XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17-22, 2006, Rio de Janeiro, Brazil

DYNAMIC METHOD OF THE THERMAL CONDUCTIVITY MEASUREMENT

Mladen Bezjak
Civil engineering Institute of Croatia, Zagreb, Croatia, mladen.bezjak@igh.hr

Abstract: This work deals with the dynamic thermal conductivity (λ) measurements by hot wire method. This measurements are performed at unstationary state conditions (short time). Very thin (0,05 mm) manganin wire is supplied by DC electrical power and produces a thermal impulses which pass through the testing sample. Dynamics of this process depends on the type of the sample material. Temperature, electric power and time elapsed during the process are measured parameters from which thermal conductivity could be calculated. Measurement results are compared with those gained by the standard guarded hot plate (stationary) method. Testing samples are made from isotropic and homogenous materials.

Keywords: hot wire, thermal conductivity, temperature

1. INTRODUCTION

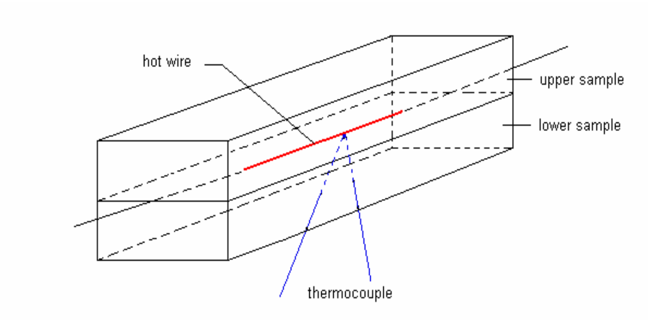


Fig. 1. Measuring system

Measurements are performed on the two equal [(1)] extruded polystyrene samples with small thermal conductivity and on the two equal thermal mortar samples with significantly higher thermal conductivity. Each of them are already been tested by the guarded hot plate method.

The wire is made of manganin because electrical resistance of the manganin changes only 0,05 % from the temperature changes up to 100 °C. According to this, it could be presumpted that the electrical power which generates heat flux through the samples shall be stable and that the heat flux is influenced only by the thermal properties of the samples material.

The wire is positioned in the middle between the two equal samples and the diameter of the wire is 0,05 mm, so it could be presumpted that thermal energy is transfered only by conduction.

[(1)] Equal means that the samples have same dimensions and are made of the same material (equal thermal properties)

Chrom-Nickel (type K) thermocouple is positioned near wire (less then 1 mm) and in respect to the thickness of the samples it could be said that distance between thermocouple (measuring point) and wire is neglecteable. Thermocouple joint diameter is 0,5 mm.

Before testing, measurment samples should be dried into ventilated oven up to constant mass (until two weights do not differ more than 0,5 % during the period of 24 hours).

This is twodimensional unstationary temperature field that could be represented by Fourieres partial differential equtation of the second kind [1]:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c \cdot \rho} \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \qquad (1)$$

where:

$a = \frac{\lambda}{c \cdot \rho}$	diffusivity coefficient	$(m^2 \cdot s^{-1})$
T	temperature	(K)
t	time	(s)
c	specific thermal capacity	$(J \cdot kg^{-1} \cdot K^{-1})$
ρ	density	$(kg \cdot m^{-3})$
λ	thermal conductivity coefficient	$(W \cdot m^{-1} \cdot K^{-1})$

Mathematical solution is [1]:

$$\Delta T(x, y=0, t) = \frac{\dot{q}_L}{4 \cdot \pi \cdot \lambda} \int_0^t \frac{1}{t} \cdot e^{-\frac{x^2}{4 \cdot a \cdot t}} dt \qquad (2)$$

where:

$\dot{q}_L$ is thermal flux density per lenght of the wire $(W \cdot m^{-1})$

$$\dot{q}_L = \frac{P}{L} = \frac{U \cdot I}{L}$$

P is electrical power on the wire (W), U is DC voltage (V), and I is DC current (A).

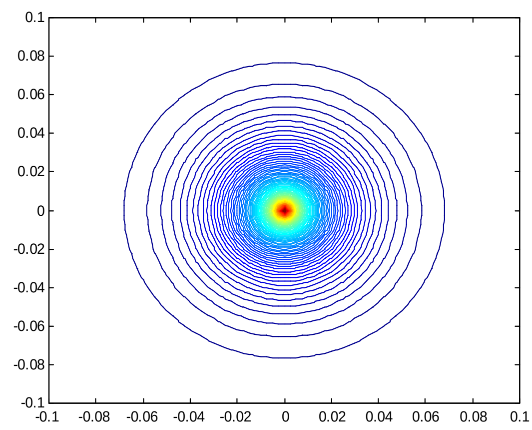


Fig. 2. Themperature field through the sample (MATLAB)

Since the thermocouple is positioned very close to the wire, it follows that the distance from wire to measurement point (point under observation) is neglecteable ($x \rightarrow 0$), and solution is:

$$T(t_2) - T(t_1) = \frac{P}{L \cdot 4 \cdot \pi \cdot \lambda} \ln \frac{t_2}{t_1} \qquad (3)$$

where:
$T(t_1)$ and $T(t_2)$ are temperatures measured at the times t_1 and t_2, respectively

2. MEASUREMENT EQUIPMENT

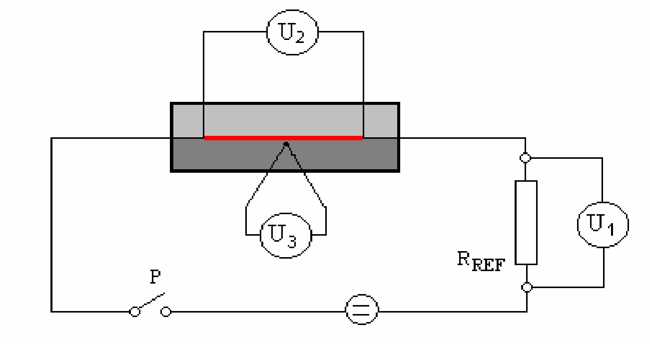


Fig. 3. Measurement equipment

Measurement equipment consists of:
- manganine wire, length L = 0,165 m
- voltmeter with several chanels scanner that measures voltages on the referent resistance (U_1), hot wire (U_2) and thermocouple (U_3),
- referent resistance (R_{REF} = 1 Ω),
- thermocouple with conversion polynom (given by the manufacturer):

[T = $-1{,}265\cdot10^{-18}\cdot U_3^5 + 1{,}358\cdot10^{-14}\cdot U_3^4 - 2{,}402\cdot10^{-11}\cdot U_3^3 - 6{,}250\cdot10^{-7}\cdot U_3^2 + 2{,}585\cdot10^{-2}\cdot U_3 - 9{,}383\cdot10^{-3}$, where U_3 is expressed in μV.]
- battery (4,5 V) and switch,
- digital contact thermometer. This termometer mesures temperature changes of the table where the all contact joints are mounted, and from which scanner and voltmeter colect the measured data. During the measurements, this temperature was stable within 0,5 °C.
- digital thermohygrometer. This termometer mesures air temperature and relative humidity during the measurements.

The battery gives a DC voltage (U_1) measured on the referent resistance (R_{REF}), so that according to Ohm law, DC current could be calculated:

$$I = \frac{U_1}{R_{REF}} \qquad (A)$$

Since, electrical power on the wire length is:

$$P = \frac{U_1}{R_{REF}} \cdot U_2 \qquad (W)$$

The equtation (3) becomes:

$$T(t_2) - T(t_1) = \frac{U_1 \cdot U_2}{R_{REF} \cdot L \cdot 4 \cdot \pi \cdot \lambda} \ln \frac{t_2}{t_1}$$

where $T(t_1)$ and $T(t_2)$ are thermocouple temperatures calculated from conversion polynom from measured voltages (U_3) at the times t_1 and t_2.
Finaly, thermal conductivity could be calculated as:

$$\lambda = \frac{U_1 \cdot U_2}{R_{REF} \cdot L \cdot 4 \cdot \pi \cdot (T(t_2) - T(t_1))} \ln \frac{t_2}{t_1} \qquad (4)$$

Time until thermal impulse reaches the edges of the samples culd be determined from (2), when $x = \frac{d}{2}$ [2]:

$$t_K < \frac{d^2}{16 \cdot a} \qquad (5)$$

where:

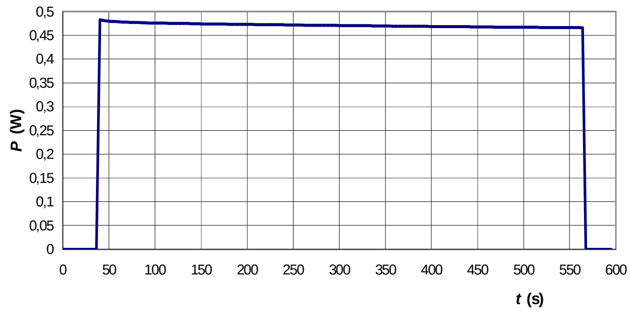
d	thickness of the both samples	(m)
a	themperature diffusivity	($m^2 \cdot s^{-1}$)

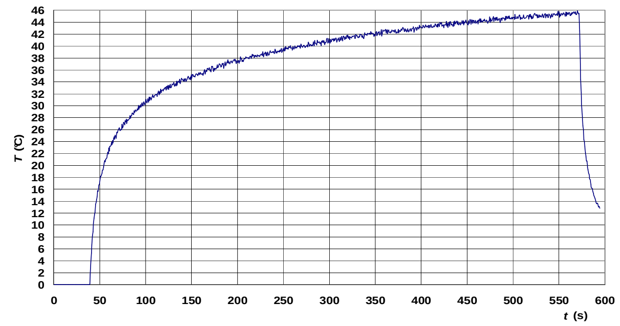
Time interval, from which thermal conductivity is going to be calculated (4), should not exceed t_K !

3. MEASUREMENTS

3.1 Thermal conductivity measurements of the extruded polystyrene

Measurement results are shown by the diagrams and the table as follows:



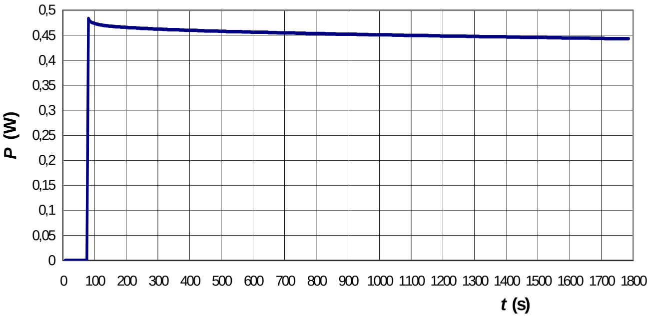


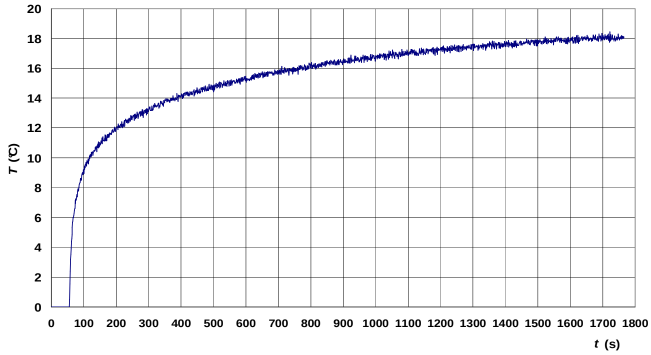
U_1 (V)	U_2 (V)	R_{REF} (Ω)	L (m)	$T(t_1)$ (°C)	$T(t_2)$ (°C)	d (m)
0,1131	4,135	1	0,165	43,1	44,5	0,07991
a ($m^2 \cdot s^{-1}$)	**t_K (s)**	**t_1 (s)**	**t_2 (s)**	**λ ($W \cdot m^{-1} \cdot K^{-1}$)**		
$7{,}7\cdot10^{-7}$	520	402	502	0,03592		

Time interval (t_1, t_2) is chosen from the biggest lineary part of the curve and is shorter than t_K !

3.2 Thermal conductivity measurements of the thermal mortar

Measurement results are shown by the diagrams and the table as follows:





U_1 (V)	U_2 (V)	R_{REF} (Ω)	L (m)	$T(t_1)$ (K)	$T(t_2)$ (K)	d (m)
0,1136	4,0393	1	0,165	15,5	17,8	0,07259
a ($m^2 \cdot s^{-1}$)	t_K (s)	t_1 (s)	t_2 (s)	λ ($W \cdot m^{-1} \cdot K^{-1}$)		
$2{,}2 \cdot 10^{-7}$	1490	600	1400	0,08153		

Time interval (t_1, t_2) is chosen from the biggest lineary part of the curve and is shorter than t_K !

3.3 Thermal conductivity measurements by the guarded hot plane method

Measurements are performed at the three measurement points, from which the regression line was calculated. From this regression line thermal conductivity of the measured sample could be calculated for every temperature within temperature interval (0°C, 55°C) [4].

Thermal conductivity measurements performed by guarded hot plate method give following results:

Extruded polystyrene, d = 79,91 (mm):

$$\lambda_{GHP} = 0{,}000171 \cdot T + 0{,}035561$$

Thermal mortar, d = 72,59 (mm):

$$\lambda_{GHP} = 0{,}000224 \cdot T + 0{,}078187$$

where:

λ_{GHP} - thermal conductivity measured by the guarded hot plate method ($W \cdot m^{-1} \cdot K^{-1}$)

T - temperature (°C). $0\,°C \leq T \leq 55\,°C$

EXTRUDED POLYSTYRENE

During the measurements, mean values of the air humidity and temperature were: 36,8 % and 18,1 °C.
For this temperature, thermal conductivity measured by guarded hot plate method and represented by the regression line could be calculated as:

$$\lambda_{GHP} = 0{,}000171 \cdot 18{,}1 + 0{,}035561 = 0{,}03866 \ (W \cdot m^{-1} \cdot K^{-1})$$

THERMAL MORTAR

During the measurements, mean values of the air humidity and temperature were: 40,4 % and 20,6 °C.
For this temperature, thermal conductivity measured by guarded hot plate method and represented by the regression line could be calculated as:

$$\lambda_{GHP} = 0{,}000224 \cdot 20{,}6 + 0{,}078187 = 0{,}08280 \ (W \cdot m^{-1} \cdot K^{-1})$$

4. CONCLUSION

Thermal conductivity measurements by the hot wire (dynamic) and by guarded hot plate (stationary) methods are compared with following table:

	HOT WIRE	GUARDED HOT PLATE	DISSIPATION (%)
	Thermal conductivity, λ ($W \cdot m^{-1} \cdot K^{-1}$)		
EXTRUDED POLYSTYRENE	0,03582	0,03866	- 7,3
THERMAL MORTAR	0,08153	0,08280	- 1,5

From the following results, and with the presumtion that guarded hot plate method (as stationary and standard method) is more accurate than the hot wire method, could be said that dissipation of the results between extruded polystyrene and thermal mortar is significant. Temperature diffusivity (a) of the extruded polystyrene is higher than that of thermal mortar and density of the thermal mortar is more than 10 times higher than density of the extruded polystyrene. So than, time for thermal impulse pass through extruded polystyrene sample is much shorter then time for thermal impulse pass through thermal mortar sample of the same or similar thickness. As a consequency, in the case of extruded polystyrene, chosen time interval (t_1, t_2) is too short and significantly contributes to the general error.

In respect to the quality of the measurements, invariability of the electrical power that the wire is suplied with, should be higher because mathematical solution of the problem presumpts that heat flux through the sample per lenght of the wire is constant.

Temperature measurements should be performed in the different way, because the thermocouple is not dimensionless and contributes to unstability of the temperature field during the measurements.

5. LITERATURE

[1] Conduction of heat in solids, Carslaw and Jaeger, second edition, Oxford, 1959.

[2] Generalised hot wire method for thermal conductivity measurements, P. Prelovšek i B. Uran, J. Phys. E: Sci. Instrum., Vol. 17, Great Britain, 1984.

[3] Introduction to Heat Transfer, F. P. Incropera & D. P. DeWitt, Wiley & Sons INC., 1996.

[4] EN 12667, Thermal performance of building materials and products - Determination of thermal resistance by means of guarded hot plate and heat flow meter methods - Products of high and medium thermal resistance (EN 12667:2001)