XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

EXAMPLE OF CALCULATING CORRECTIONS AND MEASUREMENT UNCERTAINTY IN THE PROCEDURE OF MEASURING SPHERES

Vedran Mudronja[1], *Sanjin* Mahović[2], *Biserka Runje*[3], *Srđan Medić*[4], *Gorana Baršić*[5]

[1] Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia, vedran.mudronja@fsb.hr
[2] Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia, sanjin.mahovic@fsb.hr
[3] Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia, biserka.runje@fsb.hr
[4] Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia, srdjan.medic@fsb.hr
[5] Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia, gorana.balic@fsb.hr

Abstract – Metrology practice confirms that very often by carrying out the corrections the result of measurement may contain greater error (deviation) than in the case of no corrections at all. Therefore, it is often assumed that the value of correction is zero, with assumed influences on the uncertainty of correction included in the uncertainty of the measuring results. In the field of length metrology the correction cannot be avoided in case of temperature influences or actions of the measuring force during the measurement procedure. Numerous experiences show that temperature correction is a questionable procedure due to a whole number of unknowns, and in case of more demanding measurements and the related more demanding uncertainties of measurement, one resorts to achieving of standard temperature conditions. However, the measurement results have to be corrected in case of the action of the measurement force, especially because the value of correction often exceeds the value of uncertainty of measurement. An example of measuring the sphere diameter, described in this work, fully supports the mentioned claim.

Keywords: measurement sphere, uncertainty of measurement, Monte Carlo simulations

1. INTRODUCTION

In case of applying the measurement procedure in which the object of measurement is subjected to a certain measuring force, it is impossible to avoid the correction of measurement results because of elastic deformations since the value of correction often exceeds the amount of the uncertainty of measurement of the measurement results.

In case of insufficient laboratory experiences and the related “fixed” items for reliable performance of correction, there is always certain doubt whether the correction has been performed properly and whether the uncertainty of correction has been “properly measured”. Comparison measurement can give efficient answer to this question. This paper presents a case in which the correction value exceeds the amount of measurement uncertainty due to the action of the measuring force. Further in the text, more detailed explanations are given regarding the calculation of correction and measurement uncertainty for a Rubin sphere of 1 mm diameter.

2. RESULTS OF SPHERE MEASUREMENT

Table 1 shows the results of comparison measurements of Rubin spheres of diameters 1 mm, 3 mm, 4 mm and 5 mm of the LABCOM group from 2004 and 2005. It should be noted that in 2005 the certificates of calibration indicated apart from the result the values of correction as well. The LABCOM (Laboratory Cooperation in Measurements) Group is an informal group of laboratories organized with the aim of cooperation in carrying out the comparison measurements (once a year) and experience exchange in calibration procedures. The group consists of 8 accredited laboratories from five European countries. The work of the Group includes also the Laboratory for Precise Measurement of Length (LFSB), where the authors are employed

Table 1. Measurement results for diameters of sphere from 2004 and 2005

No.	Laboratory	2004.		2005.					
		d, mm	*U*, µm	*d*, mm	*U*, µm	*d*, mm	*U*, µm	*d*, mm	*U*, µm
1	A	0,99960	0,25	3,00050	0,25	3,99990	0,25	5,00010	0,25
2	B	1,00000	0,50	3,00040	0,50	3,99960	0,50	5,00090	0,50
3	LFSB	1,00100	0,40	3,00049	0,60	3,99977	0,60	5,00098	0,60
4	C	0,99940	0,70	3,00050	0,50	3,00000	0,50	5,00090	0,50
5	D	0,99980	0,20	3,00030	0,20	3,99970	0,20	5,00090	0,20
6	E	-	-	3,00061	0,16	3,99991	0,16	5,00115	0,16
7	F	-	-	2,99950	0,80	3,99900	0,80	5,00030	0,80

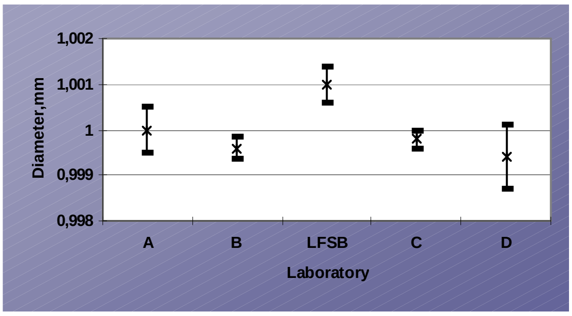

Figure 1: Graphical presentation of measurement results for 1 mm sphere from 2004

Table 1, i.e. the graphical presentation in Figure 1 (results of measuring 1mm spheres from 2004), shows that the result of LFSB has shifted to a positive regarding other four laboratories.
Strictly taken, this could be interpreted

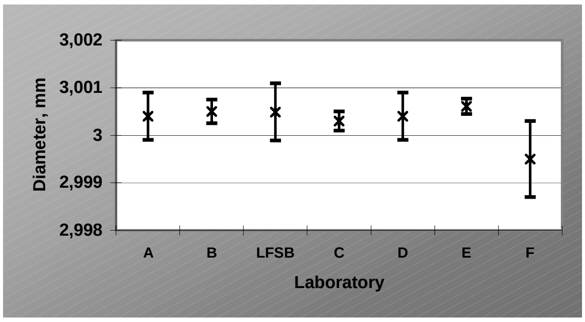

Figure 2: Graphical presentation of the measurement results for 3 mm sphere from 2005.

by the fact that LFSB has carried out the correction due to the action of the measuring force, whereas other laboratories did not do it (positive shift of LFSB results).

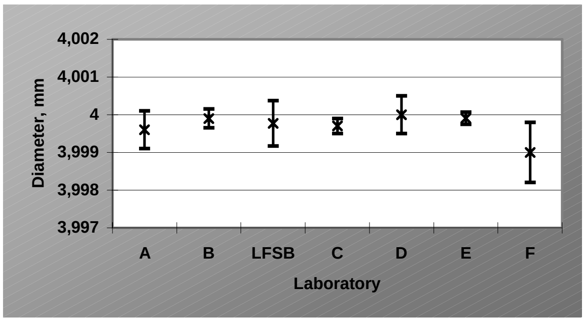

Figure 3: Graphical presentation of the measurement results for 4 mm sphere from 2005.

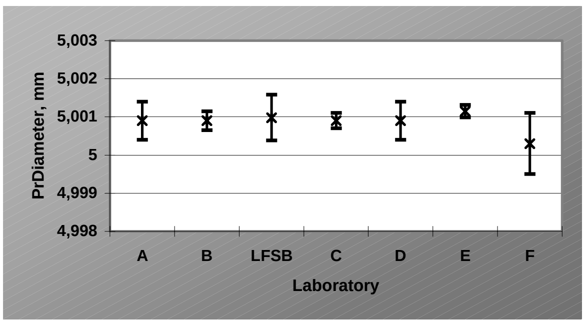

Figure 4: Graphical presentation of the measurement results for 5 mm sphere from 2005

Figures 2, 3 and 4, which present the results for the spheres of diameters 3 mm, 4 mm and 5 mm from 2005 show that all the results are at a uniform level as different from the results in 2004. This could be interpreted if all the laboratories had carried out the correction procedure due to the influence of the measuring force. The exception is the result of the laboratory F which did not participate in the 2004 comparison.

3. CALCULATION OF UNCERTAINTY OF MEASUREMENT

The uncertainty of measurement was calculated by Monte Carlo method (MCS) with M = 100000 simulations. The basic characteristics of the MCS method can be listed as follows:

- input values defined by various functions of probability density (pdf);
- probability density functions of input values are combined and form the experimental pdf measured (output) values;
- the estimate of the output value, and the interval for a certain probability P are estimated from the experimental pdf output values.

<u>Mathematical measurement model:</u>

$$d_x = d_{ix} + \delta d_{ix} + \delta d_T + \delta d_E + \delta d_A \qquad (1)$$

d_x	-	Actual sphere diameter at 20 °C
d_{ix}	-	Measured sphere diameter
δd_{ix}	-	Correction for the error of indication of the measuring device
δd_T	-	Correction due to temperature effects
δd_E	-	Correction due to elastic deformation
δd_A	-	Correction due to Abbe error

<u>Uncertainty of repeatability of measuring the sphere diameter</u> $u(d_{ix})$

The estimate of uncertainty of repeatability of measuring the sphere diameter contains the influence of the sphere geometry (deviation from the shape), as well as the influence of repeatability of the measuring device, and uncertainty of determining the reference value on the measuring device. It has been found that the standard uncertainty of the repeatability of measuring the sphere diameter amounts to: $u(d_{ix}) = 0{,}15$ µm.

Uncertainty of correction due to error of measuring device reading $u(\delta d_{ix})$

The uncertainty of correction due to the error of the measuring device reading $u(\delta d_{ix})$ results from the manufacturer's specifications and amounts to:

$$U(\delta d_{ix}) = (0{,}35 + 1 \cdot d)\ \mu\text{m},\ d \text{ u m} \tag{2}$$

Assuming the rectangular distribution the standard uncertainty of correction due to the error of the measuring device reading amounts to:

$$u(\delta d_{ix}) = (0{,}20 + 0{,}58 \cdot d)\ \mu\text{m},\ d \text{ u m} \tag{3}$$

Uncertainty of temperature correction $u(\delta d_T)$

The uncertainty assigned to temperature correction (expression 4) consists of the uncertainty of measurement of sphere temperature and the measuring scale temperature $u(\overline{\delta t})$ and the uncertainty of knowing the amounts of the coefficients of temperature expansion $u(\overline{\alpha})$.

$$\delta d_T = d \cdot \overline{\alpha} \cdot \overline{\delta t} \tag{4}$$

where:

d	-	Nominal sphere diameter
$\overline{\alpha}$	-	Average linear coefficient of temperature expansion of the sphere and measuring scale
$\overline{\delta t}$	-	Average temperature deviation of the sphere and measuring scale from 20°C

Standard uncertainty was determined in the amount of $u(\delta d_T) = 1 \cdot d$ μm, d u m

Uncertainty of correction due to elastic deformations $u(\delta d_E)$

During the procedure of measuring the sphere diameter, there is contact between two flat surfaces and the sphere. In the measuring system the probes with flat carbide surfaces are used, whereas the sphere is made of Rubin. The known Hertz formulae yield the amount of deformation for the realized contact according to the following expression:

$$\delta d_E = 0{,}8255 \cdot \sqrt[3]{\frac{F^2}{r}\left(\frac{1-\mu_1^{\,2}}{E_1} + \frac{1-\mu_2^{\,2}}{E_2}\right)^2} \tag{5}$$

where:

E_1 – Young′s modulus for stylus tip material , N/m^2

E_2 – Young′s modulus for sphere material, N/m^2

r – sphere radius, m

F – measuring force, N

μ_1 – Poisson′s ratio for stylus tip material

μ_2 – Poisson′s ratio for sphere material

The values of the Poisson's coefficient and Young's modulus depend on the composition of the material, and on the specific process of material hardening. According to the literature data, it may be assumed that the uncertainty of the mentioned constants amounts to about 3%.

The input values x_i for the sphere of diameter 1mm are defined in the probability density functions $g(x_i)$ as presented in Table 2. The probability density function of the output value δd_E has been simulated by the MCS method with M = 100000 simulations.

Table 2: Input values and probability density functions in simulation of value δd_E for sphere of 1mm diameter

Input value x_i		Probability density function $g(x_i)$
Measuring force	F	Normal distribution (M; 2,33 N; 0,001 N)
Sphere radius	r	0,5 mm
Young′s modulus for stylus tip material	E_1	Rectangular distribution (M; $5{,}335 \cdot 10^{11}$ N/m^2 ; $5{,}665 \cdot 10^{11}$ N/m^2)
Young′s modulus for sphere material	E_2	Rectangular distribution (M; $4{,}171 \cdot 10^{11}$ N/m^2; $4{,}429 \cdot 10^{11}$ N/m^2)
Poisson′s ratio for stylus tip material	μ_1	Rectangular distribution (M; 0,2231; 0,2369)
Poisson′s ratio for sphere material	μ_2	Rectangular distribution (M; 0,2813; 0,2970)

For the sphere of nominal diameter 1mm the correction due to elastic deformation amounts to 0,90 μm, and the estimated standard deviation 0,008 μm. The output value δd_E is within the interval:

[(δd_E)$_{0,025}$ = 0,885 μm; (δd_E)$_{0,975}$ = 0,913 μm] with P = 95%.

Uncertainty of correction due to Abbe error $u(\delta d_A)$

In the set measuring system the correction value of Abbe error equals zero, δd_A = 0, whereas the uncertainty of Abbe error $u(\delta d_A)$ is not zero. Based on the testing of the device and experience of the laboratory, Abbe error in the amount of ±0,05 μm is estimated.

The uncertainty $u(\delta d_A)$ assuming rectangular distribution of probability, thus, amounts to:

$$u(\delta d_A) = 0{,}028\ \mu\text{m}.$$

Probability density function of output value d_x

The probability density function of the output value d_x has been simulated by MCS method with M = 100000 simulations. The input values x_i for the sphere of nominal diameter of 1mm are defined by the probability density functions $g(x_i)$ as presented in Table 3.

Table 3. Input values and pdf in simulating the value d_x for 1 mm sphere

Input value x_i		Probability density function $g(x_i)$
Measured sphere diameter	d_{ix}	Normal distribution (M; 1,000 mm; 0,15 µm)
Correction for the error of indication of the measuring device	δd_{ix}	/Rectangular distribution (M; -0,20 µm; 0,20 µm)
Temperature correction	δd_T	Normal distribution (M; 0 µm; 0,001 µm)
Elastic deformation correction	δd_E	Triangular distribution (M; 0,885 µm; 0,913 µm)
Abbe error correction	δd_A	Rectangular distribution (M;-0,05 µm; 0,05 µm)

Probability density function $g(d_x)$ for sphere of nominal diameter 1 mm is shown in Figure 5.

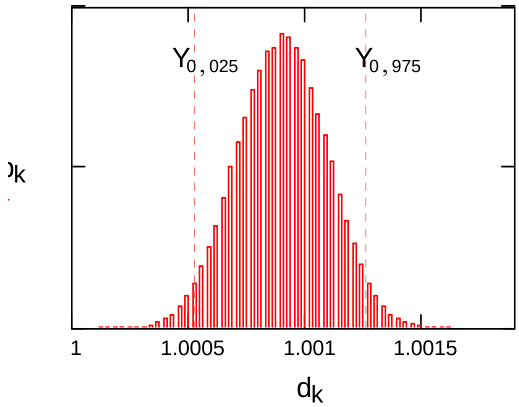

Figure 5. Probability density function $g(d_x)$ for 1 mm sphere

The estimated standard deviation of the output value $g(d_x)$ for the sphere of nominal diameter 1mm amounts to 0,191 µm. The output value d_x is within the interval:

$$[(d_x)_{0,025} = 1{,}00053 \text{ mm}; (d_x)_{0,975} = 1{,}00127 \text{ mm}] \text{ with } P = 95\%$$

Table 4 shows the summarized overview of the uncertainty elements, correction values and the respective standard uncertainties, and the value of the expanded uncertainty of measurement in measuring the diameter of a Rubin sphere of 1mm.

Table 4: Uncertainty budget for 1mm sphere

Input value x_i		Type of uncert.	Correc, µm	$u(x_i)$ µm
Measured sphere diameter	d_{ix}	A	0	0,15
Correction for the error of indication of the measuring device	δd_{ix}	B	0	0,20
Temperature correction	δd_T	B	0	0,001
Elastic deformation correction	δd_E	B	0,9	0,014
Abbe error correction	δd_A	B	0	0,03
Expanded uncertainty U = 0,40 µm; k=2 ; P= 95%				

It may be noted that the value of correction due to elastic deformation (δd_E = 0,9 µm) is greater than the total uncertainty of measurement (U = 0,4 µm), and that in no way can the correction be "hidden" within the uncertainty of measurement.

4. CALCULATION OF CORRECTIONS DUE TO ELASTIC DEFORMATIONS

The amounts of corrections and the respective uncertainties for the measuring force F = 2,33 N, which have been implemented in the concrete cases by LFSB for Rubin and carbide, are presented in Table 5.

Table 5: Correction value (F=2,33 N)

Sphere diameter mm	Material: Rubin Correct. µm	Material: Rubin Correct. Uncertain, µm	Material: Carbide Correct. µm	Material: Carbide Correct. Uncertain., µm
0,1	1,94	0,030	1,8	0,028
0,2	1,54	0,024	1,43	0,022
0,3	1,34	0,021	1,25	0,019
0,4	1,22	0,019	1,13	0,018
0,5	1,13	0,018	1,05	0,016
1	0,90	0,014	0,83	0,013
3	0,62	0,010	0,58	0,009
4	0,57	0,009	0,53	0,008
5	0,53	0,008	0,49	0,008
10	0,42	0,007	0,39	0,007

It should be noted that the uncertainties of corrections, which are components of the total uncertainty of measurement, are substantially smaller than the corrections themselves. The problem may arise in measuring (performing correction) of the small diameter beads (up to 1mm).

This is presented in diagram in Figure 6 where there is high curve gradient (correction) for the force F = 2,33 N for the sphere diameters smaller than 1mm.

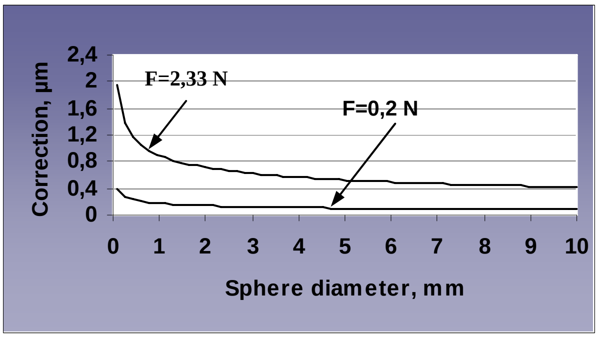

Figure 6: Graphical presentation of the amount of correction for different sphere diameters

A much better situation (more reliable correction) is in case of applying smaller measuring force (F = 0,2 N). The uncertainty of correction, namely, in the area of sphere diameter of up to 1mm with F = 2,33 N significantly depends on knowing the value of the Poisson's coefficient and the module of elasticity. Reliable knowing of the values of the mentioned constants in the range of ± 3% from the nominal value has no major influence on the uncertainty of correction, which cannot be stated if the values of constants are in the range of ± 20% of the nominal value. (Figure 7.)

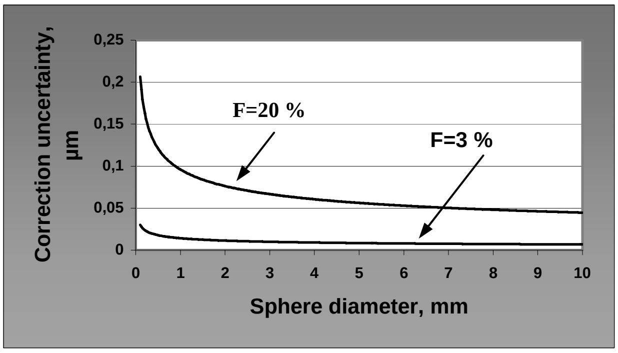

Figure 7: Graphical presentation of the amounts of uncertainty of correction (*F* = 2,33 N)

5. CONCLUSION

This work indicates only theoretically the issues related to the calculation of corrections and the uncertainty of measurement in measuring the spheres. The validation of the calculation is possible only by comparison with other laboratories, which in the concrete case has been performed.

Only by moving out of one's own laboratory, i.e. by comparison with the others, can one gain confidence in the mathematical procedures and literature claims of various parameters which are used in performing corrections, i.e. procedures of estimating he uncertainty of measurement. The improvement of the measuring procedure, i.e. reduction of the uncertainty of measurement is as a rule a time-consuming task, requiring a lot of patience and cooperation with others. Uncertain uncertainty estimated "roughly by heart" results only in a poor image of the laboratory and brings damage.

REFERENCES

[1] M. G. COX and P. M. HARRIS, "The planned supplemental guide to the GUM: Numerical methods for propagation distributions, Euromet Length Workshop, October 2001.

[2] B. AČKO; V.: MUDRONJA: Bilateral comparison of gauge block measurement by mechanical comparison between the Slovenian and Croatian national laboratories, Transactions of FAMENA, XXVI-1, 2002, pp.43-50.

[3] MUDRONJA,V.: “Il ruolo del Centro di Taratura SIT nel sistema metrologico croato”, Tutto_Misure, Anno II, N.02, 2000. pp.181.

[4] MUDRONJA, V.; RUNJE, B.; BARŠIĆ, Z.: “Reproducibility of Measurement Results of Gauge Blocks over a Period of 15 Years”, Proceedings of the XVI IMEKO World Congress, Volume V, TC-7, TC-8,TC-11, Vienna, Austria, 2000. pp.277-280.

[5] BERTOZZI, R.; GAMBERI, L.; MUDRONJA V.: Un confronto sperimentale tra laboratori accreditati, Tutto_Misure, Anno IV, N.01, 2002. pp.85-89.