XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

DETERMINING THE CHARACTERISTICS OF SENSORS DISPLACEMENT IN INSTRUMENTS FOR ROUNDNESS MEASUREMENTS

Dariusz Janecki [1], *Stanisław Adamczak* [2], *Marek Cabaj* [3]

[1] Kielce University of Technology, Kielce, Poland, djanecki@tu.kielce.pl
[2] Kielce University of Technology, Kielce, Poland, adamczak@tu.kielce.pl
[3] Kielce University of Technology, Kielce, Poland, marek.cabaj@wp.pl

Abstract: In the paper "A new approach to the examination of the displacement sensor characteristics" published in Measurement, vol. 28, pp. 261-267", Żebrowska-Łucyk proposes a new, interesting method for measuring the characteristics of displacement sensors. The method consist in exciting the sensor tip with a sinusoidal signal having a known amplitude. In this paper, the authors modify the method so that it is possible: (1) to conduct measurements by means of a typical sensor tip with a circular cross-section, (2) to eliminate the necessity of possessing an additional measuring eccentricity standard, (3) to determine the sensor characteristics on the basis of non-closed roundness profiles.

Keywords: sensor characteristics, roundness measurement.

1. INTRODUCTION

Measurements of roundness or cylindricity are generally preceded by centering and leveling of an object on the measuring table. It is necessary to make the object axis coincide with the table axis of rotation (for devices with a rotary table) or with the sensor axis of rotation (for devices with a rotary sensor). In this way, measurements can be performed in small measuring ranges and with a higher accuracy. Also, it enables to eliminate the geometrical nonlinearities resulting from the object eccentricity [2] and the nonlinearities of the measuring sensor characteristic.

The centering and leveling operations are time-consuming and performed in stages. They require good manual skills of an operator, as well. They are particularly troublesome if we are to assess cylindricity or roundness of a non-closed profile as a circle sector [3], for instance, the cross-sections of rolling bearing races. As a result of the development of electronics, it is possible to apply measuring systems with high resolution. Digital methods of signal processing help to eliminate the geometrical nonlinearities. Thus, the centering and leveling of a workpiece could not be necessary if the characteristic of the measuring sensor is known.

The knowledge of sensor characteristics can also be utilised in measurements of the surface of rotary objects with varied diameters, e.g. cones, barrels, as well as in comparative measurements of object dimensions, e.g. radii, [3]. Moreover, by applying an electronic system of signal shifts, we are able to perform accurate measurements at various points of the sensor characteristic, rather than in the vicinity of a point where the sensor is calibrated.

The new method for measuring the characteristics of a displacement sensor proposed in Ref. [1] involves exciting the sensor tip with a sinusoidal signal with a known amplitude. It is essential to determine the relationships between the coefficients of the sensor characteristic and the coefficients of the trigonometric Fourier series of a measured signal. The method requires applying a special standard consisting of two cylindrical discs arranged eccentrically. Moreover, it is necessary to use a sensor tip with a rectangular cross-section to cause exact sinusoidal excitation.

This work proposes a few modifications to the above method so that it can be used with instruments for measuring roundness and cylindricity. The method should also be adequate for measurements of non-closed profiles without applying extra standards or tips. The modifications allow employing a tip with a circular cross-section and determining the sensor characteristic on the basis of a profile being the circle sector. As the procedures of the sensor characteristic determination and the sensor calibration are separated, an ordinary roundness standard or any cylindrically shaped object with a slight roundness deviation can be used to analyze the sensor deflections.

2. MODELS OF SENSORS EMPLOYED IN INSTRUMENTS FOR ROUNDNESS MEASUREMENTS

Let R_w denote an indication of a measuring sensor. In an ideal case, when the sensor characteristic is linear, the sensor indications are in proportion to the changes in the object radius R_m, thus

$$R_m = kR_w .$$

In the above equation, the coefficient k is the calibration coefficient, which is determined during the sensor calibration by using standards with known values of deviations, the standards being regular equipment of the measuring device. We employ chamfered standards, standards with an oval cross-section, roughness standards with a known value of the coefficient Ra, and grooved standards where the groove depth is known. The sensors can also be calibrated applying gauge blocks. In practice, the sensor characteristic is nonlinear. It is best to approximate it by means of the polynomial:

$$R_m = k(R_w + a_1 R_w^2 + a_2 R_w^3 + ... + a_{n-1} R_w^n) \quad (1)$$

The number n will be called the order of approximation. The objective is to establish the coefficients a_1, a_2,..., a_{n-1} of polynomial (1). Assume that the sensor characteristic determination and the sensor calibration are performed independently of each other. Thus, for further considerations, we can assume that the calibration coefficient is a known quantity. For the sake of simplicity, it is assumed to be equal to one:

$$k = 1 . \quad (2)$$

According to [1], the coefficients of the sensor characteristics are established by measuring an object with a small roundness deviation placed eccentrically in relation to the axis of rotation. Assume that the value of eccentricity is selected in such a way that the sensor tip displacement during a measurement comprises the entire measurement range. A displacement of the sensor tip during a measurement depends on the angle φ of the table rotation (for devices with a rotary table) or the sensor (for devices with a rotary sensor) as well as the sensor structure and position in relation to the workpiece. Now, consider instruments with a rotary table. Let XYZ be a system of Cartesian coordinates where the Z axis coincides with the table axis of rotation. Assume for simplicity that the sensor tip has spherical shape. Then, let R denote a sum of the workpiece radius and the sensor tip radius. The equation of the cylinder area for a predetermined angle of rotation φ has the following form

$$(x - e_x \cos\varphi - e_y \sin\varphi)^2 + (y - e_x \sin\varphi + e_y \cos\varphi)^2 = R^2 , \quad (3)$$

where e_x, e_y are coefficients of the workpiece cross-section center for the zero value of the angle φ. The position of the object in the XY plane is shown in Fig. 1.

Consider three cases of operation of instruments with a rotary table.

CASE 1. During a measurement, the sensor tip moves along a straight line perpendicular to the axis of rotation Z (see Fig. 2). Assume that the straight line of the sensor tip displacement is described with the following equations

$$x = R + R_m - d , \; y = 0 , \quad (4)$$

where d is a value of the sensor indications at zero values of e_x and e_y. In order to determine the relationship $R_m(\varphi)$, we need to solve a system of equations (3), (4), hence

$$R_m(\varphi) = d - R + e_x \cos\varphi + e_y \sin\varphi + \sqrt{R^2 - (e_x \sin\varphi - e_y \cos\varphi)^2} \quad (5)$$

This type of sensor is employed, for example, in the MDL-10 device designed and used at the FŁT Kraśnik S.A, Poland to measure the roundness of cross-sections of races of large bearing rings [3].

The configuration shown in Fig. 2 is not suitable for measuring roundness of inner rings or holes. In such a case, it is recommended to apply a sensor with a rotary arm, which ensures better access to the workpiece.

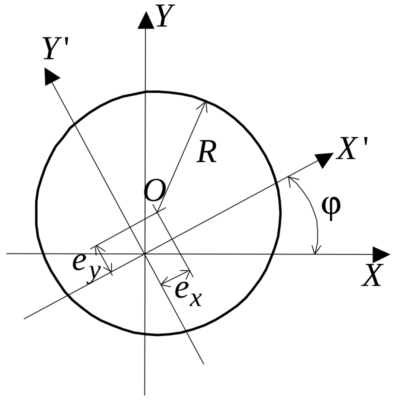

Fig. 1. Position of the workpiece in the XY plane (the X'Y' system is associated with the rotary table, and O denotes the workpiece center).

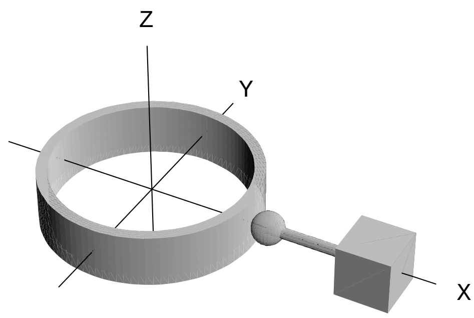

Fig. 2 Measuring sensor with a tip moving along a straight line.

CASE 2. The sensor tip moves around a circle lying in a plane intersecting the axis of rotation Z (see Fig. 3). In addition, assume that the position of the arm is parallel to the Z-axis at zero values of e_x and e_y. Denote the length of the sensor arm by L. From the assumptions we can see that the senor tip moves around a circle described by the following equations

$$z = L - L\cos\alpha, \; x = R + L\sin\alpha, \quad \alpha = \arctan\frac{R_m - d}{L}, \; y = 0, \quad (6)$$

where α is the angle between the sensor arm and the Z axis. Solving the system of equations (3), (6) we obtain the same relationship between radius changes R_m and the angle φ. As opposed to case 1, however, the model (1) includes now the nonlinearities connected with the sensor arm rotation. The described sensor system is commonly used in devices for measuring roundness and cylindricity.

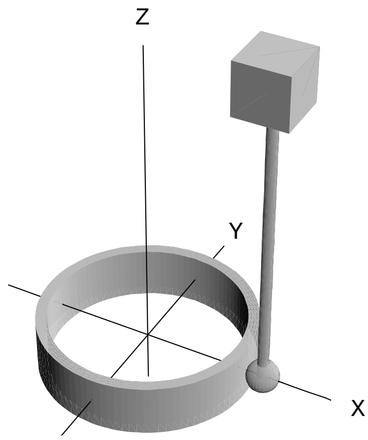

Fig. 3. Sensor with a rotary arm where the tip moves around a circle lying in the plane including the axis of rotation.

CASE 3. The sensor tip moves around a circle lying in a plane perpendicular to the axis of rotation Z (see Fig. 4). Assume that the arm is parallel to the Y axis at zero values of e_x and e_y. From the assumptions we conclude that the sensor tip moves around a circle described with the following equations

$$y = L - L\cos\alpha,\ x = R + L\sin\alpha,$$
$$\alpha = \arctan\frac{R_m - d}{L},\ z = 0, \tag{7}$$

where α is the angle between the sensor arm and the Y axis. The relationship between the shift R_m and the angle φ can be established by solving the system of equations (3), (7). The apparent form of the relationship $R_m(\varphi)$ is omitted due to its complexity. The sensor configuration presented here is used, for instance, in the Rotary-Talysurf by Taylor Hobson. The device is employed for measuring roundness and roughness in race cross-sections of bearing rings with a diameter up to 70 mm.

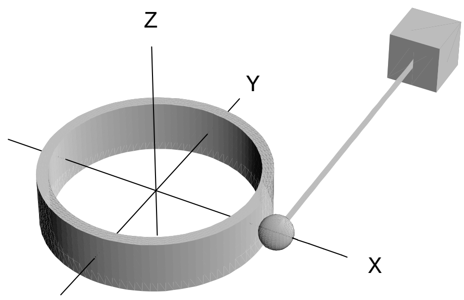

Fig. 4. Sensor with a rotary arm where the tip moves around a circle lying in a plane perpendicular to the axis of rotation.

In each case, the relationship $R_m(\varphi)$ can be written as

$$R_m(\varphi) = d + f(e_x, e_y, \varphi), \tag{8}$$

with $f(0,0,\varphi) = 0$. By performing a numerical analysis for selected values of e_x, e_y, R and L, we can show that in spite of a completely different analytical form, the properties of the function f are very similar in each of the three cases being considered. In the expansion of f into the Fourier series, the first and the second harmonic components are predominant.

3. DETERMINING THE COEFFICIENTS OF THE SENSOR CHARACTERISTIC

The sensor characteristic can be determined basing on a sequence of profile values $\{R_{wi}\}$ established for different values of the angle of rotation $\{\varphi_i\}$, $i = 1,2,...,M$. Let us formulate the following index characterizing the goodness of fit of the sensor characteristic and the measuring data

$$J = \sum_{i=1}^{M}\left(R_{wi} + a_1R_{wi}^2 + ... + a_{n-1}R_{wi}^n - d - f(e_x, e_y, \varphi_i)\right)^2. \tag{9}$$

The index J is a function of the parameters $a_1,...,a_{n-1}, d$, e_x, e_y. Their values can be determined by formulating the optimization task

$$\min_{a_1,...,a_{n-1},d,e_x,e_y} J(a_1,...,a_{n-1},d,e_x,e_y), \tag{10}$$

the best solution of which is iteration using the following algorithm.

STEP 1. Assume that the initial values are

$$e_x^o = 0,\ e_y^o = 0. \tag{11}$$

STEP 2. By applying the approximation

$$f(e_x, e_y, \varphi_i) \cong f(e_x^o, e_y^o, \varphi_i) + f'_{e_x^o}\cdot(e_x - e_x^o) + f'_{e_y^o}\cdot(e_y - e_y^o), \tag{12}$$

where $f'_{e_x^o} = \partial f(e_x^o, e_y^o, \varphi_i)/\partial e_x^o$, $f'_{e_y^o} = \partial f(e_x^o, e_y^o, \varphi_i)/\partial e_y^o$, the index J is written in the form of

$$J = \sum_{i=1}^{M}(v_i + w_i\theta)^2,\ \ v_i \in \mathbf{R}^1,\ \ w_j, \theta \in \mathbf{R}^{n+2}, \tag{13}$$

where

$$v_i = R_{wi} - f(e_x^o, e_y^o, \varphi_i) + f'_{e_x^o}\cdot e_x^o + f'_{e_y^o}\cdot e_y^o, \tag{14}$$

$$w_i = [R_{wi}^2\ ...\ R_{wi}^{n+2}\ \ -1\ -f'_{e_x^o}\ \ -f'_{e_y^o}], \tag{15}$$

$$\theta = [a_1 ... a_{n-1}\ d\ e_x\ e_y]^T. \tag{16}$$

STEP 3. The approximate values of parameters θ are assessed using the following relationship

$$\theta = -\left(\sum_{i=1}^{M} w_i^T w_i\right)^{-1}\left(\sum_{i=1}^{M} w_i^T v_i\right). \tag{17}$$

STEP 4. If the solution is not sufficiently accurate, assume that $e_x^o = e_x$, $e_y^o = e_y$ and return to Step 2.

4. SIMULATION RESULTS

The aim of the experiments was to provide ready procedures for determining the coefficients of the sensor characteristic using (11)-(17), to perform simulations for the exaplary data, and to assess the speed of the algorithm convergence. The calculations were conducted with the aid of the *Mathematica* program and presented in the *Mathematica Notebook* form. Due to space limitations, most results of the indirect calculations are hidden. Consider the sensor configuration in Fig. 4. First, derive a formula for the function $f(e_x, e_y, \varphi)$. By solving Eqs. (3) and (6), we determine $\sin\alpha$ and $\cos\alpha$.

```
In[1] := sol1 =
           Solve [
            {(x – ex Cos [φ] – ey Sin [φ])^2 +
                (y – ex Sin [φ] + ey Cos [φ])^2 == R^2,
             y == L – L C, x == R + L S, 1 == S^2 + C^2},
            {x, y, S, C}];
```

Four different solutions are obtained. Note that for $e_x = e_y = 0$ the angle of inclination of the sensor arm equals zero. Thus, $\sin\alpha = 0$ and $\cos\alpha = 1$. Now, it is crucial to check which of the solutions satisfies this condition.

```
In[2] := FullSimplify [{S, C} /. sol1 /. {ex → 0, ey → 0},
           {R > 0, L > 0}]
```

Out[2]= $\left\{\{0, 1\}, \left\{-\frac{2LR}{L^2+R^2}, -1+\frac{2L^2}{L^2+R^2}\right\}\right\}$

It is clear, that the first solution corresponds to the sensor configuration presented in Fig. 4. We shall determine the function f. Note that its analytical form is quite complex.

```
In[3] := f = L S /. sol1 [[1]];
         f /. {ex → e_x, ey → e_y} // FullSimplify //
          TraditionalForm
```

Out[4]= $$((L - \sin(\varphi)\,e_x + \cos(\varphi)\,e_y)\sqrt{((-e_x^2 + 2\,(R\cos(\varphi) + L\sin(\varphi))\,e_x + 2\,L\,R - e_y\,(2\,L\cos(\varphi) - 2\,R\sin(\varphi) + e_y))(e_x^2 - 2\,(R\cos(\varphi) + L\sin(\varphi))\,e_x + 2\,L\,R + e_y\,(2\,L\cos(\varphi) - 2\,R\sin(\varphi) + e_y)))} - (R - \cos(\varphi)\,e_x - \sin(\varphi)\,e_y)\,(2\,L^2 + e_x^2 - 2\,(R\cos(\varphi) + L\sin(\varphi))\,e_x + e_y\,(2\,L\cos(\varphi) - 2\,R\sin(\varphi) + e_y)))/(2\,(L^2 + R^2 + e_x^2 - 2\,(R\cos(\varphi) + L\sin(\varphi))\,e_x + e_y\,(2\,L\cos(\varphi) - 2\,R\sin(\varphi) + e_y)))$$

Also, it is necessary to determine the partial derivatives of the function f.

```
In[5] := Dfex = D[f, ex]; Dfey = D[f, ey];
```

Now, assume that the values of the coefficients of the sensor characteristic are, for example

```
In[6] := a1 = 0.01; a2 = –0.1; a3 = 0.03;
```

and that

```
In[7] := R = 20; L = 60;
```

To generate a sequence of samples of a measured profile we need an inverse characteristic of the sensor.

```
In[8] := sol2 = Solve[x + a1 x^2 + a2 x^3 + a3 x^4 == y, {x}];
```

In this case, too, we obtain four solutions. In order to check which of them is adequate, we use the fact that the characteristic goes through point (0,0).

```
In[9] := (x /. sol2) /. y → 0
Out[9]:= {2.85933 – 2.408 i, 2.85933 + 2.408 i,
          –2.38534, 4.44089 × 10^-16}
```

Thus, the inverse characteristic is described by the fourth solution.

```
In[10] := g[y_] = x /. sol2[[4]];
```

We shall generate a measured profile. Assume that we collect

```
In[11] := M = 2^10;
```

of profile samples, which are uniformly distributed in the angle range $\varphi \in (-\pi/4, \pi/4)$:

```
In[12] := φℓ = π Range [–M / 4, M / 4] / M // N;
          Rmℓ = 0.1 + f /. {ex → 0, ey → 1} /. φ -> φℓ;
          Rwℓ = g[Rmℓ];
```

Then, the algorithm determining the coefficients of the characteristic is implemented for $n = 5$.

```
In[15]:= n = 5;
         θ = Table [0, {n + 2}];
In[17]:= For [i = 1, i ≤ 3, i++,
          v =
           Rwℓ –
            (f – Dfex ex – Dfey ey /.
               {ex → θ[[n + 1]], ey -> θ[[n + 2]]} /.
             φ -> φℓ);
          w = Join [Table [Rwℓ^i, {i, 2, n}],
             {–1 + 0 φℓ, –Dfex, –Dfey} /.
               {ex → θ[[n + 1]], ey -> θ[[n + 2]]} /.
              φ -> φℓ];
          θ = –Inverse [w.Transpose [w]].(w.v);
          Print [θ]]
          {–0.0139876, –0.099286, 0.0347172,
           –0.00166182, 0.0764533, –0.00160353, 0.996441}
          {0.00999956, –0.0999998, 0.03,
           –1.93249 × 10^-8, 0.0999993, 3.44822 × 10^-7, 1.}
```

$\{0.01, -0.1, 0.03, 9.16771\times10^{-10}, 0.1, -2.93367\times10^{-8}, 1.\}$

Note that the proper solution was found as early as in the third iteration of the algorithm.

5. SENSOR CHARACTERISTIC MEASUREMENT

The method described above was used to determine the characteristic of an inductive sensor in the Rotary Talysurf. A cylinder with $R = 19$ mm for which roundness deviation did not exceed 0.2 μm was employed as a standard of roundness. The measuring range for the device sensor was ±200 μm. This device ensures measurement of a profile in a range of angle variations of up to 210 degrees. Figure 5 shows a sequence of samples of a profile used for determining the characteristic. Note that the sensor deflection comprises the entire available measurement range . The sensor characteristic was approximated with a polynomial of the $n = 4$ order. Figure 6 presents the error of nonlinearity of the identified sensor characteristic. We can see that the error reaches 1% of the measuring range.

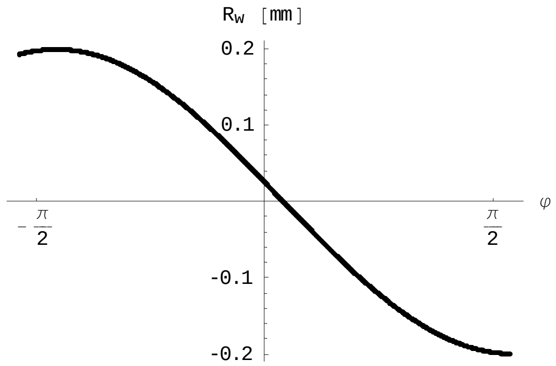

Fig. 5. Sequence of profile samples used for determining the sensor characteristic.

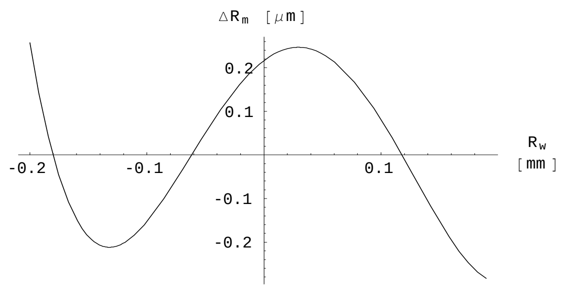

Fig. 6. Error of sensor nonlinearity $\Delta R_m = R_m - R_w$.

6. CONCLUSIONS

The work discusses a method for determining the characteristics of a displacement sensor used in devices for roundness and cylindricity measurement. An advantage of this method is that it is not necessary to apply additional standards or laboratory stands. Measurements are performed directly on the instrument using its regular equipment.

The method accuracy is affected by the assumptions concerning the sensor configurations. One of the requirements is that the line of the sensor tip displacement and the axis of rotation of the sensor system presented in Fig. 2 should intersect. Before each measurement, it is essential to set the sensor precisely, otherwise the measured profile of an object being in a eccentric position will be greatly affected by the shift value of the sensor displacement line in relation to the rotation axis [2]. The effect of this shift can be reduced by applying standards with considerable diameters, for which an excitation is more similar to a sinusoidal excitation. It is not always possible, though. For instance, while using a Rotary Talysurf, the range of the measured radii does not exceed 20 mm. The standard errors, however, have a much smaller influence on the measurement results. Moreover, it is possible to establish the standard errors before the proper measurement while setting the standard centrically. The data can be used for modeling the relationship $R_m(\varphi)$.

It is important that in a system with a rotary arm (Fig. 3) the assumed model of nonlinearity is not adequate for measurements of rotary surfaces with variations in the diameter, for instance, cones. The sensor deflection caused by a variation in the radius results in an additional displacement of the sensor tip along the Z axis. As it seems to have a considerable influence on measurement results, it needs to be included in the model. Some general information on the modeling of this type of nonlinearity can be found in Refs. [4], [5].

REFERENCES

[1] S. Żebrowska-Łucyk, New approach for examination of the displacement sensor characteristics, Measurement 28 (2000), 261-267.

[2] S. Żebrowska-Łucyk, Bezodniesieniowa metoda badania makrogeometrii powierzchni elementów obrotowych, Prace Naukowe Mechanika, z. 187, Oficyna Wydawnicza Politechniki Warszawskie, Warszawa 2001.

[3] S. Adamczak, D. Janecki, Modernizacja przyrządów do oceny struktury geometrycznej powierzchni. Cz. 3: Pomiary niezamkniętych zarysów kształtu, Mechanik 8-9/2001, 553-557.

[4] J.D. Garatt, A new stylus instrument with a wide dynamic range for use in surface metrology, Precision Engineering, July 1982, vol. 4, No. 3, pp. 145-151.

[5] S. Adamczak, D. Janecki, The complex profilometric measurement of non-closed form profiles of machine parts, Proc. International Conference on Measurement "Measurement 2003", Slovak Republic.