar}$	1,2	12,8
δW_V	7,9	-
δW_ρ	-	9,7
δV	36,7	-
$\delta\rho_L$	-	51,4
$\delta\rho_W$	10,2	-
V	-	17,5

Table 5. Uncertainty contributions for second assembling

With "$\delta\rho_W$" the uncertainty due to water density calculation and "$\delta\rho_{ar}$" the uncertainty due to air density calculation.

5. CONCLUSION

The relative uncertainty of the measurements for the solid volume was of 0.01% and 0.004% for the first and second assemblings respectively. The relative uncertainty of measurement of liquid density was 0.02% for the first assembling and 0.007% for the second assembling. This shows that the implemented hydrostatic weighing system is a simple but accurate and reliable system.

Further improvements could be done and the current technology could be used to calibrate digital density meter with resolution of 0,01 kg/m^3. The main improvement that could be made is to calibrate the volume standard with other more accurate method, such as interferometry.

The first assembling has shown to be more operational, but resulted in larger uncertainty. This mainly due to the calculation of the absolute value of the surface tension force, which may be avoided with the second assembling.

The presented method could also be used in metrology schools for understanding of calibration procedures and error analysis.

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[2] ISO/TR 20461, "Determination of uncertainty for volume measurements made using the gravimetric method."