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INDIRECT MEASUREMENT OF THE TEMPERATURE VIA KALMAN FILTER

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Abstract – *The development of an indirect measurement model for electro thermal furnace and its implementation in a reconfigurable architecture are the main issues of this article. The proposed measurement system is based on the system model, the principle of conservation of energy and the laws of thermodynamics are applied to build a model of the thermal system where the* ***Kalman*** *theory is applied for filtering and prediction of the temperatures inside of a resistive furnace. Three setup are established to evaluate the performance of the indirect measurement system: off-line implementation of the algorithm on a test platform, the indirect measurement system is connected on the furnace hardware for standalone and real time operations. The measurement algorithm can be seen as a good alternative for temperature indirect measurement systems, due to its effectiveness and simplicity during the performance evaluations.*

Keywords – *Kalman filter, error estimation, indirect measurement, furnace temperature.*

I. INTRODUCTION

The knowledge of the thermal system behavior can be evaluated by observation of certain variables. Some of these variables are very difficult to observe due to the access to insert the sensors and/or due to its high level of noise. Indirect measurement techniques based on the process models allows the evaluation of the system states by others variables measurements.

The main contribution of this work is proposed a methodology for design, analysis and synthesis of indirect measurement system. This methodology is based on a stochastic mathematical model that represents a thermal system and embedded systems.

In The Thermal System Model (section II) the Black and White Boxes Modelling methodologies are investigated for temperature measurements. The **Temperature Indirect Measurement** (section III) focus on the theoretical development of the temperature estimation; its presented the thermal system in state space description and the estimator. In **Computational and Experimental Tests** (section IV) are presented the computational results from a simulator of an indirect measurement system and the experimental results from an embedded system implemented in a reconfigurable architecture that executes the indirect measurement system tasks. In the last section is presented the **Conclusion and Remarks** of the design, analysis and synthesis of the indirect measurement device.

II. THERMAL SYSTEM MODELS

The development of the thermal system model is performed based on the thermodynamics concepts of heat transfer and fluids mechanics. The discussion is made upon a furnace, Fig. 1, under temperature T_1 and the temperature T_2 of an object inside of this furnace. The electrical resistor R is the heat source.

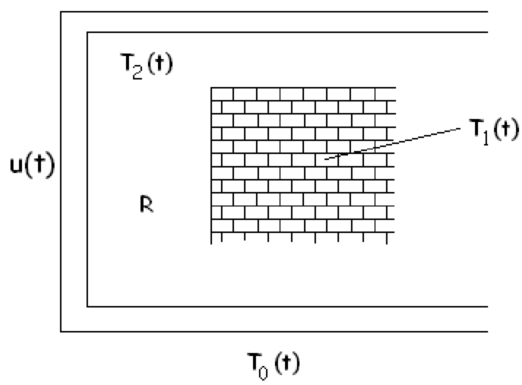

Fig. 1. The experimental thermal system sketch

The **Black Box Thermal Model** (subsection A) is a very useful identification methodology for temperature monitoring and control, in which all objects inside the furnace are associated with a sensor. The **White Box Thermal Model** (subsection B) is a well fitted identification methodology for indirect measurements; in this case, the model are based on electrical and thermal laws of nature to estimate the temperature of a furnace internal object by variations of the furnace temperature that is driven by a resistor.

A. Black Box Thermal Model

The dynamic thermal system model is represented by a difference equation,

$$y_k = -a_{n-1}y_{k-1} + ... - a_0 y_{k-n} + b_{n-1}u_{k-1} + ... + b_0 u_{k-n}, \quad (1)$$

where y_k and u_k are discrete variables, y_k represents the temperature $T_1(t)$, and $u(t)$ represents the excitation of the continuous time heat source. According to [2], a second-order equation is sufficient to represent the dynamic behavior of the temperature in the object. The structure of the model can be written as,

$$y_k = -a_1 y_{k-1} - a_0 y_{k-2} + b_1 u_{k-1} + b_0 u_{k-2}, \quad (2)$$

where a_1, a_0, b_1 and b_0 are the model parameters.

B. White Box Thermal Model

The temperature white box model is based on the thermal capacitance concept. The heat capacity is the ability of storing heat. This feature has a thermal capacitance behavior, the total heat storage feature is related to a specific heat C_p and the mass of the object inside the furnace. The temperature changes as a function of time when the stored energy flows in the furnace.

The above premiss based on nature laws drives us into the following model,

$$mc_p \frac{dT}{dt} = Q_h \quad (3)$$

where c_p is the specific heat, m is the objet mass, Q_h is the heat flux. Equation (3) has a form of a flow variable that is proportional to the rate of change of an effort variable T as a function of time.

In this modelling it is assumed that the total mass satisfies the design purpose for the temperature represented as a single temperature. Consequently, the temperature gradient in the mass is neglected. The thermal conduction inside the mass is very high when it is compared with the transfer of heat at its surface.

The thermal system is shown in Fig.2 and represents one furnace of steel with primary energy supplied an electrical power source. The external dimensions of the furnace are $x = 42.00\ cm$, $y = 52.00\ cm$, $z = 39.00\ cm$ and the net weight 28.5 kg, Fig.2.

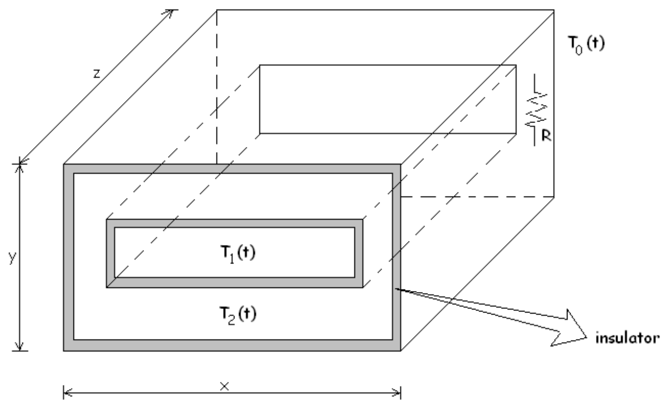

Fig. 2. Representation of the furnace with primary supplied by electric energy

The thermal energy can be stored or transferred by the temperature or range of flux heat by thermal systems. As there is difference of temperature between the two mediums, there will be transfer of heat, since the material between the mediums favors the transmission. The system model is based on the principle of energy conservation and on the laws of thermodynamics that represent heat transfer between medium 2 and medium 1 in the presence of an environmental temperature $T_0(t)$. For each medium, the resulting differential equation that represents the system behavior is expressed as a function of the temperature,

$$m_1 c_1 \frac{dT_1}{dt} = k_{12}(T_2 - T_1) \quad (4)$$

and

$$m_2 c_2 \frac{dT_2}{dt} = -k_{12}(T_2 - T_1) - k_{20}(T_2 - T_0) + u(t), \quad (5)$$

where the m_i and c_i are the mass and the specific heat of the *i*th medium, respectively, and the k_{ij} are the respective interface thermal conductances[2],[6]-[9].

III. INDIRECT TEMPERATURE MEASUREMENT

In this section is presented the indirect measurement (**IM**) models of temperature in a resistive furnace. The System Model and the *Gauss-Legendre* estimation forms are the basic elements of the **IM** system, these forms states a two steps estimation process, prediction and filtering of real signals represented as stochastic models. The System model is used to predict the variables and *Gauss-Legendre* form takes into account some measured values to improve the indirect measurement. The *Kalman* technique is used for filtering and prediction of temperature behavior in a furnace that can be controlled by a resistor. The **State Space Description** (subsection A) presents the furnace modelling based on physical laws presented in subsection B. In **Temperature State Estimator** (subsectionB) is presented the standard **Kalman** filtering application to temperature indirect measurement.

A. State Space Description

The thermal system modelled by equations (4) and (5) are represented in state space description,

$$\dot{x} = Ax + Bu \tag{6}$$

$$y = Cx \tag{7}$$

where A and B are constant nxn and nxm parameter matrices, respectively[10]. The system output y is gived by a linear combination of the states and C is a constant pxn matrix. Substituting the vector of state variables x with $x_1 = T_1$ and $x_2 = T_2$ in the Equations (4) and (5). In the matrix form,

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} \frac{-k_{12}}{m_1 c_1} & \frac{k_{12}}{m_1 c_1} \\ \frac{k_{12}}{m_2 c_2} & \frac{-k_{12}-k_{20}}{m_2 c_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_2 c_2} \end{bmatrix} u + \begin{bmatrix} 0 \\ \frac{k_{20}}{m_2 c_2} \end{bmatrix} T_0 \tag{8}$$

B. Temperature State Estimator

The **Kalman** filter algorithm produces an optimal estimative of the temperatures inside the furnace, and it minimizing the uncertainties about the prediction of the temperature. Considering $\hat{x}_k^- \in \mathbb{R}^2$ *a priori* estimation of the state variable at step k given knowledge of the system prior to step k, and $\hat{x}_k \in \mathbb{R}^2$ is *a posteriori state* estimation at step k given the measured value of $y_k \in \mathbb{R}$. *A priori* and *a posteriori* estimation of the errors,

$$e_k^- = x_k - \hat{x}_k^- \tag{9}$$

and

$$e_k = x_k - \hat{x}_k. \tag{10}$$

A priori estimation of the covariance error,

$$P_k^- = E[e_k^- e_k^{-T}] = E[(x_k - \hat{x}_k^-)(x_k - \hat{x}_k^-)^T] \tag{11}$$

and *A posteriori*,

$$P_k = E[e_k e_k^T] = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T]. \tag{12}$$

The prediction of the measured value,

$$y_k^- = C_k \hat{x}_k^-. \tag{13}$$

A posteriori estimation of $\hat{x}_k$ is a linear combination of an *a priori estimate* $\hat{x}_k^-$ and the measured $(y_k - y_k^-)$,

$$\hat{x}_k = \hat{x}_k^- + K_k(y_k - y_k^-). \tag{14}$$

The innovation $(y_k - y_k^-)$ reflects the discrepancy between the predicted and the actual measurements. The **Kalman** gain K_k is computed for each step k as a function of the model covariances,

$$K_k = P_k^- C^T (C P_k^- C^T + R)^{-1} \tag{15}$$

Is described to follow the code intermediate to temperature estimation. Is considered in this code, xhatmin(k) and xhat(k) with the estimate state *a priori* ($\hat{x}_k^-$) and *a posteriori* ($\hat{x}_k$), respectively and Pminus(k) the *a priori estimate error covariance* , P_k^- and gain(k), the gain *Kalman* K_k.

Code intermediare:

```
1. input: xhatmin(0) and Pminus(0)
2. For(k, 0, n) {
3. yminus(k) = C(k)* xhatmin(k)
4. gain(k) = (Pminus(k) * mtrans(C(k) * minv(C(k) * Pminus
* mtrans(C(k)) + R(k))))
5. xhat(k) = xhatmin(k) + (gain(k) * (y(k) - yminus(k)))
6. P(k) = (id(n) - (gain(k) * C(k))) * Pminus(k)
7. xhatmin(k+1)= A(k) * xhat(k) + G(k)*u(k)
8. Pminus(k+1) = (A(k) * P(k) * mtrans(A(k))) + Q(k)}
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From the linear stochastic difference equation $x_{k+1}^i = A_k^i x_k^i + B_k^i u_k^i$, and $y_k^i = C x_k^i + v_k^i$, is considered that the random vectors u_k and v_k that represent the process and measurement noise, respectively, [1],[3]-[5],[7],[10]-[19]. The random vectors are assumed to be independent of each other with normal probability distribution $p(u_k) \sim N(0, Q_k)$, $p(v_k) \sim N(0, R_k)$, $E(u_k v_i^T) = 0$, Q_k, if $i = k$ or 0 if $i \neq k$, $E(v_k v_i^T) = R_k$, if $i = k$ or 0 if $i \neq k$.

IV. COMPUTATIONAL AND EXPERIMENTAL TESTS

The computational results (subsection B) are obtained from a simulator of the proposed indirect measurement system. These results are used to analysis and synthesis of digital devices based on concepts that are presented in Temperature Indirect Measurement, section III. The Experimental Results the embedded system is the core of indirect measurement device. The embedded system is implemented in a reconfigurable architecture that is used to perform the main tasks of indirect measurement. The *Kalman* algorithm performs the temperature prediction of an cube of aluminum, with weight of 19.50 g, inside of the furnace, Fig. 2. The available measured output are used to support the state observer with temperature estimation that are not directly measured.

A. *Numerical State Space Description*

The numerical state space description of the furnace state is representation of the real system. This representation is obtained by substituting in the state space description, Equation (8), the values approximately for m_1c_1, m_2c_2, k_{12} and k_{20} are 3.88, 3049.5, 2.08 and 0.46, respectively.

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -0.54 & 0.54 \\ 0.00068 & -0.00083 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{3049.5} \end{bmatrix} u + \begin{bmatrix} 0 \\ \frac{0.46}{3049.5} \end{bmatrix} T_0 \tag{16}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{17}$$

The continuous state state space equations are mapped into his equivalent state space difference equations, the discrete model accomplishes the prediction step of **Kalman** filtering algorithm. In terms of the furnace temperatures, the discrete model performs the temperature T_1 indirect measurement of an object that is inside of the experimental thermal system, Fig. 1.

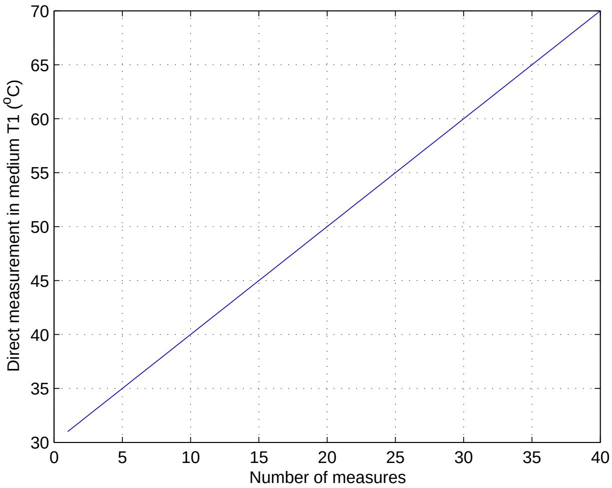

Fig. 3. T_1-Direct temperature measurements

B. *Computational Results*

Two simulations are performed as a function of the system covariance, Q, and measurement covariance, R, to achieve a better convergence of filter gains. These results are compared with the sensor measured temperature values of the furnace T_2, Fig. 4.

In the first simulation, the system covariances elements are $q_{ii} = 1$ and measurement covariance are $r_{ii} = 0.0001$. In

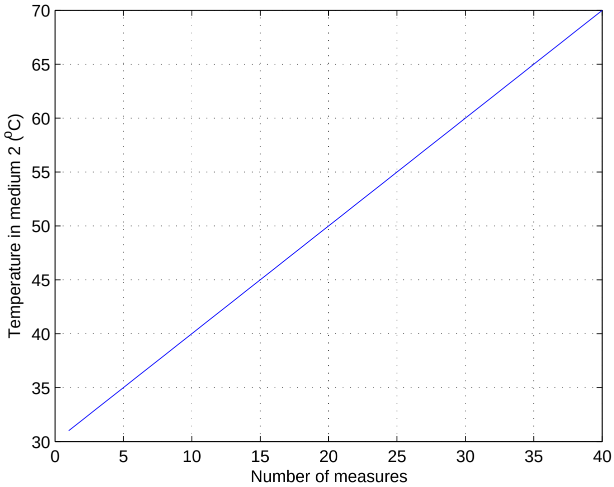

Fig. 4. T_2- Temperature sensor measurements - 40 samples

the second simulation,$q_{ii} = 10$ and $r_{ii} = 0.01$. The latest simulation presented a better performance when is compared with the measured temperature values of the furnace T_2 of Fig. 4. The Fig. 5 show the **Kalman** gain evolution, as can bee seen, the steady value is reached around fifteen sampling time intervals.

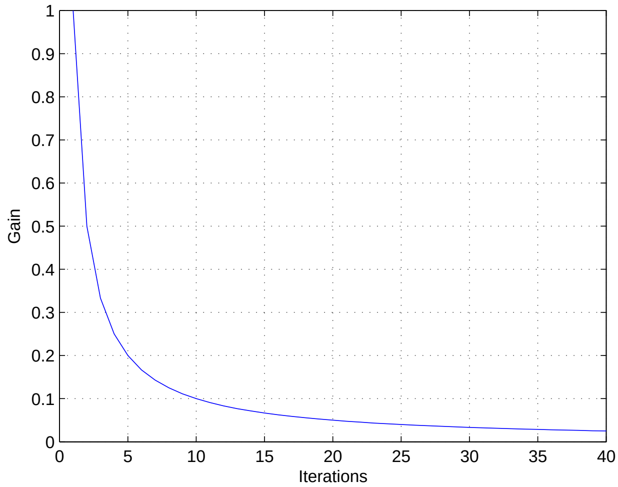

Fig. 5. Kalman evolution

C. *Experimental Setup*

The measured temperature values initially were obtained by integrated-circuit temperature sensor, $LM35DZ$, inserted in the medium 2 conform Fig. 6.

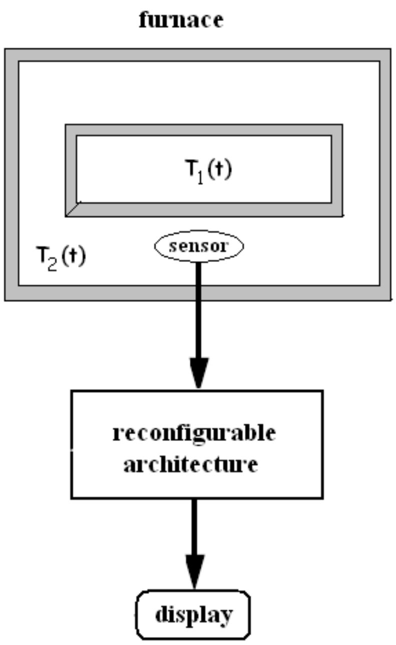

Fig. 6. The block diagram of the indirect measurement of the temperature in medium 1

C.1 Embedded **IM** System

A reconfigurable architecture software is used to implement the core of **IM** device, micro controller PSoC, version 4.0, CY8C26443 model, Liquid Crystal Display - LCD module and ADC module. One I/O port line of the reconfigurable architecture is used for output of the effective temperature value in medium 1. This line is connected from module LCD of the architecture to the external display. The PSoC general features are 24 MHz frequency, 3.0 - 5.25 V tension, 16 $kBytes$ flash memory, RAM 256 Bytes, digital block 8 (Timers, Counters, PWM, etc) and analog block 12 (ADCs, DACs, Programmable Filters, Comparators, etc). The Kalman filter algoritm is developed and processed in C language for this architecture. Indirect measurement temperature values are visualized through a LCD.

Using C language, version 4.5, Kalman's filter processing can be carried through. With this tool, temperature values of the furnace for 40 samples was indirect measurement. Filter performance is verified and compared with measurement values for sensor of the temperature. After that, Kalman's algorithm development through C language for PSoC. This program development is limited to value formats and available commands for PSoC C language, beyond memory capacity offered by the version. The indirect measurement of the temperature, in this design, was visualized through a LCD.

C.2 The Embedded Code

A Programmable System on Chip (PSoC) is used in this project for Kalman filter implementation. The PSoC available memory capacity was considered. The Kalman filter algorithm was developed in PSoC C language, considering 40 samples.

The intermediate code to temperature estimation as presented after Equation takes as input specification, Equations (6)-(7) and also the descriptions of the noise characteristics and filter parameters. The generated code is executed in C language and MATLAB. The algorithm code was developed in 148 lines of C code, using 233 $Bytes$ of the SRAM. Each floating point sample occupy 4 $Bytes$ and the integer variables occupy 2 $Bytes$. Even if the basic architecture employs 8 bits registers, the application was implemented with a 16 bits words. The algorithm main C code file has a size of 7 $kBytes$. The auxiliary .rom and .hex files took 32 $kBytes$ and 36 $kBytes$, respectively.

C.3 Experimental Results

The values obtained from the direct and indirect measurements of the object's interior inside the furnace presented some discrepancies in some instants of time. It is necessary to consider the factors that contributed for such discrepancies, as: the initial conditions, such as the mass of the material, the specific heat and thermal conductance not properly adjusted; and the heat dissipation between the material and the furnace, because the material was not properly solid. A comparative analysis performed between those two measurements (direct and indirect measurement) verified that the average error of the absolute error values in the indirect measurement is calculated as shown below:

$$\overline{E} = \frac{1}{n}\sum\nolimits_{j=1}^{n} |T_{1md(j)} - T_{1mi(j)}|, \qquad (18)$$

with T_{1md} as direct measurement in medium 1, T_{1mi}, indirect measurement in medium 1 and n, number of the measures (n = 40). The average error of the indirect measure inside the material is of $1.47\ ^oC$, approximately.

V. CONCLUSION AND REMARKS

In this paper had been presented the development of indirect measurement device, based on model, for temperatures in a small furnace. A mathematical model that represents a thermal system is assembled in the state space description and the **Kalman** theory is used for filtering and prediction of temperatures behavior in a furnace.

The performance of proposed estimation algorithm was verified computational by simulations and experimental implementations. The algorithm was implemented in a low-cost reconfigurable architecture, an AD Converter block receives the temperature values from sensors located inside the furnace and performs the estimation temperature processing of an object without associated sensors. For monitoring and control requirements match, the proposed indirect measurement device had shown as an alternative for optimal estimation of temperatures.

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REFERENCES

[1] Robert L. Woods Lawarence.*Modeling and Simulation of Dynamic Systems*. 1997.
[2] Raymond G. Jacquuot. *Modern Digital Control Systems*.Marcel Decker, Inc. 2 edition, New York, 1994.
[3] Gene F. Franklin, J. David Powell and Abbas Emami-Naeini. *Feedback Control of Dynamic Systems*. 4 edition, 2002.
[4] Katsuhiko Ogata. *Engenharia de Controle Moderno*. Prentice Hall do Brasil, 3 edition, 1997.
[5] William L. Brogan. *Modern Control Theory*. 6 edition, 1990.
[6] João Viana Fonseca Neto. *Filtros Digitais para Controle Temperatura de uma Estufa*. São Luís, 1 edition, abril 2005.
[7] William L. Luyben and Michael L. Luyben. *Essentials of Process Control Chemical Engineering*.1997.
[8] Adrian Bejan. *Transferência de Calor*. São Paulo, 4 edition,1996.
[9] Hélio Creder. *Instalações de Ar Condicionado*. Rio de Janeiro, 4 edition, 1990.
[10] Alexander Weinmann. *Uncertain Models and Robust Control*. Wien, New York, 1 edition, 1991.
[11] Eli Brookner. *Tracking and Kalman Filtering Made Easy*. John Wiley & Sons, INC.1998.
[12] Charles K. Chui and Guanrong. *Kalman Filtering with Real-Time Applications*. Volume 17 of Springer Seeries in Information and Sciences. Springer Verlag, New York, 2 edition, 1991.
[13] Hwei Piao Hsu. *Theory and Problems of Probability, Random Variables, & Random Process*. MacGraw-Hill, New Jersey, 20 edition, 1997.
[14] Haward Hutchings. *Interfacing with C*. Newnes Publishing Staff, Boston, 1 edition, 1995.
[15] Robert Grover Brown and Patrick Y.C. Hwang. *Introduction to Random Signals and Apllied Kalman Filtering*. John Wiley & Sons, New York, 10 edition, 1997.
[16] Greg Welch and Gary Bishop. *An introduction to the kalman filter*. University of North Carolina at Chapel Hill Chapel Hill, pp 1-16, April 2004.
[17] Yu Kweng Lin and Guo Qiang Cai. *Probabilistic Structural Dynamics*. Times Roman, New York, 5 edition, 1995.
[18] Maria M. Seron, Julio H. Braslavsky and Graham C. Goodwin. *Fundamental Limitations in Filterinf and Control*. Springer, February 2004.
[19] R. E. Kalman. *A new approach to linear filtering and prediction problems*. Transactions of the ASMEÜ Journal of Basic Engineering - Research Institute for Advanced Study, pp 35-45, 1960.