XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

ACCELERATED MULTISINE IMPEDANCE SPECTRUM MEASUREMENT METHOD DIRECTED AT DIAGNOSIS OF ANTICORROSION COATINGS

Marek Niedostatkiewicz[1], *Romuald Zielonko*[2]

Gdańsk University of Technology, Faculty of Electronics, Telecommunication and Informatics,
Dept. of Metrology and Electronic Systems, Gdańsk, Poland
[1] niedost@eti.pg.gda.pl
[2]zielonko@pg.gda.pl

Abstract: The paper presents a method of impedance spectrum measurement based on multisine signal stimulation of an object and response analysis by triangle window filter-banks. The method is directed at anticorrosion coatings diagnosis. It enables an acceleration of impedance spectrum measurement. The results concerning measurement time and impedance spectrum accuracy are discussed.

Keywords: impedance spectroscopy, multisine measurement signals, anticorrosion coatings diagnosis.

1. INTRODUCTION

Impedance spectroscopy is currently an interdisciplinary tool applied in many scientific, industrial and medical applications, mostly for the parameter identification of objects and phenomena modelled by electric circuits (e.g. anticorrosion coatings, dielectric materials, biomedical objects). The parameters are calculated by fitting the object model to the impedance spectrum, which is usually measured point-by-point in a wide frequency range. In the case of anticorrosion coating diagnosis, the necessary range of impedance spectrum begins from low or very low frequencies (order of mHz or µHz) [1], thus implicating a very long measurement time.

The impedance spectrum of linear objects can be measured in one cycle via multisine stimulus, according to the superposition principle. It allows reducing the spectrum measurement time. However, the conventional multisine approach to DFT method [2] is inconvenient to implement in practice, due to several problems, caused by period of multisine. It is possible that a coherently sampled multisine measurement with proper DFT window length would last longer than point-by-point, single frequency method.

To circumvent these disadvantages the Goertzel filter-bank method [3] has been proposed by the authors for measuring the impedance spectrum of non-stationary object. For acceleration of stationary objects measurement [4] the digital filter bank analysis of multisine signals has been investigated. Early results have confirmed the possibility of measurement time reduction.

In this paper, an improved method based on a filter-bank with triangle window low-pass FIR filters is presented. It is faster than the methods currently used and it can be implemented in practice, particularly for diagnosis of anticorrosion coatings.

The idea of the method (Fig.1) is described in the paper as well as analysis of measurement accuracy and time reduction ratio. The results are discussed in comparison to the single frequency DFT method for analysing a spectrum with both linearly and logarithmically distributed frequencies.

2. PURPOSE

The main purpose of developing impedance spectrum measurement methodology is the reduction of measurement

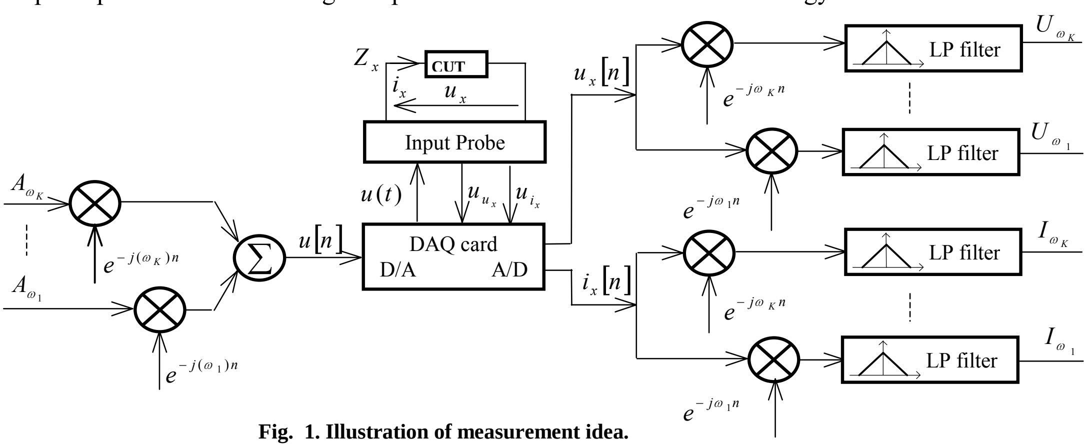

Fig. 1. Illustration of measurement idea.

time, even at the cost of accuracy. That need is motivated by several technical and economical reasons. Firstly, field measurements lasting hours are both unpractical and very expensive. Secondly, there is a need for impedance spectrum measurement of dynamic or quasi-dynamic objects, where conventional, time consuming methods are not reliable due to object's characteristic variations during measurement.

3. REFERENCE METHODS

3.1. Single frequency DFT method

The impedance spectrum measurement time of proposed solution will be compared with single frequency point-by-point spectrum measurement. Although modern single frequency DSP based impedance analysers use many methods [5], the measurement time will be compared with, DFT method. It was proven [6], that one period of sinusoidal stimulus is sufficient for the measurement, so the DFT method is one of the fastest. Time necessary to measure N-point impedance spectrum is not less then sum of periods of frequencies of interest in spectrum:

$$t_{meas.} \geq \sum_{i=1}^{N} T_i = \sum_{i=1}^{N} \frac{1}{f_i} \cdot \tag{1}$$

Thus, measurement of spectrum containing low and very low frequencies (order of mHz) will be very time consuming.

3.2. DFT with multisine stimulus spectrum measurement

The multisine stimulation is the underlying idea of accelerating linear objects impedance spectrum measurement. The linear object's response to multisine signal is a sum of responses, acquired in one measurement cycle. Correctly calculated spectra of stimulus and response contain bins corresponding to frequencies of interest. However, the conventional (and common) approach based on DFT transformation and multisine does not guarantee measurement acceleration, due to period of multisine. In order to obtain coherent sampling and avoid leakage effect, the DFT window length has to be integer multiple of signal period. As the multisine period is the least common multiple (LCM) of periods of all multisine components, in worst case, for arbitrary set of frequencies, the multisine measurement time would be:

$$t_{meas.} \geq \prod_{i=1}^{N} T_i = k \prod_{i=1}^{N} \frac{1}{f_i},\ k \in \mathrm{Z} \cdot \tag{2}$$

An alternative DFT solution with non-coherent sampling and windowing requires averaging several DFT results, thus lengthening the measurement rapidly. By comparing (2) and (1), it can be seen, that the usefulness of multisine - DFT method can be easily undermined.

4. FILTER BANK BASED METHOD

4.1. Filter bank measurement idea

The problems of accelerating impedance spectrum measurements were discussed in [4]. The measurement idea of an improved method proposed in this paper is shown in Fig.1. The measurement system consists of a PC with a DAQ card and an Input Probe.

The *K*-component multisine stimulation signal $u[n]$, characterised by pulsations ω_k, and complex amplitudes $A\omega_k$ is synthesised in summation node in a sample-by-sample manner, thus relieving from necessity of storing very long multisine period.

The DAQ and input probe apply the stimulus voltage to the circuit under test (CUT) and extracts the signals proportional to impedance voltage u_{ux} and current u_{ix}. Both signals are sampled simultaneously producing the digital forms $u_x[n]$ and $i_x[n]$, which are analysed by two identical, digital filter banks. Filter bank channels are built from a modulator and low-pass filter. Modulator shifts the spectrum in frequency domain, according to modulation principle. The spectral component corresponding to modulating frequency is shifted to DC. Its complex amplitude can be extracted by low pass filter present in every filter bank channel [7]. Filter-banks calculate the complex amplitudes $U\omega_k$ and $I\omega_k$ of frequency components, corresponding to frequencies present in stimulus. Complex division of these values allows calculating impedance spectrum of an object for a given set of frequencies.

4.2. Object under test

The method is directed at chemical applications, particularly for diagnosis of anticorrosion coatings modelled by electrical circuits (multielement two terminals). The method was tested on exemplary 3 elements two terminal:

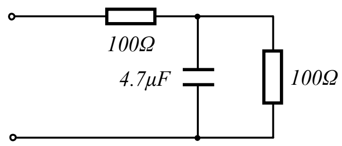

Fig. 2. Circuit Under Test.

Such an object is used in electrochemistry to compare different impedance spectroscopy methods. Its impedance has a form of:

$$Z(s) = \frac{(R_s R_p C_p)s + (R_s + R_p)}{(R_s R_p)s + 1} \cdot \tag{3}$$

The circuit parameters (R_s=R_p=100Ω, C_p=4.7μF) have been chosen to obtain significant changes in magnitude

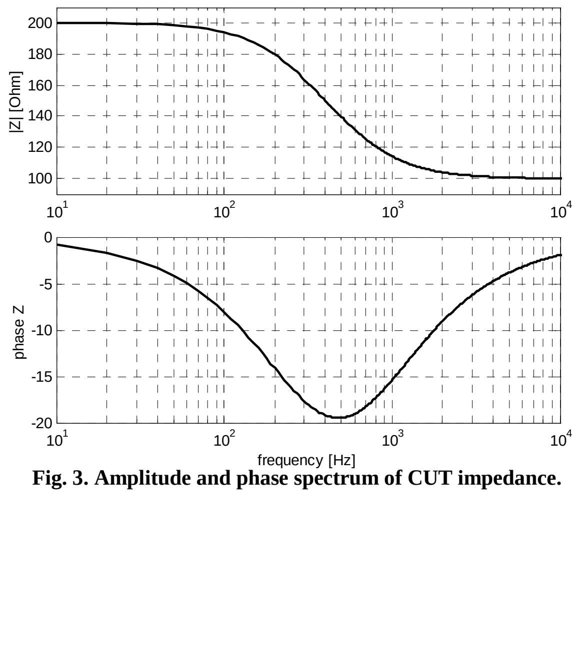

Fig. 3. Amplitude and phase spectrum of CUT impedance.

and phase of circuit's impedance spectrum in a 2-decade frequency range, from 100 Hz to about 13 kHz.

4.3. Multisine stimuli used

The method has been examined with 7 test signals covering the frequency range specified above; 5 signals are build from 14 sines and 2 signals are build from 25 sines. The sampling frequency has been assumed 128kHz. The distribution of frequencies is presented in Fig. 4.

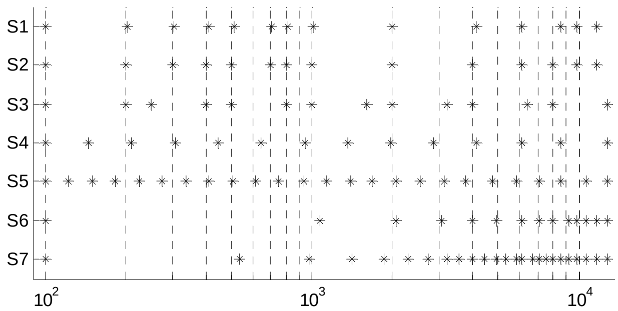

Fig. 4. Distribution of frequencies in multisines.

The signals S1, S2 and S3 are the same as in [4]; S1 is a sum of 2 linearly spaced multisines, each linear part covering 1 decade. None of components is in relation with each other – none of periods is a multiplication of other period. S2 is a LCM-optimised version of S1 – 8 of 14 components are in relation – the longest period T_1 is an integer multiple of 7 other periods.

The frequencies in signal S3 were formed by summing two geometric progressions, so frequency distribution is quasi logarithmical. An interesting (and desirable) feature of signal S3 is that it has a low value of LCM of periods, equal to $2T_1$. The doubled T_1 of lowest frequency component is an integer multiple of periods of all sines in this multisine which allows to use averaging filters efficiently.

Contrary to signals S1-S3, which have been especially designed, the frequencies of further signals were simple calculated as an arithmetic / geometric progression on selected frequency range with 14 or 25 components. S4/S5 are signals with linear frequency distribution and S6/S7 are signals with geometrical frequency distribution.

It can be seen in Fig.4, that signals with linear distribution of frequencies have less components in 1st decade then other signals and the difference between two lowest frequency multisines is bigger. For these signals, it will be easier to achieve filter bank selectivity enough to accelerate spectrum measurement.

Table 1. Sum of periods for stimulation signals.

Signal	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**
N	14	14	14	14	25	14	25
type	Lin+lin	lin+lin LCM	log+log LCM	log	log	lin	Lin
$\sum T_i$	3504	3551	3558	4089	6947	1676	2096

For logarithmic distribution of frequencies of interest in spectrum, the difference between frequencies of LF sines is very low – there will be a need for a selective (which mean long) filter, undermining the idea of measurement acceleration. However, the conventional point-by-point measurement time (1) of logarithmic signal is also much longer than for other signals, due to big amount of long period (low frequency) components and small number of short period (high frequency) sines.

4.4. Triangle window low-pass filter

The average value at output of modulator (Fig. 1) is complex amplitude of multisine component with frequency equal to modulating frequency. A digital, low pass filter calculates that value. Although the task seem to be frequency domain related, time-domain optimised filters with sharp step response and fast impulse response are used – in order to accelerate measurements, the value must be calculated as soon as possible, in a predictable time. To fulfil these requirements, finite impulse response (FIR) filters have been used, due to their performance, stability, and settling time equal to filter length.

In preliminary investigations, rectangle window MA filters has been chosen, described by equation:

$$y[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]. \tag{4}$$

Their frequency characteristic is:

$$H(j\omega) = \sin\left(\frac{\omega M}{2}\right) / \left(\frac{\omega M}{2}\right). \tag{5}$$

MA filters can be implemented recursively, thus circumventing the main FIR disadvantage – amount of calculations required due to long filter kernel:

$$y[n] = \frac{1}{M}x[n] - \frac{1}{M}x[n-M-1] + y[n-1]. \tag{6}$$

However, their spectral selectivity has been insufficient, so the triangle window filters have finally been introduced. The triangle window filter frequency response is given by equation:

$$H(j\omega) = \sin^2\left(\frac{\omega M}{4}\right) / \left(\frac{\omega M}{4}\right). \tag{7}$$

The normalized frequency response of both filters in presented in Fig. 5. Although a triangle window filter has wider main lobe than MA, it has better attenuation of higher frequencies, desirable in case of multisine signal with a wide frequency range.

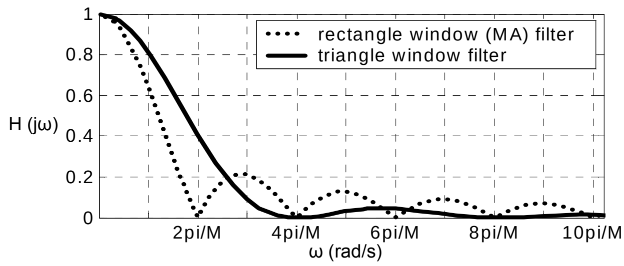

Fig. 5. Rectangle and triangle window filter characteristics.

The triangle window, being the pulse response of M-length filter (7) is a convolution of two rectangular widows of $M/2$-length:

$$h_{triangle}(t) = h_{rectangle}(t) * h_{rectangle}(t). \tag{8}$$

It means, that the M length triangle filter can be implemented very efficiently – instead of time domain convolution of signal with triangle window kernel, same result is obtained by cascade connection of two $M/2$-length rectangular filters. In practice, the modulator outputs are filtered twice with a recursively implemented (6) *M/2*-length MA filter.

Filter characteristics are heavily dependant on filter length. The longer is filter kernel, the better selectivity and roll-off is achieved. However, the filters with *M*-long kernel stabilise after *M* samples, so the measurement time must be at least *M* samples long. In order to achieve measurement acceleration, the value of *M* must be lower than single-frequency measurement time (1) being sum of periods in signal. Moreover, preliminary experiments have shown, that the problem with filter selectivity usually occurs for lowest frequency component (frequency f_1, period T_1) of multisine. As that component is hard to filter-out, it is desirable for the filter length to be a multiplication of T_1.

For every stimulation signal used (S1-S7) the method has been tested for several values of *M* calculated with different approaches. First 3 values of *M* were set to achieve certain measurement acceleration, as compared to single frequency DFT method. Filter length was equal to:

$$M_{1,2,3} = k\sum_{i=1}^{N} T_i,\ \ k_{1,2,3} = \{\, 50\%, 75\%, 100\% \,\}, \quad (10)$$

thus giving the possibility of shortening measurement time twice, 25% or no accelerating at all. Further lengths of *M* were assumed as consecutive (but not greater than sum of periods) multiples of T_1:

$$M_{4,5,\dots} = mT_i \le \sum_{i=1}^{N} T_i,\ \ m_{4,5,\dots} = \{\, 1,2,3\dots \}. \quad (11)$$

4.5. *Simulation*

The simulations have been performed in Matlab environment. The s-domain model of CUT with assumed parameter values has been programmed. The stimulation signals S1-S7 have been prepared and the CUT responses to stimulus have been resolved with transient state simulator. Both stimulus and response were modulated, and consequently, filtered for several filter lengths. In order to compare the approach presented with previous works, both MA and triangle window filter has been used. Complex division of 2 filter banks corresponding outputs calculated the impedance spectrum values for frequencies present in multisine. The calculated magnitude and phase was compared with theoretical values obtained analytically (3).

5. RESULTS AND DISCUSSION

The results of simulation are presented on 3D plots of impedance magnitude and phase for frequencies present in stimulation signal. The filter stabilization period is not shown for clarity. Although the values should be constant after filter stabilization, the fluctuations caused by limited filter selectivity are present in every plot. The time axis is scaled in ms.

The accuracy of method for different signals and filters has been compared using maximum relative error of impedance magnitude and maximum relative error of impedance phase as comparison criteria. These values were calculated for every frequency of a given stimulus during experiment time, according to formulas:

$$\delta_{magnitude} \equiv \max_{i=1\dots N}\left\{\frac{|Z_{theor}(\omega_i)| - |Z(\omega_i,t)|}{|Z_{teor}(\omega_i)|}\right\}\cdot 100\%, \quad (12)$$

$$\delta_{phase} \equiv \max_{i=1\dots N}\left\{\frac{\Phi(Z_{theor}(\omega_i)) - \Phi(Z(\omega_i,t))}{\Phi(Z_{teor}(\omega_i))}\right\}\cdot 100\%. \quad (13)$$

These criteria are estimators of method usefulness, as the 3D plots show that results' fluctuations have unexpected, but periodic character, repeatable during experiment time. The criteria allow estimating the worst-case scenario – one cannot expect the value of magnitude and phase to be calculated with worse accuracy than specified.

5.1. *Spectrum estimation errors for signals S1-S3*

New method based on triangle window filter-bank has been tested with signals S1-S3 and compared to MA filter-bank method described in [4]. Table 2 presents results for MA filter. Table 3 contains results for triangle window filter.

Table 2. Spectrum estimation error [%], signals S1-S3, MA filter

		Measurement time compared to DFT				
rectangle window		36% M=T1	50%	72% M=2T1	75%	100%
S1	magn.	6.53	25.37	5.45	14.61	12.78
	phase	50.92	126.80	36.65	90.39	42.43
S2	magn.	2.02	28.77	0.66	14.41	12.38
	phase	59.32	163.57	24.18	47.68	55.14
S3	magn.	82.68	25.51	0.002	0.12	10.54
	phase	88.09	120.03	3.31	64.89	58.41

It can be seen, that for filter lengths *M* other than multiples of T_1, the MA filter calculates spectrum with insufficient accuracy (errors >10%). It leads to interesting results: for the *M* equal to period T_1, the method is 64% faster than point-by-point method and magnitude error is less then 7%. If the *M* value is chosen to achieve 50% measurement acceleration, the magnitude error is about 25% – despite longer measurement, errors increase.

Moreover, it can be pointed out, that with MA filter, significantly better results are obtained for signal S2 being the LCM optimised version of S1, where T_1 is a multiple of 7 other components' periods. Unfortunately, for all signals MA filters calculate impedance angle with error > 24%, which is unsatisfactory.

Table 3. Spectrum estimation error [%], signals S1-S3, triangle filter

		Measurement time compared to DFT				
triangle window		36% M=T1	50%	72% M=2T1	75%	100%
S1	magn.	38.33	4.98	0.54	0.74	1.21
	phase	262.19	22.38	2.79	4.30	6.44
S2	magn.	41.72	4.25	0.52	0.73	1.08
	phase	443.55	23.82	2.87	2.78	6.21
S3	magn.	357.65	11.14	5.63	4.42	1.46
	phase	396.94	37.78	22.03	17.39	6.39

The distinctive feature of stimulus S3 is that impedance can be calculated precisely with MA filter of length similar to LCM of all periods in multisine ($2T_1$), by averaging integer number of all periods. It allows accelerating measurement by 28%. However, any other length of filter rapidly increases spectrum estimation errors.

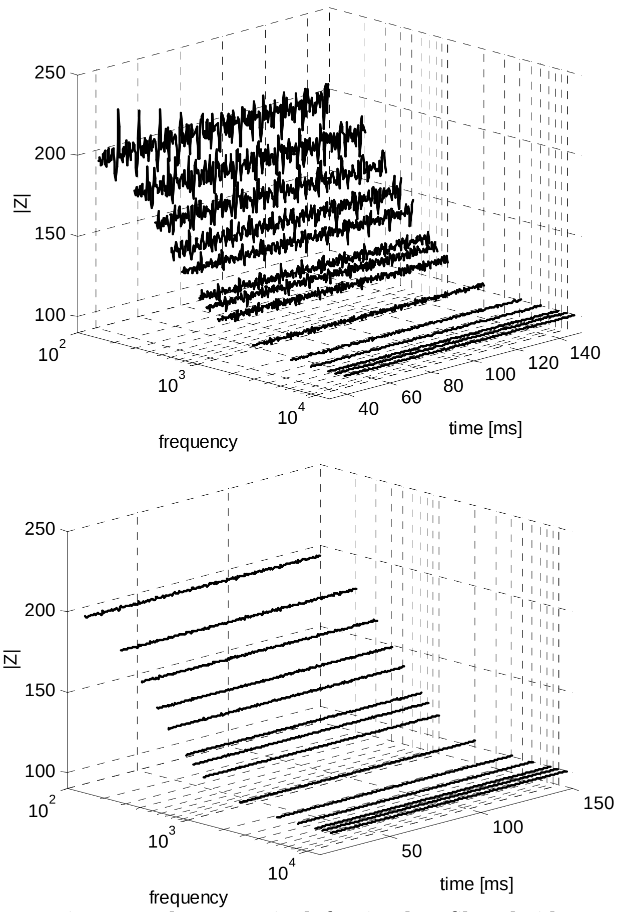

Fig. 6. Impedance magnitude for signal S1, filtered with MA (upper plot) and triangle window filter (lower plot).

The application of triangle window filters improved the selectivity of filter bank method. Both magnitude and phase of impedance are calculated with smaller error than with MA filter. Especially, the impedance angle reconstruction is much more precise, e.g. for signal S2 and $M=2T_1$, phase error decreases almost 10 times, from 24.18% to 2.87%.

Moreover, triangle window filter reduces the stress on stimulus optimisation: contrary to MA method, both signals S1 and S2 produce comparable results – the signal does not need to be LCM-optimised. Another positive feature is that although keeping filter length close to multiple of T_1 is still beneficent, the difference in accuracy for measurement time 72% and 75% is not as dramatic as for MA. It gives the possibility of fluent optimising of filter length against criteria of accuracy or measurement acceleration.

Of course, for the specific signal S3, the triangle window filter gives worse results than MA with $M=LCM\ (T_i)$, as the filtering is less accurate than averaging all periods. However, it must be kept in mind, that this signal has a very specific and hard to obtain frequency distribution.

It also must be stated, that signals S1-S3 were designed to achieve good results with MA filter-bank. Further investigations were conducted for signals with simple logarithmical or linear distribution of frequencies.

5.2. Spectrum estimation errors for signals S6-S7 with linear distribution of frequencies.

The method has been examined with test signals S6-S7. These multisines cover the frequency range linearly with 14 and 25 sines. The frequencies were calculated automatically and form the arithmetic progression. The results are presented in Tab.4 (14 frequencies) and Tab.5 (25 frequencies).

Table 4. Spectrum estimation error [%], signal S6

Signal S6 14 linear	Measurement time			
	50%	75%	76% M=T1	100%
magnitude	6.41	0.76	0.78	1.35
phase	50.30	3.46	3.50	10.70

Sum of periods for signal S6 is rather low (Tab.1), due to linear distribution of frequencies – most of components are in higher decade. Period T_1 is 76% of sum of periods. In order to achieve 50% measurement acceleration, the triangle window filter with M less then period T_1 must be used. However, even using such unselective filter in this case does not increase errors radically, as the lowest frequencies in spectrum are distant from each other.

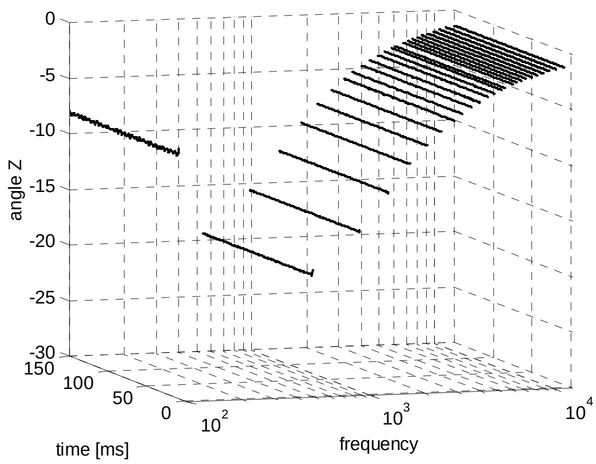

Fig. 7. Impedance angle for signal S4, triangle window filter.

The method accuracy (mostly phase reconstruction accuracy) has lowered, as the number of linearly distributed frequencies in multisine was increased to 25.

Table 5. Spectrum estimation error [%], signal S7

Signal S7 25 linear	Measurement time			
	50%	55% M=T1	75%	100%
magnitude	1.61	1.50	1.87	0.57
phase	6.59	9.87	12.90	3.35

Moreover, an interesting feature can be noticed: the more linearly distributed frequencies are in multisine, the better compression of measurement time can be achieved (as the selectivity of filter $M=T_1$ is sufficient for multisine method, while the sum of periods becomes longer).

5.3 Spectrum estimation errors for signals S4-S5 with logarithmic distribution of frequencies.

Consequently, the method was tested with signals S4-S5, which frequencies form the geometric progression, thus covering logarithmically the specified frequency range with 14 and 25 sines.

It can be seen that for logarithmical distribution of frequencies, the filter selectivity becomes a very important issue, as low frequency components are close to each other. Although acceleration of measurement is possible, the accuracy is lower than for other signals examined.

Table 6. Spectrum estimation error [%], signal S4

Signal S4 14 logarithmic	Measurement time				
	31% M=T1	50%	63% M=2T1	75%	100%
magnitude	410.00	42.73	7.45	5.03	1.587
phase	1247	130.36	52.63	29.61	9.403

In case of 25 multisine components (signal S5), the amplitude calculation error was 4.47% and phase error was 29.96%, for measurement time equal to 75% of conventional point-by-point method.

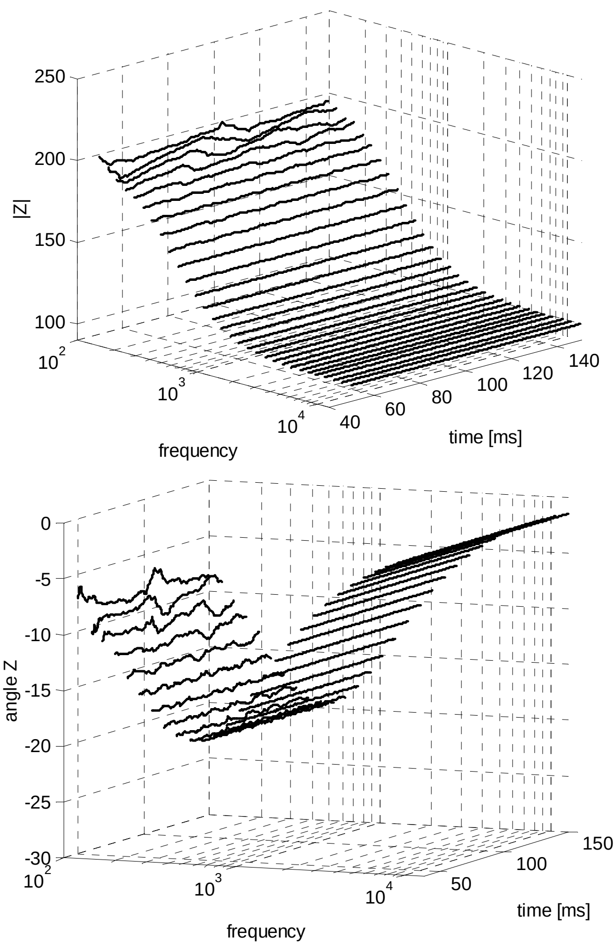

Fig. 8. Impedance magnitude (upper plot) and impedance angle (lower plot) for multisine with logarithmic distribution of 25 frequencies, 25% measurement acceleration.

6. CONCLUSIONS

Numerical simulations have confirmed the possibility of accelerating measurement of anticorrosion coatings impedance via multisine stimulation and filter-bank analysis. Application of triangle window filters has improved the frequency selectivity of filter-bank channels in comparison to previously investigated MA filters. New approach allows to use arbitrary linear or logarithmical set of frequencies in the multisine signal, making implementation of the method easy – there is no need to optimise distribution of frequencies.

Generally, MA filter should not be used as the spectrum estimation errors are far beyond acceptable level (> 10% magnitude and >50% phase). The only case where MA filters are more suitable than triangle window filters is the signal like S3, for which LCM of periods is significantly less than sum of periods. However, that distribution of frequencies is rare and difficult to obtain.

The results have shown, that the method accuracy and the reduction of measurement time is dependent on the distribution and number of frequencies in the spectrum. The measurement acceleration ratio in case of stimulus with many linearly distributed frequencies can reach up to 50%, while measurement of other signals could be accelerated about 20%, in comparison to single frequency DFT method.

Although the method and system are oriented at anticorrosion coating diagnosis, they can be useful for the acceleration of parameter identification of other objects and phenomena modelled by electric circuits.

REFERENCES

[1] P. L. Bonora, F. Deflorian , L. Federizzi : “Electrochemical impedance spectroscopy as a tool for investigation under paint corrosion”, III Int. Symp. on Electrochemical Impedance Spectroscopy, Nieuwport, Belgium, 1995.

[2] D. Smith, “The acquisition of Electrochemical Response Spectra by On-Line Fast Fourier Transform”, Analytical Chemistry, vol. 48, February 1976

[3] G. Lentka, M. Niedostatkiewicz, – “The Goertzel filter-bank usage in the non-stationary impedance measurement”, Proc.13th Int. Symp. IMEKO TC-4, Athens 2004.

[4] M. Niedostatkiewicz , R. Zielonko : – “Accelerated impedance spectrum measurement via multisine perturbation and digital filter banks”, Proc.14th Int. Symp. IMEKO TC-4, Gdynia 2005.

[5] M. Sedlacek, “Digital measurement of phase difference of LF signals – a comparison of DSP algorithms”, Proc. Of XVII IMEKO World Congress, Dubrovnik 2003.

[6] J. Hoja, G. Lentka – ”Low Cost Impedance analyzer for diagnostics of non electrical objects”, Proc.11th Int. Symp. IMEKO TC-4, Lisbon 2001.

[7] J. McClellan, R. Schafer, M. Yoder, “DSP First a Multimedia Approach”, Prentice-Hall 1996