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OPTICAL DIFFERENTIATION PHASE MEASUREMENT USING A MODIFIED MULTI-STEP BIAS-SHIFTING METHOD

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Abstract: An optical differentiation phase measurement system using a modified multi-step bias-shifting method is proposed. The signal obtained by multi-step bias shifting is divided into two groups in this new method. The effectiveness of the new method was demonstrated using a computer simulation.

Keywords: phase measurement, wavefront, optical differentiation.

1. INTRODUCTION

Phase measurement of an optical wavefront is of considerable importance for a number of applications in optical testing and material diagnostics. One method to measure phase is by performing optical differentiation using a differentiation Fourier filter.[1-6] This method is simple in construction and is suitable for wavefronts having a wide range of phase variations. It can be used for obtaining the three-dimensional appearance of a transparent phase object and for measuring optical components, etc. We previously proposed a two-step bias-shifting method for differentiation phase measurement that can correct the influence of non-uniformity of the wavefront's intensity.[7] Furthermore, we demonstrated that the error caused by the signal noise can be corrected using the multi-step bias-shifting method.[8] The error was reduced by increasing the step number. However, the number of steps is limited by the signal to noise ratio. If the bias shift is lower than the noise, the noise and the non-uniformity of the light intensity cannot be distinguished resulting in an increase in the error.

In this present paper, we propose a new type of multi-step bias-shifting method. The position of the differentiation filter is shifted laterally several times in the same manner as the standard multi-step bias-shifting method. However, the data are divided in two groups and the errors caused by the noise and non-uniformity of the light intensity are corrected. Computer simulations were used to prove the effectiveness of this method.

2. THEORY

The optical arrangement of the proposed system is shown in Fig. 1. This is the same arrangement that was used for the two-step and standard multi-step bias-shifting methods.[7,8] The wavefront on the object plane is measured. A differentiation filter is placed on a linear stage that is controlled by a computer. The wavefront is focused onto the differentiation filter by lens L (focal length: *f*). The transmission coefficient of the filter is proportional to the position along the differentiation direction *x*.

If the complex amplitude of the transmitted beam of the optical wavefront $i_1(x, y)$ is represented by

$$i_1(x,y) = A(x,y)\cdot \exp[i\phi(x,y)], \tag{1}$$

then the phase derivative can be expressed by the following approximation:[7]

$$\frac{\partial\phi(x_2,y_2)}{\partial x_2} \approx \frac{2\pi m}{\alpha A_0 \lambda f}\sqrt{I(x_2,y_2)} - a_0, \tag{2}$$

where $I = i_2{}^2$ is the intensity of the light on the image

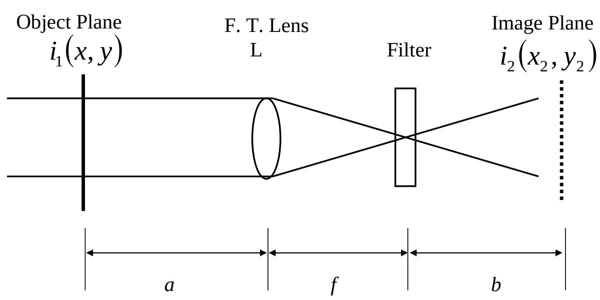

Fig. 1 Optical arrangement of the optical differentiation phase measurement system using the bias-shifting method. The filter can be shifted.

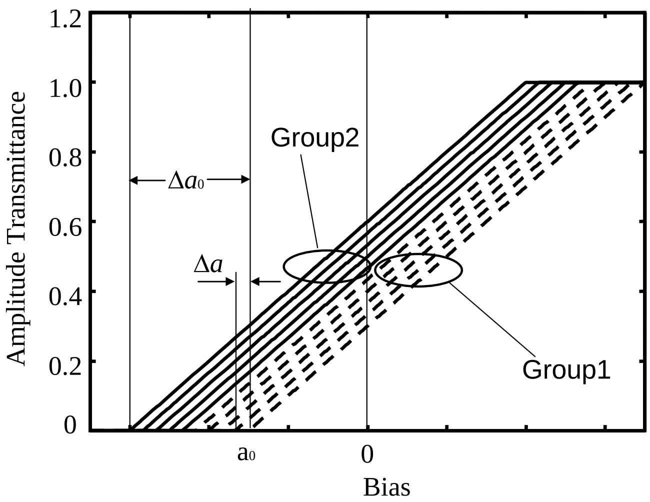

Fig. 2 Shifted filter positions. Group 1: broken lines. Group 2: solid lines.

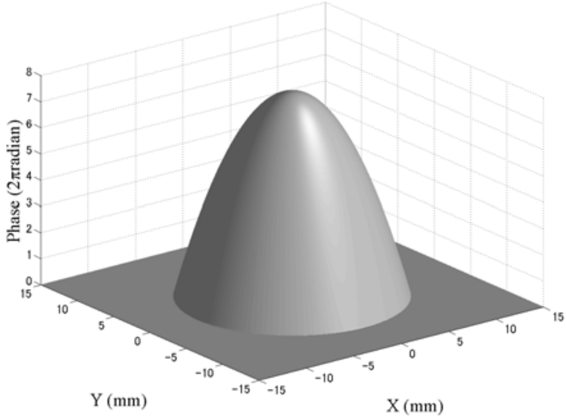

Fig. 3 Phase profile of the wavefront used in computer simulations.

plane, A is the absolute amplitude of the wavefront, and $\phi(x, y)$ is the phase. In addition, m is the magnification of the image, α is the proportional coefficient of the filter, λ is the wavelength of the light beam, and a_0 is the bias value.

If the bias is shifted laterally N-1 times as $a_0+n\Delta a$, the phase derivative can be expressed as

$$\frac{\partial \phi_n(x_2, y_2)}{\partial x_2} \approx \frac{2\pi m}{\alpha A_0 \lambda f}\sqrt{I_n(x_2, y_2)} - a_0 - n\Delta a \cdot \qquad (3)$$

Therefore, the signal noise can be corrected using the method of least squares as follows.

$$\frac{\partial \phi}{\partial x_2} \approx \frac{2\pi m}{\alpha A_0 \lambda f}\frac{\sum\sqrt{I_n}}{N} - \frac{N-1}{2}\Delta a - a_0 \cdot \qquad (4)$$

Furthermore, the error caused by the non-uniformity of the absolute amplitude of the light source or absorption along the light beam path is corrected using the measurements of the shifted signals with the base bias shift of $a_0 + \Delta a_0$. The filter positions are shown in Fig 2. The phase derivative is calculated using the equation below.

$$\frac{\partial \phi}{\partial x_2} \approx \frac{\sum\sqrt{I_n}\Delta a_0}{\sum\sqrt{I_n'} - \sum\sqrt{I_n}} - \frac{N-1}{2}\Delta a - a_0, \qquad (5)$$

where I_n is the signal intensity of group 1(broken lines). I_n' is the signal intensity of group 2(solid lines), which is the case when the base bias is shifted.

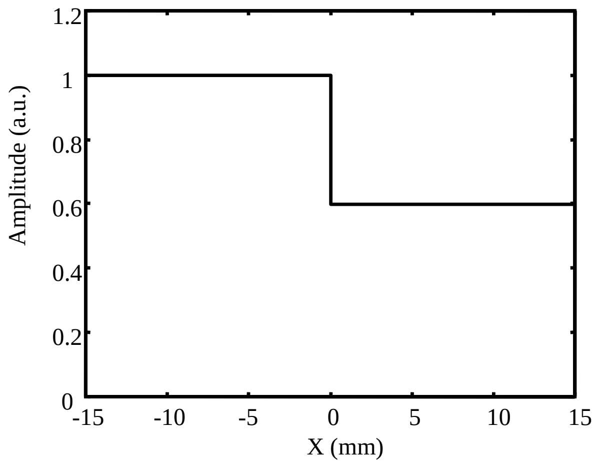

Fig. 4 Intensity profile at y=0 of the light source used in computer simulation.

3. RESULTS

A computer simulation was performed to investigate the characteristics of this method. A spherical wave having a radius of curvature of 10 m and a solid angle of $10^{-7}\pi$ sr was used in the simulation (Fig. 3). The maximum phase derivative is $1.5\times 2\pi$ rad/mm. The parameters of the optical components used in the simulation were m = 1, f = 15 cm, λ = 633 nm, and α = 1 mm^{-1}. The wavefront has an absolute amplitude variation of 40% as shown in Fig. 4.

In the simulation for the standard multi-step bias-shifting method, the initial focal point is 0.3 mm and the filter is shifted nine times to the focal point at 0.6 mm. Therefore, the step length is 0.3/9 mm and N = 10. In the case of the modified multi-step bias-shifting method, the initial focal point is 0.3 mm, the step length for the filter shift is 0.3/9 mm, N=5 and the base bias shift is 0.3/9*5 mm. Therefore the step length for the filter shift and the total number shifts are the same for both methods. Figure 5 shows the simulated signal intensities at the image plane in which random noise

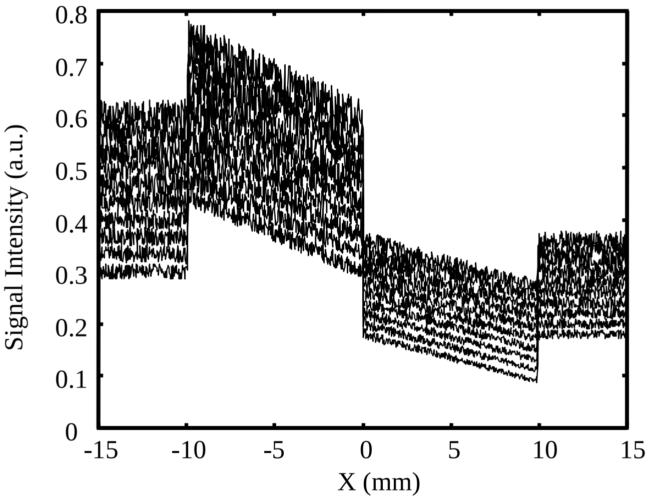

Fig. 5 Simulated signal intensities across the image plane at y=0. Signal noise is ±5%.

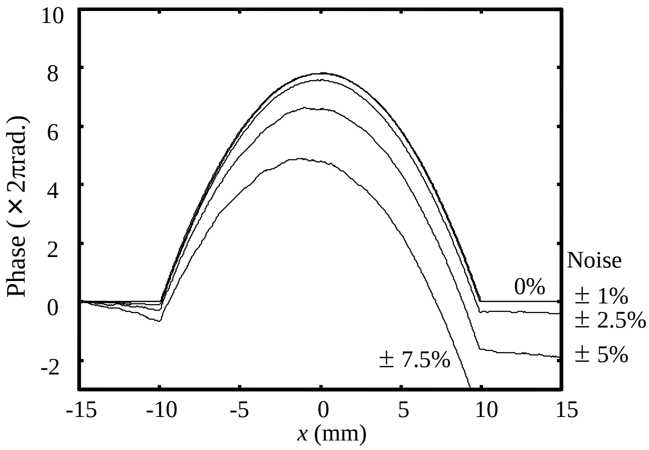

Fig. 6 Phase profiles calculated for the standard multi-step bias-shifting method at y = 0. The noise levels are 0% (ideal phase profile), ±1%, ±2.5%, ±5%, and ±7.5%.

having an intensity of ±5% is introduced. Because of the high noise level, the signal intensity ranges for adjacent signals overlap.

In the case of the standard multi-step bias-shifting method, the phase derivative is calculated using the following equation.[8)]

$$\frac{\partial \phi}{\partial x_2} = \frac{\left\{ \sum \sqrt{I_n} \sum n\sqrt{I_n} - \frac{N(N-1)}{2} \sum I_n \right\}}{N \sum I_n - \left(\sum \sqrt{I_n} \right)^2} \Delta a - a_0 \qquad (6)$$

The error caused by noise and the non-uniformity of the absolute amplitude of the light source or absorption along the light beam path (A_0) is corrected by the method of least squares simultaneously applied to the 10 signals in the standard method. Figure 6 shows the phase profiles calculated for the standard multi-step bias-shifting method at y = 0 for the noise levels of 0% (ideal phase profile), ±1%, ±2.5%, ±5%, and ±7.5%. The calculated phase profiles for low noise levels are close to the ideal phase profile. The error becomes large, however, for high noise levels.

In the case of the modified method, the noise is corrected by the method of least squares separately for the five signals from group 1 and the five signals from group 2, and the error caused by the non-uniformity A_0 is corrected by the calculated phase derivatives for group 1 and group 2. Figure 7 shows the calculated phase derivatives for the standard and the modified multi-step bias-shifting methods with a signal noise of ±5%. The noise of the phase derivative profile for the standard method is smaller than that for the modified method. This is because the number of signals in the standard method used for the method of least squares calculation is larger than that used for the modified method.

Figure 8 shows the phase profiles calculated by the standard and the modified multi-step bias-shifting methods at y = 0 with a signal noise of ±5%. The ideal curve is also shown. The curve obtained with the modified method is much closer to the ideal curve than the one obtained with the standard method.

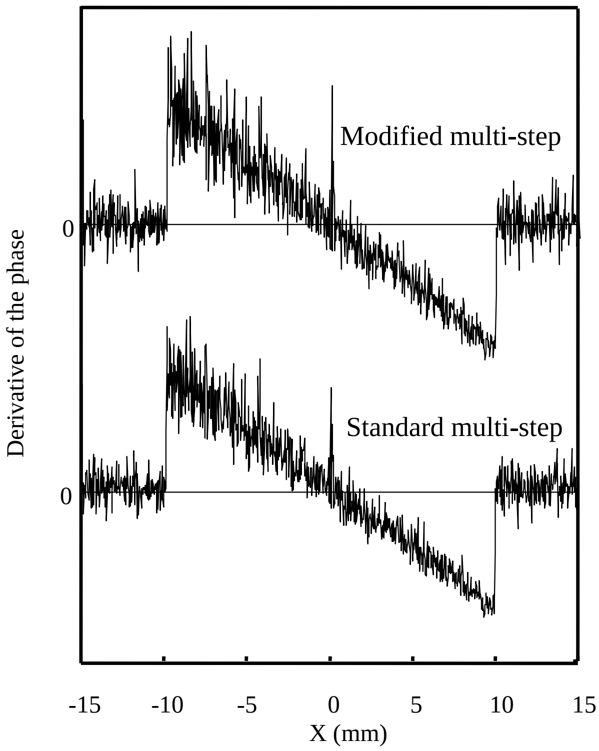

Fig. 7 Calculated phase derivatives for standard multi-step bias-shifting method and modified multi-step bias-shifting method. Signal noise is ±5%.

Figure 9 shows the phase profiles calculated for the modified multi-step bias-shifting method at y=0. The noise levels are 0% (ideal phase profile), 1%, 2.5%, 5%, and 7.5%. The error resulting from the signal noise is smaller than that for the standard multi-step bias-shifting method shown in Figure 6.

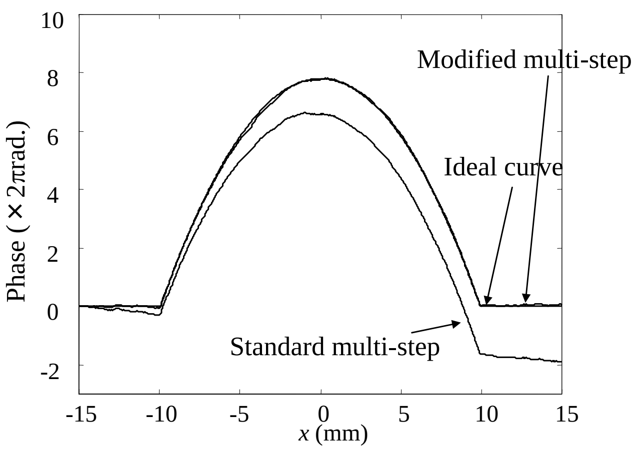

Fig. 8 Phase profiles calculated by the standard and the modified multi-step bias-shifting methods at y = 0. The ideal curve is also shown. Signal noise is ±5%.

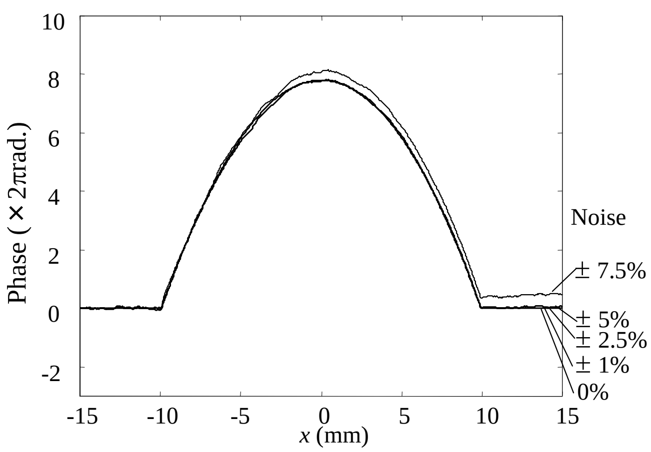

Fig. 9 Phase profiles calculated for the modified multi-step bias-shifting method at y = 0. The noise levels are 0% (ideal phase profile), ±1%, ±2.5%, ±5%, and ±7.5%.

4. CONCLUSION

In conclusion, a new phase measurement technique based on the phase differentiation method has been proposed. The position of the differentiation filter is shifted laterally several times. The data are divided into two groups and the errors caused by the noise and non-uniformity of the light intensity are corrected for each group separately. For the standard multi-step method the number of the steps is limited by the signal to noise ratio. However, using the modified method it is possible to decrease the errors by further increasing the step number. The effectiveness of the proposed method was successfully demonstrated by computer simulation.

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