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# RECOMMENDATIONS ON THE MEASUREMENT OF CYLINDRICAL GEARS WITH COORDINATE MEASURING MACHINES 

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The accurate metrological analysis of the proposed measuring software enables to determine the errors of gears measurement with CMM, as well as to formulate recommendations on the chiose of the appropriate CMM with known uncertainty for the metrological task of gear measurement, where the gear is manufactured in certain accuracy class. Criterion of the metrological correctness allows that the final error would not exceed $10 \%$ of tolerance (or in some cases $20 \%$ of tolerance). In order to perform full metrological analysis of the measuring software accuracy it is needed to carry out several simulations that would enable to determine particular errors and their influence on the final measuring result. The analysis includes:
$>\quad$ analysis of all errors occuring in the measuring process,
> analysis of the influence of particular errors on the measured deviations,
determination of the relations describing particular errors,
determination of the uncertainty of the gear accuracy evaluation.

## 1. ACCURACY OF THE DETERMINATION OF PARTICULAR DEVIATION ON MEASURED GEAR

The accuracy of particular deviation of the measured gear may be described by formula:

$$
\Delta()=\Delta_{x 0, y 0}+\Delta_{p o z}+\Delta_{k o r},
$$

where:
$\Delta()$ - error of particular deviation measurement (i.e. involute profile, basic radius, tooth thickness, pitches, tooth line),
$\Delta_{x 0, y 0}$ - error of the coordinate system center determination,
$\Delta_{p o z}$ - error of the positioning of the probe tip during
the contact with the measured surface,
$\Delta_{k o r}$ - error of the correction applied in the algorithm.
The criterion of the metrologically correct measurement was following: maximal acceptable error should not exceed $10 \%$ of tolerance ( 0.1 T ) for the accuracy class of measured gear.

### 1.1. The error of the coordinate system center determination

$$
\Delta_{x 0, y 0}=f\left(N_{\mathrm{pkt} \text { pom }}, E, \Delta_{\mathrm{ksztaltu}}\right),
$$

where: $N_{\text {pkt pom }} \quad$ - number of the measuring points steadily distributed on the measured circle,
$E\left(M P E_{E} ; U_{3}\right) \quad$ - uncertainty of the CMM,
$\Delta_{\mathrm{kszalatu}} \quad$ - form deviations (ovality and 3-angular lobing as most common and most affecting the accuracy of circle center location determination).

The coordinate system of the measured gear is created with its basic geometrical elements. During technological process, the gear is formed around the establishing orifice or shaft of the gear, i.e. aroung the actual rotation axis of the gear. Hence, the reference element for the coordinate center establishment in correct coordinate measurement should be the same element. The $z$-axis is determined through three measurements of circles on three different levels of the orifice or shaft. The 3D line built on the centers of those orifices constitutes the $z$-axis, perpendicular to the main plane of the gear.

The minimizing of the error during the measurement of the centers of those circles is crucial for the accurate determination of the coordinate system. Any error made during this process will affect all the measured values. The numerous simulations had been performed for the measurement of circle and determination of its center's coordinates, in order to evalueate the accuracy of the determination.

The form deviations and CMM's uncertainty has been simulated. The following conditions have been assumpted:
$>$ accuracy of $\mathrm{CMM}-M P E_{E}(E)$ given by the producer,
$>$ tolerance of the circle corresponded with classes $5,6,7$ and 8 of accuracy,
$>$ form deviations: ovality and 3-granity as most undesirable, with various weight of influence; the simulations have proven that most undesirable situation occurs when the wieght of both deviations is equal - then the final form deviation is maximal,
$>$ random error distribution generated by CMM and measuring head,
$>$ the total sum of all assumed and simulated error should not exceed the tolerance of given accuracy class (the circle is made according the given class requirements).
Simulations have been performed for the circles of $10 \div 250 \mathrm{~mm}$ radiuses, tolerance $\mathrm{H} / \mathrm{h}$ and $5 \div 8$ accuracy class. The coordinates of the circle center have been calculated for various numbers of measuring points dustributed steadily on the circle, and for various uncertainty values of CMM.

Table 1. The influence of CMM's accuracy and the number of measuring points on the error of the circle center determination

| $E$ | 0.5 | 1.0 | 2.0 | 2.5 | 3.0 | 4.0 | 5.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> points 0.5    <br> 32 0.2 0.4 0.7 0.9 <br> 1.2 1.8 2.2   <br> 16 0.4 1.1 1.6 2.0 <br> 2.5 3.8 4.5   <br> 8 0.7 1.5 2.4 3.1 <br> 3.8 5.4 6.3   |  |  |  |  |  |  |  |

The accuracy of the circle center coordinates determination depends on the number of collected measuring points, uncertainty of the CMM and the distribution of the form deviation (see table 1 and fig. 1).


Fig.1. Influence of the CMM's accuracy and number of measuring points on the error of circle center coordinates

Decreasing the number of measuring points and application of the less accurate CMM leads to the increase of the circle center determination. Hence, the errors appearing during this process would set conditions of application of particular CMM with known uncertainty for the measurement of gears made in certain accuracy class according to the recommendations of the standard DIN 3962.

### 1.2. The probe tip positioning terror

$\Delta_{p o z} \quad$ - uncertainty of the measuring point location, equal to the uncertainty of CMM:

$$
\Delta_{p o z}=M P E_{E}\left(E ; U_{3}\right) .
$$

The error of the positioning of the probe tip during the CMM measurement is considered to be equal to $M P E_{E}\left(E ; U_{3}\right)$, because during the measurement, the movement in all directions (all axes) takes place.

### 1.3. Correction terror

$\Delta_{k o r}$ - correction error (superposition of the errors made during correction I and correction II applied in the algorithm).

The worked out algorithm with two correction minimizes the error of the measuring points location on the measured gear surface. The errors of those algorithms do not exceed $0.2 \mu \mathrm{~m}$. It is metrologically correct to consider them as zero ( $\left.\Delta_{k o r} \cong 0\right)$ in the following analysis of the software.

## 2. THE ERROR OF PARTICULAR DEVIATIONS' DETERMINATION

The accuracy of particular deviations' determination depends on the uncertainty of CMM, number of the measuring points during the coordinate system center establishment, and the form deviations of the orifice of shaft. The errors of particular values determination are presented in the table 2. For the most commonly used in Polish industry CMMs with uncertainty $\operatorname{MPE}_{E}\left(\mathrm{E} ; \mathrm{U}_{3}\right)= \pm 2.5 \mu \mathrm{~m}$, the final error of gear measurement is $3.4 \mu \mathrm{~m}$.

Table 2. Accuracy of determination of particular gear's deviation [ $\mu \mathrm{m}$ ] described with the formula (I.1)

| $\boldsymbol{\mu m} /$ described with the formula (I.1) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mumber of points | 0.5 | 1.0 | 2.0 | 2.5 | 3.0 | 4.0 | 5.0 |
| 32 | 0.7 | 1.4 | 2.7 | 3.4 | 4.2 | 5.8 | 7.2 |
| 16 | 0.9 | 2.1 | 3.6 | 4.5 | 5.5 | 7.8 | 9.5 |
| 8 | 1.2 | 2.5 | 4.4 | 5.6 | 6.8 | 9.4 | 11.3 |

## 3. RECOMMENDATIONS ON THE CHOICE OF CMM FOR THE GEAR MEASUREMENT

The determinated errors of particular deviations measurement allow to formulate recommendation for the metrologically correct measurement of gears manufactured in particular accuracy class. In the table 3, the recommendations of CMM choice are presented for
the criterion 0.1T, dependent on the accuracy class of the measured gear. So, the CMM with uncertainty $\pm 2.5 \mu \mathrm{~m}$ enables the measurement of the gears made in 9th accuracy class (modules 1-2), 8th accuracy class (modules 2-6), 7th accuracy class (modules 6-10) and 6th accuracy class (modules over 10), with the proposed software.

Table 3. Accuracy of CMM [ $\mu \mathrm{m}$ ]that may be applied for the gear

| 0.1 T | module (over - to) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Accuracy class | $1 \div 2$ | $2 \div 3.55$ | $3.55 \div 6$ | $6 \div 10$ | $10 \div 16$ |  |
| 5 | 0.3 | 0.5 | 0.5 | 1.0 | 2.0 |  |
| 6 | 1.0 | 1.0 | 1.0 | 2.0 | 2.5 |  |
| 7 | 1.0 | 2.0 | 2.0 | 3.0 | 3.0 |  |
| 8 | 2.0 | 2.5 | 2.5 | 3.5 | 3.5 |  |
| 9 | 2.5 | 3.0 | 3.0 | 4.0 | 4.5 |  |
| 10 | 3.0 | 4.0 | 4.0 | 7.0 | 10.0 |  |

| - CMM with uncertainty MPE $_{E}=0.3 \mu \mathrm{~m}$
$\square$ - CMM with high accuracy of measurement


Fig. 2. CMM's accuracy for gear measurement (criterion 0.1T)
When the criterion 0.2 T is acceptable (table 4) the largest measuring error is allowed, and the possibility of the CMM's application may be widened with one more accuracy class (dropped line in the fig. 3). It is recommended to apply the criterion 0.1 T for the gears with module bigger than 2 . When the module is less than 2 , criterion 0.2 T may be applied because of the narrow tolerance of the gear.

Table 4. Accuracy of CMM $[\mu \mathrm{m}]$ for the gear measurement according the 0.2 T criterion

| 0.2 T | module (from - to) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Accuracy class | $1 \div 2$ | $2 \div 3.55$ | $3.55 \div 6$ | $6 \div 10$ | $10 \div 16$ |
| 5 | 1.0 | 1.0 | 2.0 | 2.0 | 3.0 |
| 6 | 1.0 | 2.0 | 2.5 | 3.0 | 3.5 |
| 7 | 2.0 | 2.5 | 3.5 | 4.0 | 6.0 |
| 8 | 3.0 | 3.5 | 4.5 | 6.0 | 10.0 |
| 9 | 3.5 | 4.5 | 7.0 | 10.0 | $\boldsymbol{\lambda}$ |
| 10 | 6.0 | 7.5 | 10.0 | $\boldsymbol{\pi}$ | $\boldsymbol{\lambda}$ |

7- more than $10 \mu \mathrm{~m}$


Fig. 3. Accuracy of CMM recommended for the gear measurement according the criterion 0.2 T

## 3. CONCLUSIONS

In the performed metrological analysis of the proposed measuring software for the gear measurement with CMM, the values of particular errors have been determined, as well as their influence on the measured values. The final error of the gear measurement performed with CMM depends on CMM's uncertainty and the accuracy of the coordinate system establishment. When the coordinate system is established according the definition, its error is minimized. The errors of algorithms and their corrections have been determined, and they have been proved to be of no influence on the final error of the gear measurement. The results of simulations confirm the metrological correctness of proposed algorithms and the measuring software for gears.

The analysis of the gear measurement errors enabled to propose recommendations on the appropriate choice of CMM with known uncertainty ensuring the appropriate measurement of gear made in certain accuracy class. The choice of the CMM according the recommendations enables the metrologically correct measurement and identification of particular geometrical parameters of gear and their deviations. The measurement of the manufacturing deviations of the gear without metrological analysis of the measuring method and applied algorithms, and the determination of final error may lead to inappropriate choice of the measuring device (CMM), and to the incomplete andmetrologically non-correct information on the accuracy of measured gear.

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