

## MEASUREMENT OF EFFECTIVE AREA RATIO OF GAS AND HYDRAULIC PRESSURE BALANCES

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**Abstract:** This paper describes a method to compare the pressure generated by gas and hydraulic pressure balances with different medium, in order to improve harmonization between a gas pressure standard and a liquid pressure standard. The pressure range compared is 0.5 MPa to 7 MPa. In the method, a gas-liquid exchanger is used and a precise pressure transducer is employed as a comparator to determine the equilibrium state between the two pressure balances. From the measurement result obtained from the method, it was shown that the effective area ratio of gas and hydraulic pressure balances was determined as a function of pressure with the estimated uncertainty in all the pressure ranges compared.

**Keywords:** pressure balance, gas pressure, hydraulic pressure, comparison.

### 1. INTRODUCTION

A pressure balance is equipment, which can generate pressure stably, and is used in many calibration laboratories as a pressure standard. The calculation method of pressure generated by the pressure balance is well known by several international regulations [1-2], and its reliability has gained high evaluation. The National Metrology Institute of Japan/National Institute of Advanced Industrial Science and Technology (NMIJ/AIST) uses a pressure balance whose characteristics were evaluated in detail, and establishes and maintains the pressure standard of Japan. Calculation of the generating pressure requires many parameters. We calibrate principal parameters to make them traceable to the national standards of mass, length, time, temperature, etc. and ensure reliability of their values. Furthermore, we compare many pressure balances that we possess and make mutual comparison in various combinations to carry out group management. Thus, we evaluate long-term stability as the standard for each pressure balance. Pressure balances are available in many types. They can be classified by their structure, usage, etc. They are classified also by the medium used. Then they are roughly classified into a gas pressure balance, which uses gas and a hydraulic pressure balance which uses liquid. Usually, gas pressure standard and hydraulic pressure standard are maintained by gas pressure balance and hydraulic pressure balance, respectively.

This paper describes a high-precision method to compare the pressure generated by gas and hydraulic pressure balances with different medium, in order to improve harmonization between a gas pressure standard and a liquid pressure standard. Concretely, we pay attention to the effective cross-sectional area of a piston-cylinder assembly, which is a device constant of pressure balances. We make comparative measurements and examine a high-precision method to determine the effective area ratio, including the pressure dependence of both gas and hydraulic pressure balances. If the effective area of either pressure balance used is known as a function of pressure, then the effective area ratio including pressure dependence determined, can give the unknown effective area of another pressure balance as a function of pressure. In order to reduce uncertainty of the effective area to be sought, it is necessary to reduce uncertainty of the effective area ratio that is determined by comparative measurement. This research adopted a method to seek the effective area ratio of both gas and hydraulic pressure balances. The method uses a gas-liquid pressure exchanger. In the method, we made our measurement by the comparator method [3], where we used, as a comparator, a high-precision pressure transducer. The pressure range compared is 0.5 MPa to 7 MPa. This paper indicates the result of the effective area ratio obtained from the method, along with its uncertainty.

### 2. PRESSURE BALANCE

The main constituent elements of a pressure balance are the piston-cylinder and the weights. If a pressure is applied to float the piston with the weights up to an appropriate position, then the pressure  $P$  generated is given by the following equation [1-2].

$$P = \frac{W}{A(P,t)} + (\rho_f - \rho_a) \cdot g \cdot h \quad (1)$$

where,  $A(P, t)$  is the effective area at pressure  $P$  and temperature  $t$  which are determined by the piston-cylinder.  $(\rho_f - \rho_a) \cdot g \cdot h$  is the pressure corrected by head difference.  $\rho_f$  is the density of medium used.  $\rho_a$  is air density.  $g$  is gravity acceleration at the location where a measurement was made.  $h$  is the perpendicular distance between the

measurement position and the reference level of pressure balance, with the measurement position is relatively lower being positive.  $W$  is the force applied to the piston, which is expressed with the following equation.

$$W = M \cdot g \cdot \left(1 - \frac{\rho_a}{\rho_m}\right) + \gamma \cdot C \quad (2)$$

where,  $M$  is the sum of the total mass of weights including the piston mass.  $\rho_m$  is the average density of the piston and weights.  $\gamma$  is the surface tension of medium.  $C$  is the circumference of piston.

Positive free deformation type or negative free deformation type piston-cylinders are easy to handle and highly reproducible. If their dependence on pressure is assumed to be linear, their effective area is given by the following equation.

$$A(P, t) = A(0, t_r) \cdot (1 + b \cdot P) \cdot \{1 + \alpha_s \cdot (t - t_r)\} \quad (3)$$

where,  $A(0, t_r)$  is the effective area at the reference temperature under atmospheric pressure.  $b$  is a pressure distortion constant.  $\alpha_s$  is the sum of  $\alpha_p$ , which is thermal expansion coefficient of the piston and  $\alpha_c$ , which is that of the cylinder. This study used pressure balances: a negative free deformation type gas pressure balance and a positive free deformation type hydraulic pressure balance. Therefore, the above-mentioned equation gives the pressure generated.

### 3. MEASUREMENT

This section describes the method to obtain the effective area ratio of a gas pressure balance and a hydraulic pressure balance by using a gas-liquid pressure exchanger.

#### 3.1. Measurement principle

Figure 1 is a schematic diagram of equipment configuration, which is used traditionally to compare gas and hydraulic pressure balances. As shown in the figure, we often use a gas-liquid pressure exchanger to directly compare gas pressure and hydraulic pressure. The calibration principle is explained briefly. Firstly, observe so that the gas-liquid boundary plane inside the exchanger may be located at the reference level of the center of exchanger. Use two variable volumes (VV) and pressurize each medium. Raise the pistons of both pressure balances. The piston of each pressure balance receives the gravity of the perpendicular direction, which is caused by the application of the load of the mass of the weights. The medium flows out between the piston and cylinder. The position falls gradually over time. In this comparative method, if the pressure generated by both pressure balances reaches equilibrium, then the boundary plane stops at the reference level and the fall rate of each piston becomes almost equal to the natural fall rate at the pressure generated. Usually before adjustment, the generating pressure of both pressure balances is not at equilibrium. Therefore, adjust the amount of small weight on either pressure balance, until the above-

mentioned equilibrium is reached. Within the exchanger, if the pressure  $P_G$  generated by the gas pressure balance is at equilibrium with the pressure  $P_L$  generated by the hydraulic pressure balance, then the following equation is given.

$$P_G = P_L \quad (4)$$

where,  $P_G$  and  $P_L$  are represented by the following equations, respectively, using the equation (1).

$$P_G = \frac{W_G}{A_G(P, t_G)} + (\rho_{fG} - \rho_a) \cdot g \cdot h_G \quad (5)$$

$$P_L = \frac{W_L}{A_L(P, t_L)} + (\rho_{fL} - \rho_a) \cdot g \cdot h_L \quad (6)$$

where, the meaning of each parameter is as explained in the foregoing paragraph. The subscripts  $G$  and  $L$  refer to parameters of the gas side and the hydraulic side, respectively.  $W_G$  and  $W_L$  are the sum of gravity due to all weights mass including the small weight and the force due to surface tension. They are calculated from the equation from (2).  $h_G$  and  $h_L$  are the perpendicular distance to the reference level of the exchanger from the reference levels of the gas and hydraulic pressure balances, respectively. From the equations (4), (5) and (6), the effective area ratio of piston-cylinder of both pressure balances is given by the following equation.

$$\frac{A_G(P, t_G)}{A_L(P, t_L)} = \frac{W_G}{W_L} + \Delta R(P, t_G) \quad (7)$$

where,  $\Delta R$  is the term to correct the effect of head difference between each reference level as mentioned above. It is given by the following equation.

$$\Delta R(P, t_G) = \frac{A_G(P, t_G) \cdot g}{W_L} \times \{(\rho_{fG} - \rho_a) \cdot h_G - (\rho_{fL} - \rho_a) \cdot h_L\} \quad (8)$$

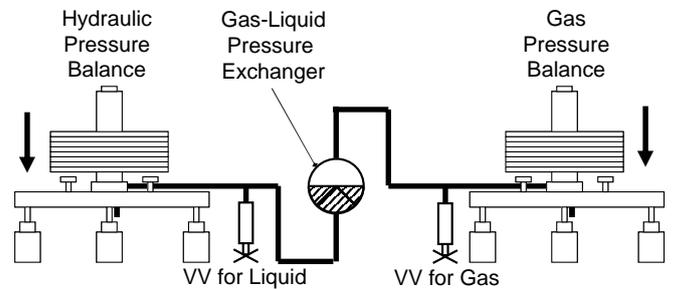


Fig. 1. Conventional comparison between gas and hydraulic pressure balances by fall-rate method using a gas-liquid exchanger. VV: variable volume.

where, if  $A_G(P, t)$  is presumed to be known, then the effective area ratio at the reference temperature is obtained, from equation (3), by the following equation.

$$\frac{A_G(P, t_r)}{A_L(P, t_r)} = \frac{A_G(P, t_G)}{A_L(P, t_L)} \times \frac{1 + \alpha_{sL} \cdot (t_L - t_r)}{1 + \alpha_{sG} \cdot (t_G - t_r)} \quad (9)$$

### 3.2. Measurement by the comparator method

The method shown in Figure 1 has an advantage of the simple configuration of equipment. However, high-precision comparison was difficult, because the equilibrium was observed, not by the generating pressure, but by the fall rate of each piston. For this measurement, we applied the comparator method [3] where we used, as a comparator, the high precision pressure transducer. Figure 2 is a schematic diagram of the measuring equipment. BV is the ball valve, which opens or closes by remote control from external air signals. Two valves open or close, and apply the respective pressure generated at the gas and hydraulic side to a high-precision pressure transducer. For all measurements made for this paper, we used a high-precision quartz Bourdon-type transducer which has high resolution of measurement.

In this measurement, the position measurement of the gas-liquid boundary plane in an exchanger is important. For example, if the measurement of the boundary plane position is different from the actual position by 1 cm, then it will cause the pressure error of about 90 Pa. This means that the measured pressure of 1 MPa results relatively in  $9 \times 10^{-5}$ . This is a rather large deviation considering the intended uncertainty. Accordingly, for this measurement, we used the system of using a CCD camera and television [4] in order to measure the position of the gas-liquid boundary plane with more accuracy than before. This system is supposed to ensure a 0.5 mm uncertainty for a boundary plane position measurement. In order to reduce the influence of the correction term expressed in equation (8), we adjusted it beforehand to minimize the difference between the reference levels of both pressure balances and that of the exchanger. In particular, the liquid density  $\rho_{fL}$  is much larger than the gas density  $\rho_{fG}$ . So, in order to reduce the uncertainty of the second term of the right side of equation (8), we adjusted it to make  $h_L$  within 0.5 mm.

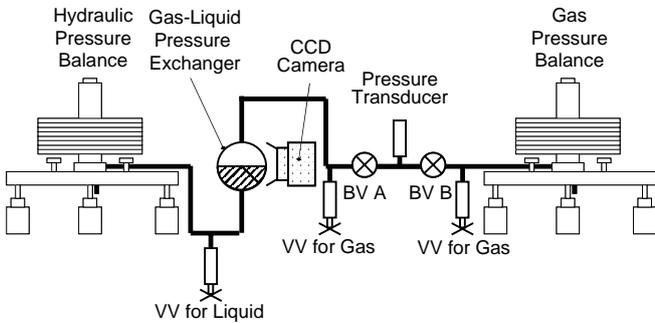


Fig. 2. Comparison between gas and hydraulic pressure balances by comparator method using a gas-liquid exchanger. VV: variable volume, BV: ball valve.

For this measurement, we used nitrogen with more than 99.9999% purity as the gas medium, and Sebacate (Di-2-Ethyl Hexyl Sebacate) as the liquid medium. The surface tension of nitrogen is negligibly small. So,  $\gamma_G = 0$  N/m. For the surface tension of Sebacate,  $\gamma_L = 0.031$  N/m. The density of both medium,  $\rho_{fG}$  and  $\rho_{fL}$  was obtained as a function of pressure  $P$  and temperature  $t$  in the pressure range compared from the following equations [5].

$$\rho_{fG} (\text{kg/m}^3) = \frac{28.01348 \cdot (P / \text{MPa} + 0.101325) \cdot 10^3}{0.9967 \cdot 8.31451 \cdot (t / ^\circ\text{C} + 273.15)} \quad (10)$$

$$\rho_{fL} (\text{kg/m}^3) = \{912.67 + 0.7521 \cdot P / \text{MPa} - 1.6448 \cdot 10^{-3} (P / \text{MPa})^2 + 1.45625 \cdot 10^{-6} (P / \text{MPa})^3\} \cdot \{1 - 7.8 \cdot 10^{-4} (t / ^\circ\text{C} - 20)\} \quad (11)$$

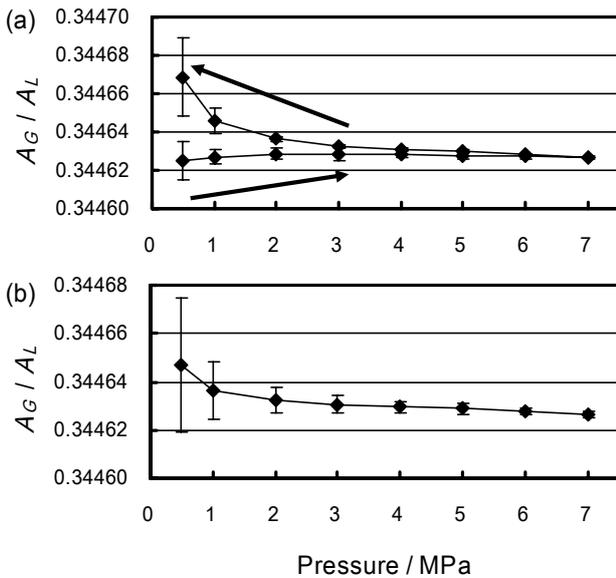
In this method, firstly, we measured alternately the generating pressure of each pressure balance by using the high-precision pressure transducer as a comparator, and obtained the pressure difference between them. Next, we found a small weight to be mounted on either pressure balance in order to obtain equilibrium from the pressure difference. Concretely, as shown in Figure 2, if valve BV A is closed and valve BV B is opened, then the high-precision pressure transducer can measure the generating pressure of the gas medium of the gas pressure balance. If valve BV A is opened and valve BV B is closed, then the pressure transducer can measure the generating pressure of the liquid medium of the hydraulic pressure balance, because it is converted into gas pressure by the exchanger. For one session of comparative measurement, we switched valves 6 times and obtained the pressure difference. The details of this comparator method are described in the reference [3]. If the pressure balance for each medium is invaded by any medium whose characteristics are different from the medium used, then the original characteristics are likely to be affected seriously. Accordingly, we watched the movement of a gas-liquid boundary plane carefully, and made adjustments/measurements. Furthermore, for each pressure point, we operated valves, adjusted pressure, raised the piston of both pressure balances to a proper position, and established equilibrium. After that, we waited for 15 minutes and started measuring.

### 3.3. Measurement results

For this measurement, we used the gas pressure balance whose effective area was about  $0.49 \text{ cm}^2$ . Mounting of a 35 kg weights enables the pressure balance to generate stably gas pressure of about 7 MPa. The hydraulic pressure balance has the effective area of about  $1.42 \text{ cm}^2$ . Mounting of a 101.5 kg weights enables the pressure balance to generate stably hydraulic pressure of about 7 MPa. Accordingly, the effective area ratio ( $A_G/A_L$ ) is about 0.345. Figure 3 is the effective area ratio of gas and hydraulic pressure balances. To obtain this result, we made comparative measurements using the exchanger and high-precision pressure transducer as shown in Figure 2, and we used equation (9). During

eight months, we made 9 round-trip measurements. We gradually increased and then decreased the measurement pressure in the pressure range of 0.5 MPa to 7 MPa. Figure 3(a) shows the average and standard deviation of results separately for pressure increase and decrease. As shown in the figure, in the lower pressure range, the value for pressure increase is different from that for pressure decrease, resulting in hysteresis. In the lower pressure range, the standard deviation is larger. The standard deviation for pressure decrease is larger than that for pressure increase. The hysteresis amount was relatively about  $6 \times 10^{-5}$  at the measured pressure of 1 MPa. We made a separate measurement and found the following. The observed hysteresis amount varies depending on the waiting time. The waiting time starts when the generating pressure of both pressure balances is stabilized and ends when data is acquired. The relation between the hysteresis amount and the waiting time is described in the discussion section.

We can estimate the uncertainty of the effective area ratio obtained from this measurement, if we consider all the parameters contained in the right side of equation (9) as uncertainty elements [6]. Here, in order to avoid the complexity beyond necessity, we neglected such elements whose influence on the result of the effective area ratio is relatively smaller than  $1 \times 10^{-6}$ . Table 1 shows the parameters whose influence is relatively larger than  $1 \times 10^{-6}$  in the range of 0.5 MPa to 7 MPa. Here,  $R_{exp}$  and  $\sigma_{exp}$  are the average and standard deviation, respectively, of the effective area ratio, which were calculated from measured values. From items shown in Table 1, we obtained the combined standard uncertainty ( $k = 1$ ). Figure 3 (b) shows the average and combined standard uncertainty ( $k = 1$ ) of the effective area ratio which was obtained from ascending and descending pressure.



**Fig. 3** The effective area ratio of gas pressure balance to hydraulic pressure balance obtained by method using a gas liquid exchanger: (a) results obtained from ascending and descending pressure, (b) average and combined standard uncertainty ( $k = 1$ ).

**Table 1** Parameters for uncertainty evaluation with standard uncertainties ( $k = 1$ ) and sensitivity coefficients

$x_i$	$u(x_i)(k=1)$	$s(x_i)$
$W_G$	$1.8 \times 10^{-6} W_G / N$	$W_G^{-1} / N^{-1}$
$W_L$	$1.8 \times 10^{-6} W_L + 1.3 \times 10^{-4} / N$	$W_G^{-1} / N^{-1}$
$h_L$	0.5 / mm	$\frac{A_G \cdot g(\rho_L - \rho_a)}{W_L \cdot R}$ / mm
$t_G$	0.2 / °C	$\alpha_{sG} / ^\circ\text{C}^{-1}$
$t_L$	0.2 / °C	$\alpha_{sL} / ^\circ\text{C}^{-1}$
$R_{exp}$	$\sigma_{exp}$	$R_{exp}^{-1}$

#### 4. DISCUSSION

In the results of measurement using a gas-liquid exchanger, we observed a large hysteresis in the ascending and descending pressure result. We will consider the causes of this. Hysteresis caused by changing direction of the generating pressure of each pressure balance used is sufficiently small and relatively 2 to  $3 \times 10^{-6}$  or less even at maximum. This was found by the preliminary evaluation of characteristics. Therefore, a cause may be the measurement using an exchanger. Concretely, the following phenomenon occurs at the gas-liquid boundary plane inside an exchanger. The increased pressure causes gas to dissolve into liquid, while the decreased pressure causes liquid to generate air bubbles.

Actually, in this measurement using an exchanger, we confirmed that, at the time of pressurization, gas (nitrogen) is dissolved into liquid (Sebacate). The confirmation method is as follows. Firstly, we pressurize the inside of the exchanger up to 7 MPa, and waited for a considerable time. Then, we reduced the pressure inside the exchanger quickly to atmospheric pressure. At this time, we observed the gas-liquid boundary plane and observed air bubble coming out of the liquid. Generally, when gas and liquid are in contact each other, pressurization causes dissolution of gas into liquid. If high pressure is kept, then the dissolved gas remains in the liquid. If the pressure is reduced, then the gas, which exceeds saturation amount becomes bubbles and comes out of the liquid. As Henry's law indicates, when gas is dissolved into liquid, the mass of dissolved gas is proportional to the gas pressure. The higher pressure causes more gas to dissolve into liquid.

When measurement is made with increasing pressure, the above-mentioned gas dissolution continues until saturation where gas-liquid equilibrium is reached at the applied pressure. Until gas-liquid equilibrium is reached, gas continues to dissolve into liquid. The gas volume decreases and the gas-liquid boundary plane in the exchanger goes up. On the other hand, when measurement is made with decreasing pressure, the above-mentioned bubble occurrence continues until gas-liquid equilibrium is reached at the applied pressure. Until gas-liquid equilibrium is

reached, bubbles continue to come out. So, the gas volume increases, and the gas-liquid boundary plane in the exchanger goes down. During measurement, we make an adjustment so that the gas-liquid boundary plane aligns with the reference level. If the liquid level goes up or down faster than this adjustment, and if the adjustment is not complete, then it causes errors in measurement results. When the high-precision pressure transducer measures the generating pressure at liquid side, these measured values have errors due to the above-mentioned measurement errors of the gas-liquid boundary plane. When the pressure ascends, they are underestimated than the actual pressure. When the pressure descends, they are overestimated than the actual pressure.

Generally when liquid is stationary without changing its volume, the principle of Pascal is viable. The pressure is spread uniformly and becomes constant everywhere in liquid. In this measurement, while gas continues to dissolve, or air bubbles continue to come out, the gas medium increases or decreases, causing the volume to change. Therefore, while the volume is changing, the generating pressure of hydraulic pressure balance does not accurately reach the high-precision pressure transducer. Thus, it causes probably the pressure gradient. Usually, if pressure changes at the end of pipe, the transfer function becomes worse accordingly as the length of the pipe is longer and the diameter of pipe is smaller. In this measurement, in order to connect a hydraulic pressure balance to an exchanger, we used a pipe of about 0.21 mm in inner diameter and about 100 cm in length. Accordingly, the time delay in pressure transfer may have affected the determination of equilibrium. Furthermore, strictly speaking, gas dissolution may have affected the values of evaluation of liquid density. Also such influence needs to be considered. However, as mentioned above, this measurement used the preliminary adjustment, and the liquid column difference to be considered is kept within 0.5 mm. So, the influence due to the change of liquid density is negligible.

In order to investigate how the influence of gas dissolution and bubble generation on measurement results change over time, we made the following measurement. In the comparative calibration of gas and hydraulic pressure balances shown in Figure 3, we waited for 15 minutes after equilibrium had been reached by pressure adjustment at each pressure point, and then acquired data. But, we changed this waiting time and made measurements. Then, we evaluated the relative difference of the effective area ratio obtained by increasing and decreasing pressures, namely, the magnitude of hysteresis. Hysteresis was large at three points of 0.5 MPa, 1 MPa, and 2 MPa. At these points, while increasing and then decreasing pressures, we obtained the effective area ratio after the lapse of waiting time of 15 minutes, 30 minutes, 60 minutes, 90 minutes and 120 minutes. The measurement procedure is the same as that mentioned in section 3.2. When pressure was increased to 2 MPa, we made a measurement. Then, we changed the pressure to 3 MPa, and kept it at 3 MPa for 12 hours. After that, we decreased the pressure to 2 MPa. We made similar measurements at the time of pressure decrease. Thus, we evaluated the relative difference of the effective area ratio

obtained at the time of increase and decrease of pressure. Figure 4 shows the average of results obtained by two round-trips of increase and decrease of pressure. This measurement shows that a longer waiting time results in a smaller amount of hysteresis. The reason is as follows. Gas dissolution and bubble generation advance over time. The dissolved gas in liquid reaches a saturation amount. It suppresses further advance of the gas dissolution and bubble generation. The change in gas volume becomes slowly over time. It is possible to use the measurement results and to obtain the relation between the waiting time and hysteresis amount, and to set the waiting time which is required to keep the hysteresis amount sufficiently small. However, if the waiting time is too long, the measurement procedure becomes less efficient. For actual measurement, the required uncertainty and allowable waiting time should be balanced and the reasonable waiting time should be determined.

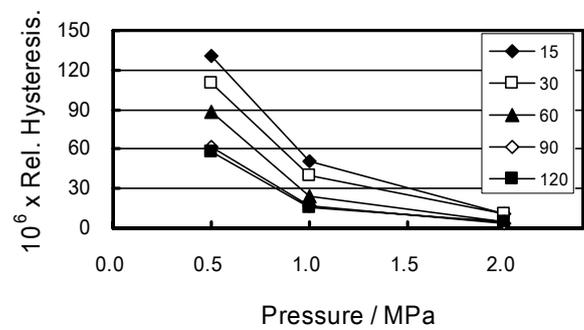


Fig. 4 Effect of waiting time on hysteresis of the effective area ratio.

## 5. CONCLUSIONS

In order to precisely compare gas pressure with hydraulic pressure, we studied how to compare the effective area ratio between gas and hydraulic pressure balances. In this study, we used a traditional gas-liquid exchanger and a high-precision pressure transducer. In the method, we used the comparator method, where we used, as a comparator, a high-precision pressure transducer. From the results, we calculated the effective area ratio of both gas and hydraulic pressure balances. The measurement result by the method showed a large hysteresis in the lower pressure range, which accompanied the ascending and descending calibration pressure. The relative combined standard uncertainty ( $k = 1$ ) of the effective area ratio is  $7 \times 10^{-6}$  or less in the pressure range of 4 MPa to 7 MPa. In the pressure range of 3 MPa or less, the relative uncertainty became bigger because of the presence of a large hysteresis. We obtained the effective area ratio as a function of pressure. The results shown in this article suggest that the method compares gas pressure balance with hydraulic pressure balance with high definition. NMIJ/AIST is currently developing other comparative methods to ensure consistency of a gas pressure standard and a hydraulic pressure standard in a wider pressure range with a smaller uncertainty.

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