XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 - 22, 2006, Rio de Janeiro, Brazil

NUMERICAL MODELLING OF PIRANI SENSOR

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Abstract: Numerical calculations of heat flux in the Pirani sens were carried out for a wide range of the gas rarefaction based on the kinetic Boltzmann equation. To take into account a non-diffuse gas-surface interaction the Cercignani-Lampis scattering kernel was used in the boundary condition. Such a kernel allowed us to study the influence of the momentum and energy accommodation coefficients on the heat transfer from the sensor wire into surrounding gas. The present results can be applied to improve the precision of pressure measurements by Pirani sensor.

Keywords: Pressure measurements, Pirani sensor, Boltzmann equation.

1. INTRODUCTION

The heat transfer through a gas confined between two coaxial cylinders having different temperatures was studied theoretically by many researchers, see e.g. Refs. [1–14]. In most of these papers the diffuse gas-surface interaction is assumed. Moreover, the ratio of radii was assume reasonable small, say about 2, because in this case the computational efforts are not so great as those for a large radius ratio, say about 100.

However, in many practical applications, i.e. in the Pirani sensor [15, 16] this ratio is about 16×10^3, i.e. it is very large. Experimental data given in Ref.[17] correspond to the radius ratio equal to 65 and 260. For such radius ratios the regime can be free-molecular with respect to the internal cylinder and at the same time it can be hydrodynamic with respect to the external cylinder. Numerical calculations of the heat flux for such geometry require great computational efforts because the region of calculation is large.

The experimental data [17] mentioned above showed that in many situations the gas-surface interaction is not diffuse. Physically, it means that the thermal accommodation coefficient is not unity. Comparing the experimental data [17] with theoretical results [18] based on the diffuse-specular scattering the accommodation coefficients were calculated. As was shown in Refs.[19–21] the diffuse-specular law does not provide a correct physical description of the gas-surface interaction. The problem is that the corresponding scattering kernel contains just one parameter α, which is usually called as accommodation coefficient without any specification. That is why if one calculates this parameter from different types of experiments, e.g. Poiseuille flow and heat transfer, one obtains quite different values of α.

The scattering kernel proposed by Cercignani and Lampis (CL) [22] contains two parameters α_t and α_n. The first one corresponds to the accommodation coefficient of tangential momentum and the second one is the accommodation coefficient of kinetic energy due to the normal velocity of molecules. Thus, applying the CL scattering kernel it is possible to distinguish the accommodation coefficients.

The aim of the present work is two-fold. First, we are going to perform calculations of the heat flux between two coaxial cylinders with a high radius ratio. An arbitrary rarefaction of gas with respect to the internal cylinder will be considered, while the regime with respect to the external cylinder is assumed to be hydrodynamic. It will be shown that under such a condition an analytical relation between heat flux for different radius ratios can be obtained. Second aim is to study an influence of the gas-surface interaction law on the heat flux by applying the CL scattering kernel.

2. STATEMENT OF THE PROBLEM

Consider two coaxial cylinders with radii R_0 and R_1 confining a gas. The axes of the cylinder coincide with the z-axis of the Cartesian coordinates. Let the temperature of the internal cylinder be $T_0 + \Delta T$ and the temperature of the external one be T_0, where $|\Delta T| \ll T_0$. We are going to calculate the temperature distribution and heat flux between the cylinders assuming the radius of the external cylinder be significantly larger than that of the internal one, i.e. $R_1 \gg R_0$.

The main quantity determining the heat flux is the rarefaction parameter referred to the internal cylinder radius and defined as

$$\delta = \frac{R_0}{\ell}, \tag{1}$$

where ℓ is the equivalent mean-free-path given as

$$\ell = \frac{\mu v_0}{P_0}, \qquad v_0 = \left(\frac{2kT_0}{m}\right)^{1/2}, \tag{2}$$

P_0 is the gas pressure in equilibrium, μ is the gas viscosity at the temperature T_0, v_0 is the most probable molecular speed at the temperature T_0, k is the Boltzmann constant, and m is the molecular mass. If $\delta \gg 1$ the regime is hydrodynamic and the Fourier law can be applied to calculate the heat flux. In this case the solution can be obtained analytically. However, if δ is not large then the Fourier law is not valid and the problem must be solved on the basis of the kinetic equation.

It should be noted that in many works the heat conductivity κ is used to define the mean-free-path ℓ. However, for the monoatomic gases the relation

$$\kappa = \frac{15k}{4m}\mu \tag{3}$$

is fulfilled with a high accuracy. Thus, the mean-free-path ℓ given by Eq.(2) can be defined in terms of the conductivity as

$$\ell = \frac{4m\kappa v_0}{15kP_0}. \tag{4}$$

The results will be presented in terms of dimensionless heat flux defined as

$$q = \frac{q'_r}{P_0 v_0}\frac{T_0}{\Delta T}, \tag{5}$$

where q'_r is the dimensional radial heat flux.

Further, the following dimensionless quantities will be used

$$r = \frac{r'}{R_0}, \qquad a = \frac{R_1}{R_0}, \tag{6}$$

where r' is the radial coordinate.

2.1. Hydrodynamic regime

In the hydrodynamic regime ($\delta \gg 1$) the Fourier law

$$\mathbf{q}' = -\kappa \nabla T, \tag{7}$$

is used. If we assume the temperature continuity conditions, i.e,

$$T = \begin{cases} T_0 + \Delta T_0 & \text{at} \quad r = 1, \\ T_0 & \text{at} \quad r = a, \end{cases} \tag{8}$$

then the temperature is easily obtained from Eq.(7) and energy conservation law

$$T(r) = T_0 + \Delta T\left(1 - \frac{\ln r}{\ln a}\right). \tag{9}$$

In this case the dimensionless heat flux q defined by Eq.(5) reads

$$q = \frac{15}{8\delta}\frac{1}{r\ln a}, \tag{10}$$

where the relation (3) has been used.

A moderate gas rarefaction can be taken into account by applying the temperature jump boundary condition, which in its general form reads [20]

$$T_g = T_w + \zeta\ell \left.\frac{\mathrm{d}T_g}{\mathrm{d}x}\right|_{x=0}, \tag{11}$$

where T_g is the temperature of the gas near a wall having the temperature T_w, x is the normal coordinate with the origin at the wall. This coefficient depends on both gas and surface, i.e. it can be different for the internal and external cylinders. Let us denote the jump coefficients on the internal and external cylinders as ζ_0 and ζ_1, respectively. Then, the jump boundary conditions for the problem in question read

$$T = \begin{cases} T_0 + \Delta T_0 + \frac{\zeta_0}{\delta}\frac{\mathrm{d}T}{\mathrm{d}r} & \text{at} \quad r = 1, \\ T_0 - \frac{\zeta_1}{\delta}\frac{\mathrm{d}T}{\mathrm{d}r} & \text{at} \quad r = a. \end{cases} \tag{12}$$

If we apply the boundary condition (12) instead of Eq.(8) and omit the terms of the order $1/\delta^2$ then the temperature distribution takes the form

$$\begin{aligned} T(r) \;=\; & T_0 + \Delta T\left(1 - \frac{\ln r}{\ln a}\left\{1 + \frac{1}{\delta}\left[\zeta_0\left(\frac{1}{\ln r}\right.\right.\right.\right. \\ & \left.\left.\left.\left. - \;\frac{1}{\ln a}\right) - \zeta_1\frac{1}{a\ln a}\right]\right\}\right) \end{aligned} \tag{13}$$

The corresponding heat flux reads

$$q = \frac{15}{8\delta}\frac{1}{r\ln a}\left[1 - \frac{1}{\delta\ln a}\left(\zeta_0 + \frac{\zeta_1}{a}\right)\right]. \tag{14}$$

The temperature jump coefficients ζ_0 and ζ_1 significantly depend on the gas-surface interaction law [20]. Thus, in case of rarefied gas the heat flux is determined not only by the heat conductivity but by the gas-interaction too. From the expressions (13) and (14) one can see that assuming $a \gg 1$ the contribution of the term containing the coefficient ζ_1 vanishes. Thus, under assumption $a \gg 1$ the surface properties of the external cylinder do not affect the temperature distribution and heat flux.

2.2. Transitional regime

In this section we consider the situation when the rarefaction parameter δ is arbitrary, while the quantity δa is large, i.e.

$$\delta a \gg 1. \tag{15}$$

Physically, it means that the mean-free-path ℓ is significantly smaller than the external cylinder radius, i.e. $\ell \ll R_1$, while the internal cylinder radius can be both smaller and larger than the mean free path. Under such a condition it is possible to relate analytically two heat fluxes q and q^* corresponding two different values of the radius ratio a and a^*, respectively.

Consider two pairs of cylinders with the radius ratio equal to a and a^*. The external cylinders have the temperatures T_1 and T_1^*, respectively. The heat flux q_r' in both situations will be the same if the temperatures T_1 and T_1^* satisfy the relation

$$T_1^* - T_1 = -\frac{q_r' r'}{\kappa} \ln \frac{a^*}{a}, \tag{16}$$

which has been obtained from the Fourier law being valid in the space $a \le r \le a^*$. Using Eq.(16) and the definition (5) the relation between the dimensionless heat fluxes is obtained as

$$\frac{1}{q} - \frac{1}{q^*} = \frac{8\delta r}{15} \ln\left(\frac{a}{a^*}\right). \tag{17}$$

Thus, if the heat flux q corresponding to the ratio a is known, then the heat flux q^* for any $a^* > a$ is immediately known. The expression (17) provides the heat flux with the relative error about $1/(\delta a)$, because the violation of the Fourier law has the order of $1/(\delta a)$.

2.3. Kinetic equation

At arbitrary values of the rarefaction parameter the Boltzmann equation [23, 24] must solved. Till now, a numerical solution of the exact Boltzmann equation requires grate computational efforts. As was shown in Ref.[25] the model equation provide reliable results with modest efforts. For the problem in question the S model [26] is most appropriate one. The linearization of this equation is described in Ref.[13], where the distribution function f is written down as

$$f(\mathbf{r}, \mathbf{v}) = f_0 \left[1 + h(\mathbf{r}, \mathbf{v}) \frac{\Delta T}{T_0}\right], \tag{18}$$

where f_0 is the equilibrium distribution function. Considering the axial symmetry the linearized S model reads

$$\begin{aligned} c_r \frac{\partial h}{\partial r} &- \frac{c_\varphi}{r}\frac{\partial h}{\partial \theta} = \delta \left[\upsilon + \tau\left(c^2 - \frac{3}{2}\right)\right. \\ &+ \left.\frac{4}{15} q c_r \left(c_r - \frac{5}{2}\right) - h\right], \end{aligned} \tag{19}$$

where $\boldsymbol{c}$ is dimensionless molecular velocity related to the dimension one as $\boldsymbol{c} = \boldsymbol{v}/v_0$, c_r is the radial component of $\boldsymbol{c}$ and c_φ is its azimuthal component. The moments of the perturbations function υ, τ and q are given as

$$\upsilon = \frac{1}{\pi^{3/2}} \int \exp(-c^2) h \, d\boldsymbol{c}, \tag{20}$$

$$\tau = \frac{1}{\pi^{3/2}} \int \exp(-c^2) h \left(\frac{2}{3}c^2 - 1\right) d\boldsymbol{c}, \tag{21}$$

$$q = \frac{1}{\pi^{3/2}} \int \exp(-c^2) h \left(c^2 - \frac{5}{2}\right) c_r \, d\boldsymbol{c}. \tag{22}$$

The quantity q is given by Eq.(5). The quantities υ and τ represent deviations of the density and temperature, respectively, i.e

$$\upsilon = \frac{n - n_0}{n_0}\frac{T_0}{\Delta T}, \quad \tau = \frac{T - T_0}{\Delta T}. \tag{23}$$

2.4. Boundary conditions

To solve the linearized kinetic equation (19) boundary conditions are needed at the cylinders. In general form such conditions are written as [24, 25]

$$h^+ = \hat{A} h^- + h_w - \hat{A} h_w, \tag{24}$$

where h^+ and h^- are the perturbations of the reflected and incident particles, respectively, h_w is the perturbation of the Maxwellian corresponding to the surface temperature, i.e.

$$h_w = \begin{cases} \upsilon_w + \left(c^2 - \frac{3}{2}\right) & \text{at} \quad r = 1, \\ 0 & \text{at} \quad r = a. \end{cases} \tag{25}$$

The operator $\hat{A}$ is expressed as

$$\begin{aligned} \hat{A} h &= \frac{1}{|\boldsymbol{c}\cdot\boldsymbol{n}|} \int_{\boldsymbol{c}'\cdot\boldsymbol{n}<0} \exp\left(c^2 - c'^2\right) |\boldsymbol{c}'\cdot\boldsymbol{n}| \\ &\times \; R(\mathbf{c} \to \mathbf{c}') h(\mathbf{c}') d\mathbf{c}', \quad \mathbf{c}\cdot\boldsymbol{n} \ge 0, \end{aligned} \tag{26}$$

where $R(\mathbf{c} \to \mathbf{c}')$ is the scattering kernel.

In Refs.[19–21] it was shown that the kernel proposed by Cercignani and Lampis [22] is more physical than the widely used diffuse-specular one. For the problem in question the kernel can be decomposed as

$$R(\mathbf{c} \to \mathbf{c}') = R_r(c_r \to c_r') R_\varphi(c_\varphi \to c_\varphi') R_z(c_z \to c_z'). \tag{27}$$

where

$$\begin{aligned} R_r(c_r \to c_r') &= \frac{2c_r}{\alpha_n} \exp\left(-\frac{[c_r^2 + (1-\alpha_n)c_r'^2]}{\alpha_n}\right) \\ &\times \; I_0\left(2\frac{\sqrt{1-\alpha_n} c_r c_r'}{\alpha_n}\right), \end{aligned} \tag{28}$$

$$R_i(c_i \to c_i') = \exp\left(-\frac{[c_i-(1-\alpha_t)c_i']^2}{\alpha_t(2-\alpha_t)}\right) \times \frac{1}{\sqrt{\pi\alpha_t(2-\alpha_t)}}, \quad i=\varphi, z \tag{29}$$

$I_0(x)$ is the modified Bessel functions of the first kind

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos\varphi)\mathrm{d}\varphi. \tag{30}$$

The Cercignani-Lampis (CL) kernel contains two accommodation coefficients: the energy accommodation coefficient corresponding to the normal component of the velocity, α_n, and the momentum accommodation coefficient corresponding to the tangential component of the velocity, α_t. If $\alpha_n = 1$ and $\alpha_t = 1$ the CL kernel corresponds to the diffuse scattering.

3. METHOD OF SOLUTION

To solve Eq.(19) new variables are introduced

$$c_p^2 = c_r^2 + c_\varphi^2, \quad \theta = \arctan\left(\frac{c_\varphi}{c_r}\right). \tag{31}$$

To eliminate the c_z dependence, the follows two orthogonal functions, ϕ and ψ, are defined

$$h = \phi(r,\theta,c_r,c_\varphi) + 2\psi(r,\theta,c_r,c_\varphi)\left(c_z^2 - \frac{1}{2}\right). \tag{32}$$

With the help of Eq.(31)-(32) we obtain

$$\mathfrak{D}\phi = \delta\left[\upsilon + \tau\left(c_p^2-1\right) + \frac{4}{15}qc_p\cos\theta\left(c_p^2-2\right)\right], \tag{33}$$

$$\mathfrak{D}\psi = \frac{\delta}{2}\left[\tau + \frac{4}{15}qc_p\cos\theta\right], \tag{34}$$

$$\upsilon = \frac{1}{\pi}\int_{-\pi}^{\pi}\int_0^\infty \exp\left(-c_p^2\right)\phi c_p dc_p d\theta, \tag{35}$$

$$\tau = \frac{2}{3\pi}\int_{-\pi}^{\pi}\int_0^\infty \exp\left(-c_p^2\right)\left[\phi\left(c_p^2-1\right) + \psi\right]c_p dc_p d\theta, \tag{36}$$

$$q = \frac{1}{\pi}\int_{-\pi}^{\pi}\int_0^\infty \exp\left(-c_p^2\right)\left[\phi\left(c_p^2-2\right) + \psi\right]c_p\cos\theta c_p dc_p d\theta. \tag{37}$$

where the following operator has been used

$$\mathfrak{D} = c_p\cos\theta\frac{\partial}{\partial r} - \frac{c_p\sin\theta}{r}\frac{\partial}{\partial\theta} + 1. \tag{38}$$

The system Eq.(33)-(34) can be solved using the discrete velocity method [27]. The numerical calculations are difficult near the free-molecule regime due a discontinuity in the distribution function. To overcome this problem, the functions ϕ and ψ are split as

$$\phi = \tilde{\phi} + \phi_0, \quad \psi = \tilde{\psi} + \psi_0 \tag{39}$$

where ϕ_0 and ψ_0 are the solution in the free-molecule regime

$$\phi_0 = \begin{cases} \phi^+\exp\left(-\frac{\delta S_0}{c_p}\right) & \text{if } |\theta| < \theta_0 \\ 0 & \text{if } \theta_0 < |\theta| < \pi \end{cases}$$

$$\psi_0 = \begin{cases} \psi^+\exp\left(-\frac{\delta S_0}{c_p}\right) & \text{if } |\theta| < \theta_0 \\ 0 & \text{if } \theta_0 < |\theta| < \pi \end{cases}$$

where

$$S_0 = r\cos\theta - \sqrt{1 - r^2\sin^2\theta} \tag{40}$$

and

$$\theta_0 = \arcsin\frac{1}{r} \tag{41}$$

Then, the kinetic equation is solved for the functions $\tilde{\phi}$ and $\tilde{\psi}$.

4. RESULTS AND DISCUSSION

The kinetic equation was solved numerically for the rarefaction parameter varying from the near free-molecule regime to the hydrodynamic regime, for some combinations of energy and momentum accommodation coefficients. To verify the numerical solution, a comparison with other works in the case of the diffuse boundary conditions and small radius ratio was performed in Table I. One can see that our data are in a good agreement with the previous results.

A more complete results are presented in Table II, for $a = 65$. It can be seen that the heat flux decrease monotonically for decreasing α_n and α_t. To verify this, a comparison between the analytical, i.e. Eq.(13), and numerical solution was performed in Table III, where the relative deviation ε, between numerical and analytical solutions are given. ε tends to zero by increasing δ,

A comparison between the numerical results with experimental data [17] is shown in Fig.(1). The discrepancy of the theoretical solution and experimental data is less than 1% for $\delta > 0.8$. For $\delta < 0.8$ the disagreement is significant because of the experimental uncertainty.

To verify the validity of Eq.(17), some data, obtained from numerical solution of the kinetic equation are presented in Table IV. These data correspond to the case of diffuse scattering of gas at the cylinders surface, i.e. $\alpha_n = \alpha_t = 1$. Comparing the values in Table 17, i.e. for the data of fifth and sixth columns, and Eq.(17), we can see that the relative difference between them is about $1/\delta a$.

TABLE I: Heat flux for $a = 2$, at $r = 1.5$, S-Model present work, BGK [2], Variational principle [7].

	q		
δ	S-Model	[2]	[7]
0.1060	0.3688	0.372	0.3695
1.0605	0.3126	0.311	0.3165
2.1215	0.2670	0.266	0.2718
4.2425	0.2058	0.204	0.2106
8.4850	0.1399	0.139	0.1436

TABLE II: Radial heat flux q for a=65 at r=1 as a function of α_n, α_t and δ

	$\alpha_t \backslash \alpha_n$	0.25	0.50	0.75	1.00
$\delta = 0.1$	0.50	0.2694	0.3337	0.3969	0.4589
	0.75	0.3171	0.3805	0.4427	0.5037
	0.90	0.3303	0.3934	0.4554	0.5162
	1.00	0.3328	0.3959	0.4578	0.5185
$\delta = 0.3$	0.50	0.2452	0.2978	0.3478	0.3950
	0.75	0.2838	0.3343	0.3821	0.4274
	0.90	0.2943	0.3441	0.3914	0.4362
	1.00	0.2963	0.3460	0.3932	0.4378
$\delta = 1$	0.50	0.1822	0.2100	0.2343	0.2553
	0.75	0.2024	0.2273	0.2491	0.2682
	0.90	0.2076	0.2317	0.2530	0.2715
	1.00	0.2085	0.2326	0.2537	0.2721
$\delta = 3$	0.50	0.1024	0.1104	0.1171	0.1223
	0.75	0.1083	0.1149	0.1205	0.1250
	0.90	0.1097	0.1160	0.1214	0.1257
	1.00	0.1100	0.1162	0.1216	0.1258
$\delta = 10$	0.50	0.03988	0.04078	0.04169	0.04236
	0.75	0.04064	0.04136	0.04210	0.04266
	0.90	0.04083	0.04151	0.04221	0.04274
	1.00	0.04085	0.04152	0.04222	0.04275
$\delta = 20$	0.50	0.02130	0.02142	0.02166	0.02184
	0.75	0.02146	0.02157	0.02177	0.02192
	0.90	0.02150	0.02160	0.02180	0.02194
	1.00	0.02151	0.02161	0.02180	0.02194

TABLE III: Relative difference ε between the Fourier and the S-Model heat fluxes for some $(\alpha_n \backslash \alpha_t)$ pairs: A=(0.25\0.5), B=(0.75\0.75) and C=(1.00\1.00).

δ	$\varepsilon(\%)$		
1	199.43	36.31	13.38
3	22.18	2.54	0.16
10	3.18	0.21	0.07
20	1.97	0.18	0.09
$(\alpha_n \backslash \alpha_t)$	A	B	C

TABLE IV: Radial heat flux q, at r=1, for some radius ratio. Diffuse case.

δ	q				
	$a = 2$	5	10	20	65
1	0.4735	0.4057	0.3627	0.3246	0.2721
2	0.4073	0.3099	0.2582	0.2192	0.1729
3	0.3568	0.2484	0.1985	0.1640	0.1258
5	0.2849	0.1761	0.1347	0.1084	0.08110
7	0.2365	0.1359	0.1016	0.08076	0.05972
10	0.1881	0.1009	0.07407	0.05832	0.04275
15	0.1399	0.07055	0.05098	0.03983	0.02900
20	0.1113	0.05419	0.03884	0.03023	0.02194
30	0.07891	0.03700	0.02631	0.02041	0.01476

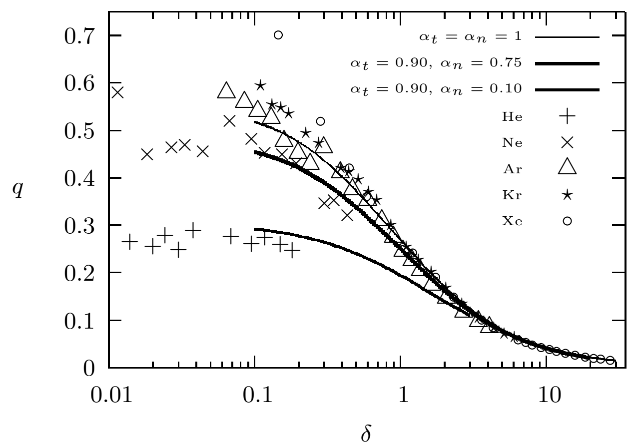

FIG. 1: Comparison between theoretical (present work) and experimental [17] heat fluxes.

5. CONCLUSION

The heat flux between to coaxial cylinders through rarefied gas was numerically calculated in the range of rarefaction parameter δ from 0.1 to 20. It was considered the non-diffuse gas-surface interaction by using the Cercignani-Lampis boundary condition. Comparing the theoretical results with experimental data accommodation coefficients were calculated for some gases.

6. ACKNOWLEDGMENTS

The authors acknowledge Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq, Brazil) for the support of their research.

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