

DETERMINATION OF OPTIMUM INPUT SIGNALS FOR CALIBRATION OF MEASUREMENT APPARATUS

Chingiz Hajiyeu

Istanbul Technical University, Faculty of Aeronautics and Astronautics,
 Maslak, 34469 , Istanbul, TURKEY, e-mail: cingiz@itu.edu.tr

Abstract. A procedure for optimal selection of sample input signals to get the best calibration characteristics of measuring apparatus is proposed. As an example the problem of optimal selection of standard pressure setters when calibrating differential pressure measurer is solved.

Keywords: measuring apparatus, calibration characteristics, input signals, least squares method.

1. INTRODUCTION

Accurate measurement is the basis of almost all engineering application, since uncertainty inherently exists in the nature of any measuring apparatus. The cost of a measuring apparatus, on the other hand, increases with its accuracy. Therefore low cost accurate measurement devices are one of the main goal of engineers. One way of reducing the cost of accuracy is the calibration process. Therefore this paper deals with the calibration of a low cost sensor with a high accurate one.

The method can be explained as follows. A sample signal whose characteristics are known priori is applied to both the low cost sensor and the high accurate sensor and outputs of both sensors are recorded. This experiment is repeated for a variety of input signals and the results are tabulated [1,2]. The calibration characteristics can be evaluated from this table. Interpolation techniques should be used when this table does not have the required data. From the practical point of view, this characteristics should be in a polynomial form. The accuracy of this polynomial depends on the noise-free data which was used to obtain the characteristics [3]. To reduce the effect of noise, excessive number of data should be used. However this requires more experiments and that will increase the cost. Thus the main question becomes the evaluation of accurate calibration characteristics with a few number of experimental data. In the existing works [1-7] for calibration of measurement apparatus the equidistant sample signals are used. On the other hand, though it is paradoxical, the application of equidistant sample signal to get the best calibration characteristic is erroneous. Therefore this work deals with the problem of optimal selection of sample signal composition with a view to get the best calibration characteristics (planning experiment problem).

2. PROBLEM STATEMENT

From practical considerations the calibration curve should be in a polynomial form as follows

$$y_i = a_0 + a_1 p_i + a_2 p_i^2 + \dots + a_m p_i^m \quad (1)$$

where y_i is the output of the low cost transducer and p_i are the outputs of the high precision transducer, a_0, a_1, \dots, a_m are the calibration curve coefficients. Measurement contains random noises in Gaussian form

$$z_i = y_i + \delta_i = a_0 + a_1 p_i + a_2 p_i^2 + \dots + a_m p_i^m + \delta_i \quad (2)$$

where z_i is the measurement result, δ_i is measurement error with zero mean and σ^2 variance.

Let the calibration curve coefficients be denoted as $\tilde{\theta} = [a_0, a_1, \dots, a_m]^T$. The coefficients in these polynomials were evaluated in [8] by the least squares method. The expressions used to make the evaluation had the form:

$$\tilde{\theta} = (\tilde{X}^T \tilde{X})^{-1} (\tilde{X}^T z) \quad (3)$$

$$\tilde{D}(\hat{\theta}) = (\tilde{X}^T \tilde{X})^{-1} \sigma^2 \quad (4)$$

where $\tilde{z}^T = [z_1, z_2, \dots, z_n]$ is the vector of the measurements;

$$\tilde{X} = \begin{bmatrix} 1 & p_1 & p_1^2 & \dots & p_1^m \\ 1 & p_2 & p_2^2 & \dots & p_2^m \\ \dots & \dots & \dots & \dots & \dots \\ 1 & p_n & p_n^2 & \dots & p_n^m \end{bmatrix} \quad (5)$$

is the matrix of the known coordinates (here, p_1, p_2, \dots, p_n are values that are producible by the standard instruments), $\tilde{D}(\hat{\theta})$ dispersion matrix of the estimated coefficients.

The values of p_1, p_2, \dots, p_n the outputs of the high accuracy transducer, should be such that the polynomial

characteristics given by (1) best approximates the real calibration characteristics. Thus the problem can be stated as follows: Find the values of p_1, p_2, \dots, p_n such that the values of a_1, a_2, \dots, a_m are optimum in a sense that a given performance criterion is minimum.

3. THE SOLUTION ALGORITHM

As mentioned above the matrix \tilde{D} can be used as a measure of the error between the low cost transducer and the high precision transducer. A performance criteria for the minimum of the matrix \tilde{D} can be selected in several ways. A scalar measure of the matrix \tilde{D}

- the *A optimality* -the trace (sum of diagonal elements) criterion,
- the *D optimality* -the generalize dispersion (determinant) criterion,
- the *E optimality*-maximal eigenvalue of matrix criterion, sum of all the elements of the matrix and etc.

It should be noted that the choice of the scalar measure of covariance matrix estimation errors is founded by human experience and intuition. The mathematical simplicity and convenience of obtaining analytical results are of importance here. For this reason A optimality criterion is used in this work, i.e

$$\min_{p_i} [\text{Tr}(\tilde{X}^T \tilde{X})^{-1} \sigma^2] \quad (6)$$

is sought. The values of p_1, p_2, \dots, p_n found by solving the above equations should be in the range $0 - p_{\max}$. Otherwise the solution is invalid.

Let the objective function is denoted as:

$$f(p_1, p_2, \dots, p_n) = \text{Tr}\{\tilde{D}(p_1, p_2, \dots, p_n)\}. \quad (7)$$

As explained above the problem is a constrained optimization problem. The objective function is a multivariable, nonlinear, continuous and has derivative in the considered interval.

Assume that the minimum of $f(p_1, p_2, \dots, p_n)$ exists for the following values of p_1, p_2, \dots, p_n

$$p^* = [p_1^*, p_2^*, \dots, p_n^*]^T.$$

In order that p^* is a minimum of (7), the following conditions should be satisfied

$$\begin{aligned} \nabla f(p^*) &= 0 & (8) \\ \nabla^2 f(p^*) &\text{ is semi positive} & (9) \end{aligned}$$

where ∇ denotes the gradient.

The extremum condition given by (8) can explicitly be written as:

$$\partial [\text{Tr}\{\tilde{D}(p_1, p_2, \dots, p_n)\}] / \partial p_i = 0, \quad (i = 1, 2, \dots, n). \quad (10)$$

When the derivatives in (10) is calculated n algebraic equations with n unknowns are obtained

$$Q_i(p_1, p_2, \dots, p_n) = 0, \quad (i = 1, 2, \dots, n), \quad (11)$$

where n denotes the number of measurements. Derivation of equation (11) for the case of $m = 2$ is given in detail in the Appendix A.

Numerical routines such as gradient descent algorithms [10] can be used. It should be noted that the sign of $\nabla^2 f(p)$ should be calculated together with $\nabla f(p)$ in order to determine whether the values found corresponds a local minimum or a local maximum. Furthermore those values which makes $\text{Tr}\{\tilde{D}(p_1, p_2, \dots, p_n)\}$ minimum should be in the range $0 - p_{\max}$. The solution set which satisfy the above conditions can be used to calculate the polynomial coefficients given in (1). This polynomial best approximates the calibration characteristics between the low cost and high precision equipment and can be used to extract the accurate values from the outputs of the low cost transducer.

4. COMPUTATIONAL RESULTS

As an example the problem of optimal selection of standard pressure setters when calibrating differential pressure measurer is solved. The results of determination of optimum input signals for the cases of $m = 2$ and $n=3, n=4$ are given below. In the calculations the following data and initial conditions are taken.

- Calculation of optimum input signals is performed for the differential pressure gage "Sapphir-22DD". The range of the transducer is $0 \leq p_i \leq 1600$ bar. The differential pressure gage errors are subjected to normal distribution with zero mean and the standard error deviation $\sigma_i = 2,6$ bar [8].

- Calibration characteristics of the measuring device (in the present case, differential pressure gage) is described by 2 order polynomial as follows:

$$y_i = a_0 + a_1 p_i + a_2 p_i^2. \quad (12)$$

- Measurement equation is written in the form

$$z_i = a_0 + a_1 p_i + a_2 p_i^2 + \delta_i, \quad i = \overline{1, n}, \quad (13)$$

where δ_i is the measurement error with zero mean and σ^2 variance.

The optimum input signals for calibration of above mentioned differential pressure gage are evaluated via equation (A4), which is presented in the appendix. The method is used to obtain the optimum coefficients of the characteristic polynomial for a selected criteria. Calculation is performed for $n=3$ and $n=4$. Closed form algebraic equations (A4) is calculated to solve the equation given in (5). The software program MATHEMATICA is used to find the optimum values of p_i^* ($i \neq 1, i \neq n$). The optimum input values and corresponding $\text{Tr}\{\tilde{D}(\hat{\theta})\}$ values for the cases of $n=3$ and $n=4$ are tabulated in Table.1 and Table.2 respectively.

Table 1. Optimum and equal calibration intervals and corresponding $Tr[\tilde{D}(\hat{\theta})]$ values for $n=3$.

Calibration intervals	Pressure values, bar			$Tr[\tilde{D}(\hat{\theta})]$ bar ²
	p_1	p_2	p_3	
Optimum intervals	0	857.1	1600	7.290072
Equal intervals	0	800	1600	7.290074
Difference,bar				0.000002

Table 2. Optimum and equal calibration intervals and corresponding $Tr[\tilde{D}(\hat{\theta})]$ values for $n=4$.

Calibration intervals	Pressure values, bar				$Tr[\tilde{D}(\hat{\theta})]$ bar ²
	p_1	p_2	p_3	p_4	
Optimum intervals	0	200	845	1600	5.319307
Equal intervals	0	533	1066	1600	6.924947
Difference,bar					1.605640

For a comparison the optimality criterion values where pressure values $p_i, i=1, \dots, n$ are equally spaced are also shown in the tables. As can be seen clearly from the presented tables the optimality criterion values where the coefficients are calculated by proposed method are smaller than that where $p_i, i=1, \dots, n$ are equally spaced. As a result, the suggested method can be used to obtain the best (with respect to selection criteria) calibration characteristics polynomial.

5. EXPERIMENTAL CHECKING OF OBTAINED RESULTS

The obtained values of optimum input signals for calibration of differential pressure gage for the case of $n=4$ are checked via actual experiments. In the experiments the following data and initial conditions are taken.

Measurements are taken via differential pressure gage "Sapphir-22DD" with the measurement range $0 \leq p_i \leq 1600$ bar. The output signal of mentioned gage is electrical signal in the unit of mV.

Calibration of differential pressure gage is made by the help of the standard measuring instrument. The standard instrument reproduced pressure signals corresponding optimum and equal calibration intervals. The holding calibration experiment results are presented in the Table 3 and Table 4 respectively.

Table 3. Calibration experiment results corresponding optimum calibration intervals.

Experiment No	p_i, bar	y_i, mV
1	0	0.0051
2	200	1192.4
3	845	7867.2
4	1600	21172.9

Table 4. Calibration experiment results corresponding equal calibration intervals.

Experiment No	p_i, bar	y_i, mV
1	0	0.0051
2	533	4097.9
3	1066	11150
4	1600	21172.9

The coefficients \hat{a}_0, \hat{a}_1 and \hat{a}_2 found by estimation algorithm (3), are given in Table 5, and their errors variances in Table 6.

Table 5. Calibration coefficients estimates.

Using calibration method	\hat{a}_0	\hat{a}_1	\hat{a}_2
Optimum calibration intervals are used	0.2271	4.9199	0.0052
Equal calibration intervals are used	-0.2431	4.9212	0.0052

Table 6. Variances of the errors of coefficient estimates.

Using calibr. method	$D_{\hat{a}_0}, bar^2$	$D_{\hat{a}_1}, bar^2$	$D_{\hat{a}_2}, bar^2$	$Tr[\tilde{D}(\hat{\theta})]$ bar ²
Optimum calibr. int. are used	5.3192	0.00007	0.27×10^{-10}	5.3193
Equal calibr. int. are used	6.9248	0.00006	0.23×10^{-10}	6.9249

As it is seen from the results given in Table 6, the values $Tr[\tilde{D}(\hat{\theta})]$ in the case of optimum calibration intervals is considerably less than in the case of equal calibration intervals. After determination of coefficients \hat{a}_0, \hat{a}_1 and \hat{a}_2 , the polynomial

$$y_i = \hat{a}_0 + \hat{a}_1 p_i + \hat{a}_2 p_i^2 \quad (14)$$

can be used as the calibration curve of differential pressure gage. The obtained calibration curves corresponding to optimum and equal calibration intervals are shown in Figure 1 and Figure 2 respectively.

In the real conditions, after obtaining of each measurements z_i , in the microprocessor of differential pressure gage the inverse problem is solved, i.e., the roots of equation (14) are found

$$\hat{p}_{(1,2)i} = \frac{-\hat{a}_1 \pm \sqrt{\hat{a}_1^2 - 4\hat{a}_2(\hat{a}_0 - z_i)}}{2\hat{a}_2} \quad (15)$$

and the root \hat{p}_{1i} is assumed as the estimation of measured pressure. The root

$$\hat{p}_{2i} = \frac{-\hat{a}_1 - \sqrt{\hat{a}_1^2 - 4\hat{a}_2(\hat{a}_0 - z_i)}}{2\hat{a}_2}$$

is negative or considerably different from value z_i .

Verifying experiment based on mentioned calibration characteristics for the above example is realized. In the experiments the standard instrument reproduced pressure signals $p_i, i = \overline{1,17}$ in the measurement interval $0 \leq p_i \leq 1600 \text{ bar}$ with the step 100 bar and the output signals of differential pressure gage z_i are registered. The holding experiment results are presented in the Table 7 of Appendix B. Using presented in the Table 7 verifying calibration experiment results, $z_i, i = \overline{1,17}$ and the equations of appropriate calibration curves the values $\hat{p}_i = \hat{p}_{1i}, i = \overline{1,17}$ are evaluated via formula (15). Then the appropriate absolute error values Δ_{abs_i} and relative error values Δ_{rel_i} are determined by means of the known expressions

$$\Delta_{abs_i} = |\hat{p}_i - p_i|; \Delta_{rel_i} = \frac{\Delta_{abs_i}}{p_i} \cdot 100\% . \quad (16)$$

The obtained calibration values \hat{p}_i and absolute and relative calibration errors corresponding to optimum and equal calibration intervals are presented in the Table 8 of Appendix B and the graphs of the relations $\Delta_{abs_i} = f(p_i)$ and $\Delta_{rel_i} = f(p_i)$ in the Figure 3 and Figure 4. As it is seen from presented results, the calibration errors corresponding to optimum calibration intervals are considerably smaller than in the case of equal calibration intervals. The experiment results confirm the correctness of the obtained theoretical results.

6. CONCLUSION

The paper shows that the accuracy of the calibration characteristics of measuring apparatus substantially depends on the values of reproduced sample input signals. A procedure for optimal selection of sample input signals to get the best calibration characteristics of measuring apparatus was proposed. For simplicity A-optimality criterion has been used for this work but also, any other criterion (D-optimality, E-optimality, etc.) can be used.

As an example the problem of optimal selection of standard pressure setters when calibrating differential pressure measurer is solved. The holding experiment results confirm the correctness of the obtained theoretical results. Further work includes calculation of the calibration characteristics where different optimality criteria's are used.

ACKNOWLEDGMENTS

This work was partially supported by the DPT-ITU Research Project No. 90148.

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APPENDIX A

Derivation of equation (10) for $m = 2$

Open the dispersion matrix of estimations errors (3). After multiplication and inverting matrix, we have

$$\tilde{D} = (\sigma^2 / \det) \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}, \quad (A1)$$

where $a_{11}, a_{12}, \dots, a_{33}$ are the algebraic minors of $\tilde{X}^T \tilde{X}$ matrix;

$$\det = n \sum_{i=1}^{n-1} \sum_{j=i+1}^n p_i^2 p_j^2 (p_i - p_j)^2 - \sum_{i=1}^n p_i \sum_{\substack{i,j=1 \\ i \neq j}}^n p_i^3 p_j (p_i - p_j) + \sum_{i=1}^n p_i^2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n p_i p_j (p_i - p_j)^2 \quad (A2)$$

is determinant of $\tilde{X}^T \tilde{X}$ matrix.

After proper mathematical transformations, we have the trace of \tilde{D} dispersion matrix in the following form

$$Tr \tilde{D} = \frac{\sigma^2}{\det} \left[\sum_{i=1}^{n-1} \sum_{j=1}^n p_i^2 p_j^2 (p_i - p_j)^2 + n \sum_{i=1}^n p_i^4 - \left(\sum_{i=1}^n p_i^2 \right)^2 + n \sum_{i=1}^n p_i^2 - \left(\sum_{i=1}^n p_i^2 \right)^2 \right] \quad (A3)$$

Having taken the corresponding derivatives one has a system of n algebraic equations with n variables:

$$\begin{aligned} & \left[-2 \sum_{j=1}^n p_i p_j^3 (p_i - p_j) + 4n p_i^3 - 4p_i \sum_{i=1}^n p_i^2 + 2n p_i - 2 \sum_{i=1}^n p_i \right] \\ & \times \left[n \sum_{i=1}^{n-1} p \sum_{j=i+1}^n p_i^2 p_j^2 (p_i - p_j)^2 - \sum_{i=1}^n p_i \sum_{\substack{i,j=1 \\ i \neq j}}^n p_i^3 p_j (p_i - p_j) \right. \\ & \left. + \sum_{i=1}^n p_i^2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n p_i p_j (p_i - p_j)^2 \right] - \left[2n \sum_{j=i+1}^n p_i p_j^2 (p_i - p_j) \right. \\ & \left. \times (2p_i - p_j) - \sum_{\substack{i,j=1 \\ i \neq j}}^n p_i^3 p_j (p_i - p_j) - \sum_{i=1}^n p_i \sum_{\substack{j=1 \\ i \neq j}}^n p_i^2 p_j \right. \\ & \left. \times (4p_i - 3p_j) + 2p_i \sum_{i=1}^{n-1} \sum_{j=i+1}^n p_i p_j (p_i - p_j)^2 \right. \\ & \left. + \sum_{i=1}^n p_i^2 \sum_{j=i+1}^n p_j (p_i - p_j) (3p_i - p_j) \right] \\ & \times \left[\sum_{i=1}^{n-1} \sum_{j=1}^n p_i^2 p_j^2 (p_i - p_j)^2 + n \sum_{i=1}^n p_i^4 - \left(\sum_{i=1}^n p_i^2 \right)^2 + n \sum_{i=1}^n p_i^2 - \left(\sum_{i=1}^n p_i^2 \right)^2 \right] = 0, (i = \overline{1, n}) \quad (A4) \end{aligned}$$

With a view to find all the possible solutions of the system one can apply numeric methods of searching that are simply realized by a computer.

APPENDIX B

Table 7. Verifying experiment results.

experiment No	p_i (bar)	z_i (mV)
1	0	0.0051
2	100	545.0231
3	200	1192.3651
4	300	1942.8624
5	400	2798.7251
6	500	3758.5748
7	600	4821.0851
8	700	5990.1285
9	800	7261.6704
10	900	8635.0752
11	1000	10113.8051
12	1100	11697.1899
13	1200	13384.1651
14	1300	15181.0295
15	1400	17076.9348
16	1500	19073.5267
17	1600	21172.8851

Table 8. Absolute and relative calibration errors corresponding to optimum and equal calibration intervals.

Input Pres., bar	\hat{p}_i , bar Opt.int	\hat{p}_i , bar Equal.int	Δ_{abs} , bar Opt. Int.	Δ_{abs} , bar Eq. int.
0	-0.045	0.050	0.045	0.050
100	100.135	100.192	0.135	0.192
200	200.023	200.053	0.023	0.053
300	299.834	299.8440	0.166	0.15
400	399.839	399.834	0.161	0.167
500	499.842	499.824	0.158	0.176
600	599.724	599.696	0.276	0.304
700	699.847	699.811	0.153	0.189
800	799.813	799.770	0.187	0.229
900	899.687	899.638	0.313	0.362
1000	999.587	999.533	0.413	0.467
1100	1099.576	1099.518	0.424	0.482
1200	1199.544	1199.481	0.456	0.519
1300	1299.834	1299.767	0.166	0.232
1400	1399.838	1399.769	0.162	0.231
1500	1499.681	1499.609	0.319	0.391
1600	1599.481	1599.407	0.519	0.593

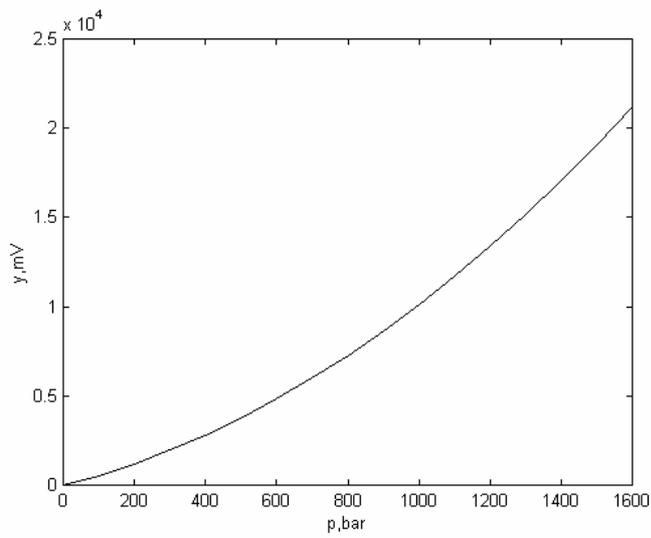


Fig. 1. Calibration curve (optimum calibration intervals are used)

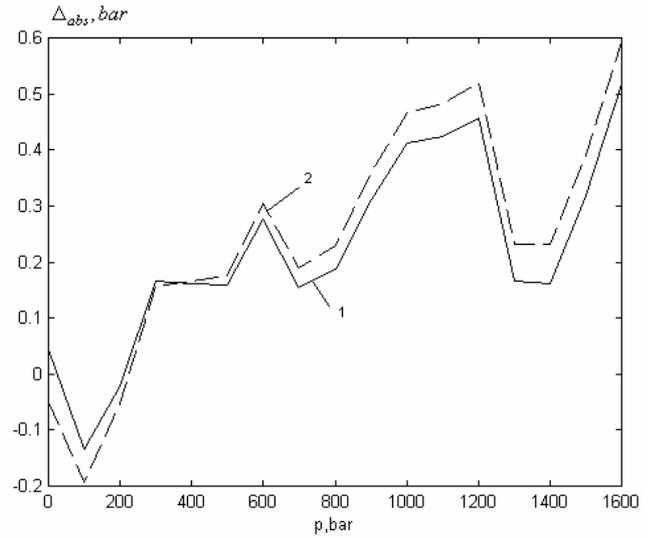


Fig. 3. Absolute calibration errors: 1- calibration was performed by using optimum calibration intervals; 2- calibration was performed by using equal calibration intervals.

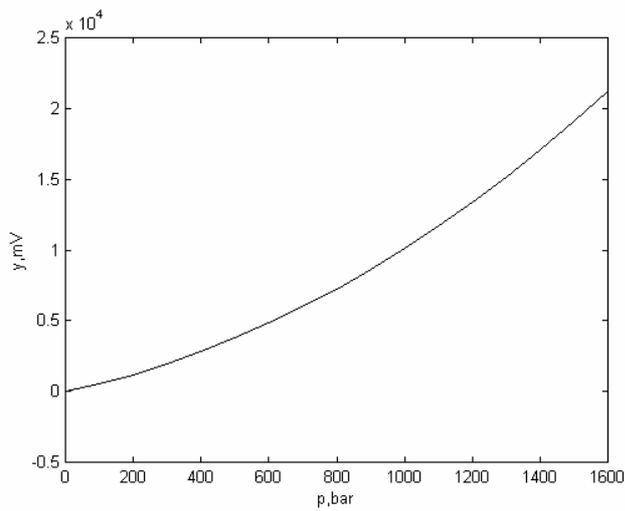


Fig. 2. Calibration curve (equal calibration intervals are used)

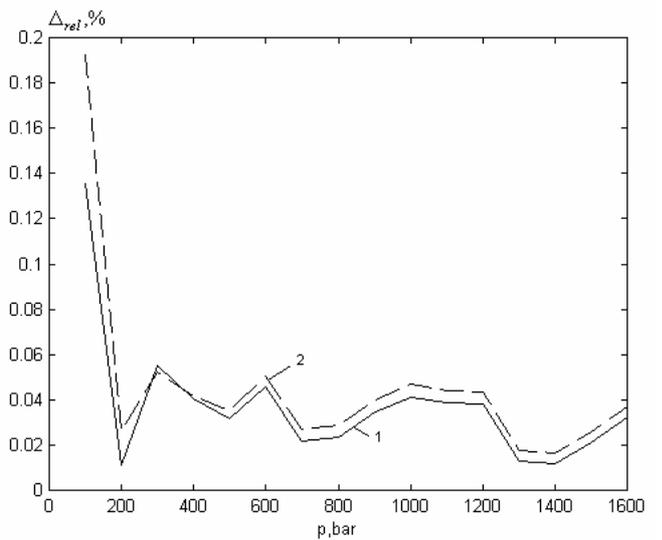


Fig. 4. Relative calibration errors: 1- calibration was performed by using optimum calibration intervals; 2- calibration was performed by using equal calibration intervals.