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PRECISION SCALES OF PLANE ANGLE: PRINCIPLES AND METHODS OF CALIBRATION

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Abstract: Precision angular scales are considered from common point of view, and principles of their construction are formulated. Methods of mutual calibration of circular scales are developed to meet these principles taking into account the redundancy factor. Multidimensional classification of the methods is proposed. The family of designs is constructed, and relations of procedure parameters of a scale carrier calibration with required accuracy of the procedure results is established.

Keywords: plane angle, circular scale, calibration.

1. INTRODUCTION

The problem of designing and performance of plane angle measurements characterized by high accuracy required and difficulty of the measuring plane fixing becomes more and more urgent in various applied fields. Examples of those are exact mechanical engineering, including machine-tool construction and production of space and sea technical equipment, and instrument making, in particular, designing and production of apparatus for precise navigation and motion control.

So, in a number of exact manufactures it is required to measure angular parameters of products with errors 1″ and less. Addition, in some cases it is required to take into account sophisticated geometrical models of controllable objects, including necessity of their exact affixment to base coordinate system. This situation takes place at coordinate measurements with small measuring bases, and also at use of models with nonlinear parameterization. An example is the problem of indirect measurements of the central angle of a detail in the form of sector of a ring. For its solve carry out direct measurements of coordinates of its arch points with the purpose of restoration of the absent center coordinates.

Besides the problem of precision angular measurements *the problem of calibration* of angular scale which is used arises. Diversified devices can serve as a carrier of angular scale and the devices can be divided into two overlapping groups. We shall attribute the device to the *first group* if, for carrying out of its calibration, we have at hand essentially more precise reference measuring instrument with the same range. In this case the information about scale under calibration receives *by direct comparison* with a known scale. As a rule, angular gauges of every kind are brought into the given group, including those which are directly used in a contour of follow-up system. As a reference measuring instrument, regular polyhedral prisms (polygons), index tables, goniometers, optical dividing heads, irregular prisms, optical wedges, angle comparators such as sine bar and other high-precision instrument of angle measurements usually serve.

If accuracy of reference instrument and device under calibration are comparable, we shall attribute it to the *second group*. Regular polyhedral prisms and manifold devices with limbs and radial lattices of various types, etc. can get into the given group. As a rule, devices of this group have the closed circular scale of a full range 0-360°.

It is necessary to note that two groups of devices described above can differ also under the form of representation of their *scale error*, including a level of error specification. So, after calibration of the 1st group device *the generalized* estimations of a variable error along scale, as one or several numbers, is used. The top border of error $\Delta=\Delta_{max}$ and various integral-averaging estimations serve by examples of such estimations. As it was already marked, procedure of calibration of such devices is rather simple and based on direct comparison with available more exact 2nd angular scale, in view of balance of accessible resources and requirements to accuracy.

Direct comparison of the 2nd group device with reference angular scale having the same accuracy will bring to roughening (doubling) of their error bound, $\Delta=2\Delta_{max}$. This doubling of an error is caused by necessity to compare angular intervals of two scales, and the contribution to a total error will be given with measurements *of both ends* of an interval. This situation is illustrated by fig. 1, on which [i] is a nominal i^{th} mark of each scale (counted from an initial mark [0]); (i) is a real i^{th} mark of each scale; d_i is a readout at comparison i^{th} marks of two scales; φ_i, ψ_i, n are errors and amount of the chosen marks.

As opposed to stated, there is an opportunity to execute the mutual calibration (known as cross-calibration) of two scales [1, 2]. Similar measurements are based on paired comparison of uniform marks of two scales with the same discreteness n, which are serially superposed with each other in several angular positions. Results of mutual calibration are estimations of individual deviations $\{\varphi_i\}$, $\{\psi_i\}$, i=0, 1, 2, …, n–1, of the chosen marks of each scale, where $\psi_0=\varphi_0\equiv 0$. As a matter of fact, after such calibration the

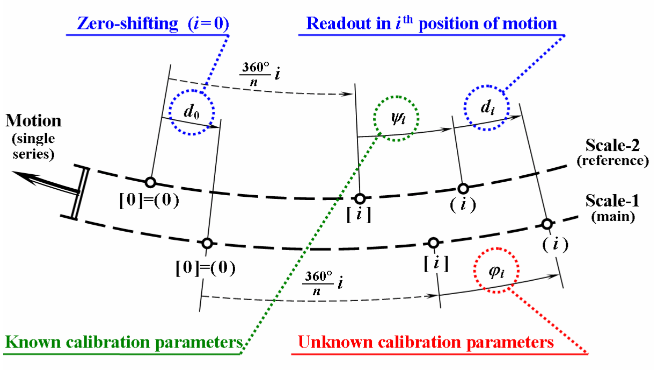

Fig. 1. Calibration of the main circular scale by direct comparison with the reference circular scale

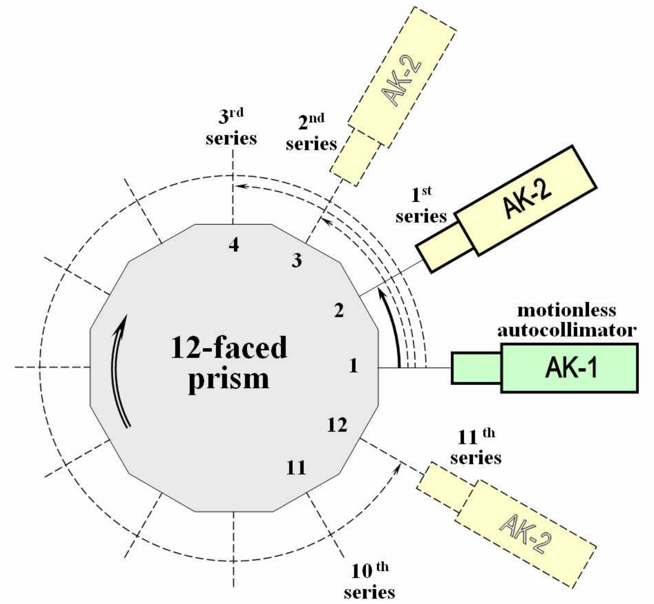

Fig. 3. Calibration scheme of 12-faced regular quartz prism

carrier of a scale becomes the multiple-valued measure similar to a polyhedral prism. It is obvious, that in contrast to direct comparison of scale under calibration and reference scale (with known $\{\psi_i\}$), the procedure of mutual calibration of two scales is much more difficult. However its doubtless advantage is absence of requirements to presence of the exact information on the reference scale errors. It is necessary to note also, that both scales participating in cross-calibration, are equal in rights among themselves. This situation is illustrated by fig. 2, on which, addition to fig. 1 notation, λ_k is residual relative shift of nominal marks (in the k^{th} series of measurements); l_i is the number of reference mark.

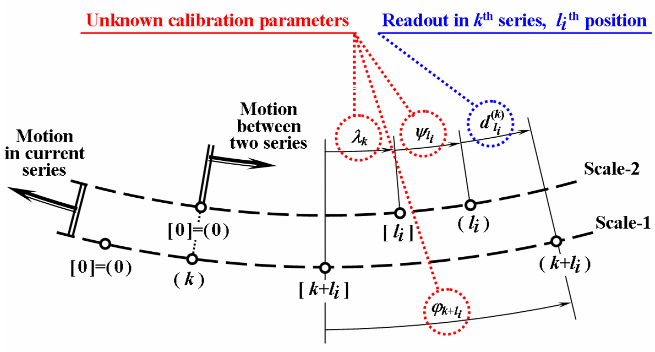

Fig. 2. Two basic motions and relationship between parameters at circular scales cross-calibration

The presentation is devoted to the problems arising in the latter case. The situation is illustrated well with the calibration of a 12-faced regular quartz prism (polygon) executed on the USSR state plane angle standard [3]. The calibration scheme of this prism represents in fig. 3. Let's comment it.

Angles between optical normals to prism faces compose *the basic circular scale* from 12 marks to be calibrated. But for all that, feature of the given scheme is *the implicit reference scale* which composes by autocollimators' axes (motionless AK-1 and mobile AK-2). At each of 11 positions AK-2 the next series of measurements is executed, turning a prism to 12 positions with step 30° and registering readout on both AK. Discreteness of readouts was 0.01″. The final accuracy of calibration was 0.05″.

Finally, mutual calibration is based on the superfluous data acquisition by repeated comparisons of scales in accordance to the drawn up design [3, 4] in conditions under strict control. Results, $\{\varphi_i\}$, $\{\psi_i\}$, are obtained by complicated processing the primary data $\{d_i\}$. The final accuracy depends on the degree of data redundancy. Such procedures are used for fulfillment of reference metrological works when result accuracy required is mostly high. Two common schemes of calibration is represented in figures above.

At the same time, modern IT-tools (data processing) make it possible to use, more widely, mutual calibration of devices of the 2nd group in case of the increased requirements for accuracy of calibration results.

2. PRINCIPLES AND METHODS OF CIRCULAR SCALE CALIBRATION. CLASSIFICATION OF MEASUREMENT PROCEDURES

The legislatively established unit of a plane angle, according to the decisions of the XIth (1960) and XIXth (1989) CGPM, is *the radian*. However this unit is of «non-measurement» nature. Really, the goniometric devices, which based on measurement of *length* of the corresponding arch and its *radius* (with the subsequent calculation of *length/radius* quotient), are not made. Therefore, in the further exposition, the plane angle is understood as «sector of the one-dimensional view» of potential observer at his presence in a point which is taken to be vertex of angle [5]. That operational definition of angle leans *on concept of rotation around the fixed axis.* Contrary to the known mathematical definition of angle (as a part of a plane), the above-mentioned definition does not use indefinitely long objects.

The definition given above generates the following *three principles* of precise angular measurements:

(i) the angular measurand is an angle of rotation of the material carrier around the fixed axis; a plane of rotation we shall name a plane of measurements;

(ii) the scale being formed during measurement is closed;

(iii) the zero mark of a scale (origin of angles) is conditional one.

Introduction of the closed (circular) scale permits to design not only one measurement, but several angles, $\mathbf{A_1, A_2, \ldots, A_n}$, at a time, for example, all angles of the regular polyhedral prism. Traditionally, in this situation, apply *the consecutive* scheme is used, i. e., *direct measurements* of each required angle, $\mathbf{A}_i$, are carried out separately and independently from each other. These measurements carry out, as a rule, by direct comparisons with reference angles (at their presence). It is shown, in the presentation, that *the parallel* scheme is more effective, when requirements to accuracy of measurement results are increased. By this scheme, instead of angles $\{\mathbf{A}_i\}$, their *linear combinations* of a kind $\mathbf{d} = \mathbf{d}\,(i, j, k, \ldots) = \mathbf{A}_i \pm \mathbf{A}_j \pm \mathbf{A}_k \pm \ldots$ are measured. Then, through the proper processing the primary data, $\{\mathbf{d}\}$, results are found including estimations of accuracy parameters [3, 4, 6-8]. Efficiency of the "parallel" scheme is caused, first of all, by significant redundancy of the primary data which allows both to raise an *accuracy of final results* and a degree of *controllability of* measurement *conditions*. In astronomy and geodesy, the similar procedures, which are known since K. F. Gauss' times, are named *measurements with a balancing*. Leading metrological schools many countries use also the following terms: *«measurements in a closed series»* or *«combined measurements»* (English); *«gesamtmessung»* (German); *«mesurages combinatoires en séries fermées»* (France); *«совокупные»* (Russian).

Till now, the "parallel scheme" had rather limited distribution (as a rule, as concrete realizations in special fields). Actually, it has not received the further development. As it was mentioned above, the scheme continues to be applied toward the calibration of precise accurate quartz prisms which is carried out on the Russian state reference standard of a plane angle using two autocollimators.

The reasons are apparently that, firstly, there are blanks in the theory and, secondly, organization and performance of such measurements are relatively more complicated, including obvious computational difficulties [3]. At the same time, as the examples of successful application of the scheme show [1-3], significant additional reserves for increase of measurement results accuracy can be used. So, the existence, in the primary data received in the course of calibration of precision accurate quartz prisms, of regular errors at a level 0.2″ is shown by similar measurements [3], which errors are unknown before. Taking account of these errors has allowed to raise accuracy of final results in 3-4 times. The modern advanced tool means (IT-tools) allow to apply more widely the specified circuit. Modern IT-tools permit to apply the specified scheme more widely.

Governing factors for the benefit of a choice of the "parallel" scheme are the following.

1) In the presentation, it is shown that, if the parallel scheme is used, the known features of errors behaviour can be taken into account more full already *at a level of measurement model*. Namely, constant systematic errors concerning which it is known only that they are present, can be included in the measurement design as preventing parameters and found together with angular measurands, $\{\mathbf{A}_i\}$.

2) Besides, if measurement procedure can be made *redundant* (it is the most important variant of the parallel scheme), the control of convergence is possible during process of experiment that leads to improvement *of accuracy parameters*.

4) Redundancy, in contrast to direct measurements, arises not by means of repetition of measurement procedure, but by more natural way, through the measurement design, due to a manifold combination of angular measurands. In so doing the number of possible combinations considerably surpasses quantity of angular measurands that allows adjusting redundancy over a wide range.

4) It is shown, in the presentation, that calibration of a circular scale by the parallel scheme always assumes presence of the 2nd auxiliary scale and is resolved in essence into comparison of two closed scales.

To carry out a choice between "consecutive" and "parallel" scheme correctly, the following classification attributes of calibration procedures are proposed depending on properties of an auxiliary scale and the chosen mutual positions of two scales at their comparison:

– uniformity (completeness) of the 2nd scale;

– quantity of mutual positions of two scales;

– quantity of reading devices (comparators);

– parameters of the used design (a degree of redundancy, sufficiency, an index of stability, symmetry, orthogonality, etc.)

The procedure of making the *adaptive* design of measurements provided with the stop rule (when the required level of result accuracy is achieved) can be proposed.

3. CONCLUSIONS

Classification of circular scales, and also the methods of their mutual calibration focused on increase of accuracy of results are proposed. The basic ways of increase of result accuracy are as follows:

– adaptive designing with the control of convergence of results;

– purposeful increase in redundancy of the primary data.

It is shown, that it is reached at the expense of:

– additional efforts on designing and the organization of the measurement procedure based on a combination of measurands;

– increase in time during which it is necessary to support of external conditions to be stable;

– growth of working hours in connection with necessity of the complicated processing experimental data demanding without fail programming of the processing procedure.

It should be pointed out that the mutual calibration of circular scales as the advanced example of the "parallel" scheme application represents *system measurements* [9].

Further it is supposed to execute the similar analysis and a substantiation of procedures potentially suitable for measurement of solid angle.

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