XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

COMPUTER SIMULATION OF THE RESPIRATORY SYSTEM IN FORCED EXPIRATION

Barbara JUROSZEK [1], *Jan STANISŁAWSKI* [2]

[1] Wroclaw University of Technology, Wroclaw, Poland, e-mail: barbara@pwr.wroc.pl
[2] Wroclaw University of Technology, Wroclaw, Poland, e-mail: jotes@pwr.wroc.pl

Abstract: The authors present a model of the respiratory system in forced expiration as an answer to the unknown pressure pulse excitation. The model has the form of a voltage divider circuit, where gas flow volume velocity is the current reaction (or the volume is the voltage reaction). The analysis is presented in time domain. The model's finding is realized in two main stages. In the first stage an *RCL* net is built. On the basis of this structure the exciting signal is found in the form that the answer to the model is $V(t)$ (or $Q(t)$), very similar to the real object's answer, in the minimum mean-square-criterion.

Keywords: forced expiratory curve, electrical analogue of the respiratory system, computer modelling.

1. INTRODUCTION

An accurate assessment of the respiratory system state is a basis for further treatment. Pulmonary function tests include flow rate and lungs volumes measurements. They are helpful in detecting breathing abnormalities. A spirometer measures the volume, the air exhaled (or inhaled) and the length of time each breath takes. During such a test the measuring structure presented in Fig. 1 is realised.

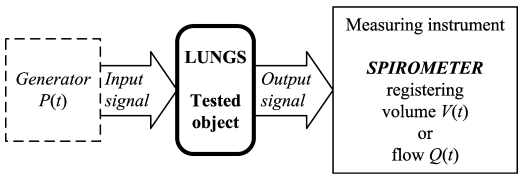

Fig. 1. The measuring structure realized during lungs testing

The pressure signal generated individually by the patient is very important. It gives the possibility of testing "the black box", which the lungs are. The measuring instrument registers the answer that is the result of composition of the testing signal and tested object's features. The correct identification of the lungs features strongly depends on two basic elements: the quality of input testing signal, as is the pressure and the quality of testing instrument, that registers the output signal (the air volume $V(t)$ or the air flow velocity $Q(t)$). It is not easy to generate the pressure $P(t)$ precisely. There are no possibilities of registering it during standard, normal testing because it is generated in the respiratory centre in the brain [1, 2]. The observer can't see if it has a proper and needed form.

A kind of testing signal standardization is the so called "forced expiration" (FVC-test). During it the patient inspires a maximum amount of the air into the lungs, waits a moment and then expires it immediately with the biggest force and velocity. He tries to present such expiration very carefully. $P(t)$ then has the form of a pressure jump. However, it is known that the exciting signal doesn't have a step form [3]. It has an exponential form and arises with different speed. In spite of this the forced expiratory test is still the most popular. The parameters measured during it are the basis of the respiratory system diagnosis.

2. THE FORCED EXPIRATION MODEL

The most popular model of the forced expiration, generally explaining it, is two element *RC* serial circuit excited by pressure jump, where *R* is a flow resistance (in the airways), *C* is a compliance of the alveolar bags. For better presentation of this model an inertial element should be taken into consideration. *L* is the inertia of the gas stored in airways and alveolar bags. In the easy model's explanation of the inertia *L* is usually neglected.

Because an excitation signal is a jump pressure, that presses the lungs, the answer, observed in the form of expired air volume or flow changes, has the form similar to the exponent [4]:

$$V(t) = FVC\left[1-\exp\left(-t/\tau\right)\right],\ Q(t) = \frac{d}{dt}V(t) \qquad (1)$$

where *FVC* is maximum air volume expired in the forced expiration (*FVC* is the abbreviation of the Forced Vital Capacity), τ is the lungs time constant ($\tau = RC$) dependent on the airway flow resistance *R* and the elastic alveoli compliance *C*.

However this model can't be used to explain the real object structure and working. *R*, *C* elements have the general meaning and are not used in further medical diagnosis yet. For this reason the expiratory signal $V(t)$ is presented in the form of samples: the collection of volumes expired in different moments of the FVC-test (e.g. the Forced Expiratory Volumes: $FEV_{0.5} = V(t)\big|_{t=0.5s}$, $FEV_1 = V(t)\big|_{t=1s}$, $FEV_2 = V(t)\big|_{t=2s}$, $FEV_3 = V(t)\big|_{t=3s}$ and the Forced Expiratory Flows: $MEF_{25\%FVC} = Q(t)\big|_{V=25\%FVC}$,

$MEF_{50\%FVC} = Q(t)\big|_{V=50\%FVC}$, $MEF_{75\%FVC} = Q(t)\big|_{V=75\%FVC}$) [5].

The forced expiration in the form of the proper model could be better and more precise instrument for the respiratory system state presentation. Such a model is presented below.

3. THE MODEL'S ASSUMPTIONS

The way of modelling presented here consists of two main subjects:

a) finding the structure of the easiest model that is built of *RCL* elements and reflects the lungs' structure (trachea, bronchi, alveoli, the air inside), having three types of the elements: rigid, elastic and inertial,
b) finding a proper form of the exciting signal.

The simplest lungs model's structure is very similar, consisting of the general *RC* structure and a hypothetical artificial exciting signal, which was mentioned earlier. The novelty of the model presented here is that:

1. in the first approximation the exciting signal has in general, hypothetical pulse form for flow answer (and step form for the volume answer),
2. the best matched *RCL* structure gives an answer similar to the smoothed real answer,
3. the final form of the exciting signal is not smooth, because it takes into consideration all fast changed parts that exist in the forced expiration curves.

The general strategy of the model synthesis is presented in the Fig. 2.

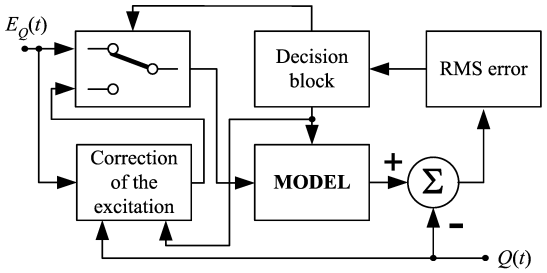

Fig. 2. The scheme of the respiratory system synthesis

The structure of the model analysed here is presented in Fig. 3.a. In the circuit synthesis six different configurations of the serial forms of *RCL*-one-port boxes were used [5, 6]: *RC* and *RL* parallel sections, and combined *RCL* section (Fig. 3.b). They were connected serially in different configurations. In principle it is not a structural model but a transmittance one which reflects the relation between the reaction in the form of flow (or volume) changes to the hypothetical generalized excitation.

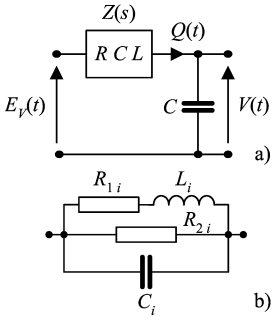

Fig. 3. The electrical model of the respiratory system: a) general structure; remarks: $E_Q(t)$ and $E_V(t)$ – the excitations (unknown), generating flow and volume curves $Q(t)$ and $V(t)$ respectively, b) elementary *RCL* section used during the respiratory system synthesis

The synthesized model approximates the dynamic features of the respiratory system at the smallest upper airways' flow resistance. The assumed normalization C = 1 F preserves the time parameters of the model and the proportions between the contents of the exponential type of reaction.

By means of this model we could determine a generalized, hypothetical excitation introducing some extra components, e.g. of the chaotic oscillations into the reaction. Finally the block *RCL*, depicted in Fig. 3.a, had the structure built of *RC* and *RL* parallel sections and combined *RCL* section. The example of such connection is presented in Fig. 4.

The general structure of the modelled system is similar to a 4-order-low-pass filter with real poles. The simulation started from the simplest form of the model (one pole transmittance function). It was the basis for the next step of modelling. In this way the second-order circuit was found, with two-pole transmittance function. This model was the basis for the next step of synthesis, etc.

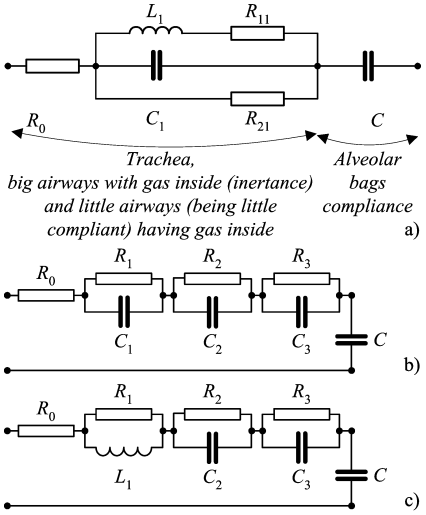

Fig. 4. An example of the lungs impedance model: a) general lungs model having resistive, compliant (capacitance) and inertial (inductance) elements, b) the structure from rigid trachea to compliant alveoli, with "swift passage", c) a similar structure with inertia of the air covered in the airways

At the beginning of simulation the assumption was made that the hypothetic excitation pulse $E_Q(t)$ is narrow. Its time duration is less then 0.2 second. It changed its form in the final stage of synthesis according to transmittance coefficients defined for the model.

For such a model let us assume that $Z_n(s) = C\,Z(s)$ is the normalized impedance and $E_n(s) = C\,E_V(s)$ is the normalized excitation. The Laplace transform of the flow $Q(s)$ of the model has the form:

$$Q(s) = \frac{E_V(s)}{Z(s) + 1/(s\,C)} = \frac{E_n(s)}{Z_n(s) + 1/s}, \qquad (2)$$

With regard to the circuit's lumped-linear-stationary features the following relation is reasonable

$$E_Q(t) = \frac{dE_V(t)}{dt}, \qquad (3)$$

The simple form of the lungs impedance model with *RCL* elements is described by:

$$Z_n(s) = \sum_{i=1}^{n} \frac{Z_i}{s + z_i} + Z_0, , \qquad (4)$$

where: Z_i – is a residuum, z_i – a negative value of the pole of $Z_n(s)$ and a real positive constant $Z_0 = Z_n(s)\big|_{s\to\infty}$.

For each section: *RC*, *RL*, *RCL* the impedance $Z_n(s)$ is completely defined by the residuum Z_i and the pole z_i:

1. for resistor: $R_0 = Z_0$,
2. for parallel *RC* section: $Z_i > 0$ and $z_i > 0$ the elements' values can be calculated as: $R_i = Z_i/z_i$, $C_i = 1/Z_i$,
3. for parallel *RL* section: $Z_i < 0$, $z_i > 0$ which was the basis for elements value calculation is: $R_i = -Z_i/z_i$, $L_i = R_i/z_i$, $Z_0 := Z_0 - R_i$,
4. for *RCL* circuit residuum and pole have complex value: $Z_i = A_i + jB_i = Z_{i+1}^*$, $z_i = x_i + jy_i = z_{i+1}^*$, where: A_i, B_i, x_i, y_i – some real constants; the *R*, *L*, *C* values of all elements are given by:

$$L_i = 2A_i^3/[y_i^2(A_i^2+B_i^2)], \quad (5)$$

$$C_i = 0.5/A_i, \quad (6)$$

$$R_{1i} = 2A_i^2(A_i x_i + B_i y_i)/[y_i^2(A_i^2+B_i^2)], \quad (7)$$

$$R_{2i} = 2A_i^2(A_i x_i - B_i y_i), \quad (8)$$

5. more complicated circuits can have different forms.

In order to limit the program solutions the assumption was made, that the synthesised *RCL* model should have positive values of elements and fulfil the condition that z_i should have positive values of real part (Re $z_i > 0$).

4. SOME DETAILS CONCERNING THE MEASURING SYSTEM

The models' structures were obtained on the basis of the group of 50 patients tests. For the experiments the measuring system was built. The basis was the spirometric flow transducer (Jaeger) connected through a 12-bit AD converter into the computer. The expiratory signal was sampled with a frequency of 1 kHz. Because the obtained signal was with interferences the zero level was defined like mean of the samples values when flow was zero, i.e. before the start of the expiration.

The moment of zero in time (t = 0 s) was found in back extrapolation procedure, according to ATS (American Thoracic Society) recommendations [7] (Fig. 5). The proper line defining point t = 0 (broken line) was found in the RMS method in the A-B part of the FVC-curve $Q(t)$. The samples in the range of 70 ms were taken into respective calculations. The points important for the further calculation are marked in Fig. 5 by squares.

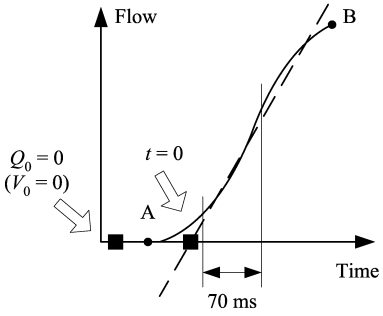

Fig. 5. The definition of the start point $t = 0$ by the back extrapolation line; the essential points are marked by squares

Finally the forced expiration was stored in the form of $Q(t)$ samples. $V(t)$ was the result of $Q(t)$ samples integration by the Simpson method. For further comparisons the basic expiratory parameters for the patients were measured: *FVC* (Forced Vital Capacity), FEV_1 (Forced Expiratory Volume in one second), *PF* (Peak Flow) and FEV_1/FVC (Tiffeneau index).

5. CALCULATIONS AND RESULTS

While looking for the solution, that was realised in six stages (comp. Fig. 6 and Table 1), the minimisation of Δ error was taken into consideration defined as:

$$\Delta = \sqrt{\frac{\sum_{i+1}^{N}\left[Q(iT) - Q_i^m\right]^2}{N}}, \quad (9)$$

where: N – number of samples, i – number of current sample, T – sampling period, $Q(iT)$ – $i^{\text{-th}}$ sample of measured flow velocity value, Q_i^m – $i^{\text{-th}}$ sample of modelling flow velocity.

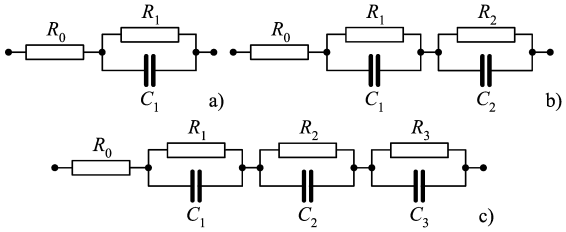

Fig. 6. The structure of the *RCL* block (here is without *L* element) of the model at different stages of modelling: a) II stage, b) III stage, c) IV stage

The modelling always started from the simplest structure, where one resistor was connected only with one capacitor. Because the preciseness of the third and fourth stage is almost the same the four poles structure was chosen as the basis for the fifth and sixth stage. For these last stages the difference was very small and the exciting signal has been found (Fig. 7). Although its form was changed two times the final RMS error became constant.

Table 1. The results of the respiratory system modelling; an example

The stage of synthesis	The characteristic of the circuit	RMS error in %
I	One pole	2.65
II	Two poles	1.68
III	Three poles	1.48
IV	Four poles	1.47
V	Four poles; the first corrections of the excitation signal	0.23
VI	Four poles; the second corrections of the excitation signal	0.23*

* comparing with stage V - not visible differences.

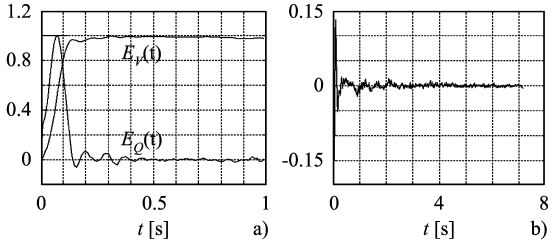

Fig 7. The forms of the exciting signals (all normalised) used during a model synthesis (an example): a) generated flow volume $Q(t)$ signal response and volume signal $V(t)$ for stage VI, b) the differences $(Q(iT) - Q_i^m)$, concerning flow velocity $Q(t)$ in modelling for four-poles function transmittance

Figure 7 presents the form of excitation signal and the errors concerning its reconstruction. As we see the corrected hypothetic signal is the composition of the smooth and variable one. The greatest differences between the real signal and model signal exist at the beginning part of the forced expiration.

Using criterion defined by eq. (9) we found the best structures that had two forms: the first – built of *RC* elements (see Fig. 4.b), the second – built of *RC* elements and one inertial (see Fig. 4.c).The exemplary values of respective elements are shown in Table 2. Because some elements are connected in pairs ($\{R_1, C_1\}$, $\{R_2, C_2\}$, $\{R_3, C_3\}$ or $\{R_1, L_1\}$, $\{R_2, C_2\}$, $\{R_3, C_3\}$) we observe a tendency that when in a pair the flow resistance R_i grows the compliance C_i diminishes. This structure is consistent with the lungs structure: upper airways are rigid (in the model it is R_0 element) and lower airways are less rigid and more compliant [8]. This is a swift passage.

Table 2. Two models calculated for one patient and the values of their elements

Model	Elements							
No 1	C	R_0	R_1	C_1	R_2	C_2	R_3	C_3
(Fig. 4.b)	1	0.91	0.91	3.37	0.16	3.5	0.06	3.31
No 2	C	R_0	R_1	L_1	R_2	C_2	R_3	C_3
(Fig. 4.c)	1	0.38	1.84	$3.14 \cdot 10^{-3}$	0.78	3.14	0.15	4.83

Taking into consideration these two presented models we have compared the relations between the models' elements and the expiratory parameters: *PF*, *FVC*, FEV_1 and FEV_1/FVC. In Table 3 we see that the linear correlation coefficients between all these parameters are not too high. The resistive and compliant elements correlate with *PF* and FEV_1 parameters. It seems to be obvious because *RC* elements decide about the lungs dynamics, that appears during forced expiration and *PF* and FEV_1 are a measure of this dynamics. Model no 1 built of only *R* and *C* elements is better than model no 2 having inertance *L*.

Table 3. The values of the linear correlation coefficient ρ for the relations between FVC-parameters and the models' parameters; the numbers in bold present the highest correlation, more than 0.7

ρ		*PF*	*FVC*	FEV_1	FEV_1/FVC
Model no 1	R_0	0.50	0.14	0.15	0.08
	R_1	0.11	0.04	0.10	0.19
	C_1	0.47	0.17	0.16	0.52
	R_2	**0.77**	0.37	0.36	**0.76**
	C_2	0.66	0.45	0.36	**0.74**
	R_3	0.65	0.14	0.11	0.56
	C_3	0.51	**0.71**	**0.76**	0.35
Model no 2	R_0	0.44	0.34	0.29	0.50
	L_1	0.18	0.09	0.21	0.25
	R_1	**0.71**	0.16	0.20	0.27
	R_2	0.42	0.38	0.34	0.66
	C_2	**0.76**	0.43	0.46	0.55
	R_3	0.46	0.24	0.29	0.14
	C_3	0.42	0.25	0.17	0.42

6. CONCLUSIONS

The forced expiration reflects the dynamics of the respiratory system. The expiratory parameters presented in the form of volumes associated with time (e.g. FEV_1) and flows (e.g. *MEF*s) are the kinds of samples of such an expiration. We suggest that they are indirect measure of the real features of the lungs as the flow resistances and compliances are. The models presented here may be a kind of instrument that will explain the real properties of the lungs in a complete way: from trachea through bronchi to the last alveolus.

Model parameters optimization starts from the model of the first order and finishes at the fourth. The model's order increasing doesn't correct substantially the modelling error. The normalized model consists of an unitary capacitor. The model de-normalization needs to measure the maximal value of the pressure in airways.

Such a model structure should be synthesized in some steps. The first main step is finding the basic *RCL* structure. This structure gives the possibility of presenting an up to, unknown form of the excitation signal. Finding it, with respectively high precision, confirms the model's structure adequacy.

Attention should be paid that the model synthesis presented here gives the possibility of dividing the respiratory system into two parts according to the structure presented in Fig. 1. The main advantage of such modelling is that we find not only the model structure and model's elements but the exciting signal as well. It is especially profitable when the forced expiration is not good, i.e. when the patient has problems with full forced expiration.

Important is the fact that the model's parameters correlate with the expiratory parameters and in the future they can be a complement of the lungs diagnosis.

REFERENCES

[1] V.K. Jain, "Optimal respirator settings in assisted respiration", Med. and Biol. Eng. And Comp., July, pp. 425–430, 1974.

[2] R.J Soto, H.V. Forster, B. Rasmussen, "Computerized method for analyzing maximum and partial expiratory flow-volume curves", J. of Appl. Physiol., Vol. 39, No. 2, pp. 315–317, 1975.

[3] B. Juroszek, "The form of the exciting signal in the forced expiratory test", In: From Measurement to Innovation. Proceedings of the XIII IMEKO World Congress. Torino, September 5–9, Vol. 3, pp. 2519–2523, 1994.

[4] M.C. Khoo, "Physiological control systems. Analysis, simulation and estimation", IEEE Press, New York 2000.

[5] B. Juroszek, "About some metrological aspects of standardization in spirometry", In: Metrology for a Sustainable Development. Proceedings of the XVIII IMEKO World Congress. Rio de Janeiro, September 17–22, 2006, (in press).

[6] B. Juroszek, J. Stanisławski, "Modelling of the lung dynamic features in forced expiration", in Polish, Joint IMEKO TC-1 & XXXIV MKM Conference 2002, Wroclaw 8-12 September, Vol. II, Wroclaw University of Technology, pp. 419–426, 2002.

[7] R.O. Crapo, "Standardization of spirometry. 1994 update", Am. J. Respir. Crit. Care Med., Vol. 152, pp. 1107–1136, 1995.

[8] J.R. Hammersley, D.E. Olson, "Physical models of smaller pulmonary airways", J. Appl. Physiol., Vol. 72, No 6, pp. 2402–2414, 1992.