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A PRIMARY ANGULAR ACCELERATION CALIBRATION STANDARD

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Abstract: Primary angular acceleration calibration standard is developed by CIMM to generate standard rotational angle, angular velocity and angular acceleration, which are traceable to the International System of Units (SI). It can be used to calibrate angular transducers, i.e. angular accelerometer, angular velocity transducer, and rotational angle transducer to obtain amplitude sensitivity and phase shift by sinusoidal vibration. This paper will introduce the mechanic system, control system and measurement system of the standard. Calibration results of angular transducer using the standard also introduced. It shows that the standard can be used in angular movement calibration in the frequency range from 0.1Hz to 200Hz with amplitude uncertainty better than 1% and phase uncertainty of 1 degree.

Keywords: angular exciter, angular acceleration, standard, grating, laser interferometer.

1. INTRODUCTION

Primary angular acceleration calibration standard is used to generate standard rotational angle, angular velocity and angular acceleration, which are traceable to the International System of Units (SI), and it can be used to calibrate angular transducers, i.e. angular accelerometer, angular velocity transducer (tachometer, gyro, etc), and rotational angle transducer to obtain amplitude sensitivity and phase shift by sinusoidal vibration. ISO 16063-15 "Methods for the calibration of vibration and shock transducers-Part15: Primary angular vibration calibration by laser interferometry" is on the stage of draft at present and Germany has established angular acceleration calibration standard several years before [1].

The primary angular acceleration calibration standard introduced in this paper is developed by CIMM. It includes three systems, i.e. mechanical system, control system and measuring system. The angular exciter is shown in Fig.1.

As shown in Fig.2, angular acceleration realized by the servo motor driving the air bearing system, the transducer is mounted on the top of angular exciter table surface. A diffraction column grating and two laser interferometers are used to measure the angular acceleration, and the table surface is used to mount transducer to be calibrated. The computer in PXI bus system control ADC module to sample the data from the interferometer and the transducer to be calibrated, an arbitrary waveform generator is used to control the angular exciter at the same time. The software analyse data and calculate the complex sensitivity of the transducer. The PXI bus instrument is a stand-along system in the primary angular acceleration calibration system, which is used to sample data from optical-electrical detectors of grating (in the case of low rotation rate), interferometers and transducer, control angular exciter and process data. It has an embedded computer, several data acquisition cards, an arbitrary waveform generator card and a digital multimeter card. The embedded computer is a high-performance PXI/CompactPCI-compatible system controller; it is also a general computer.

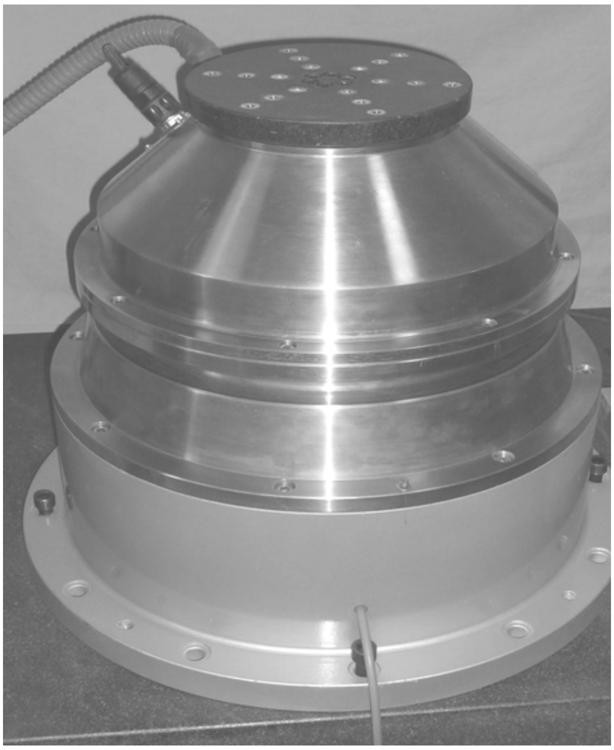

Fig.1. Photo of angular acceleration exciter

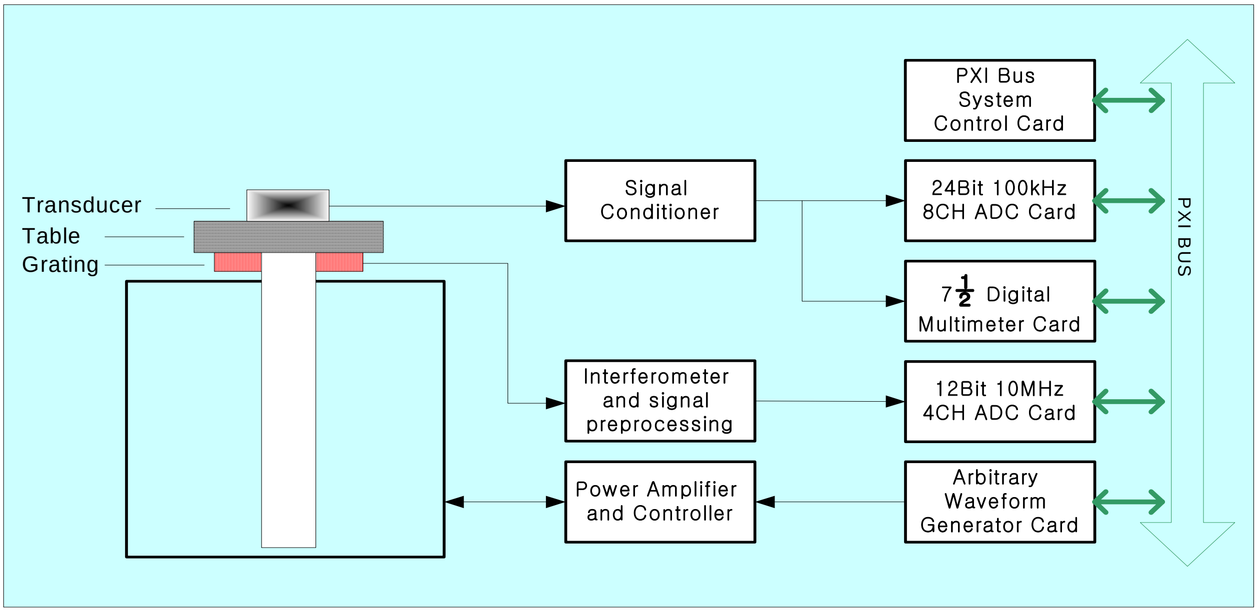

Fig.2 Layout of the standard

2. INVESTIGATION OF THE SYSTEM

2.1. The mechanical system

The mechanical system uses a servomotor driving an air-bearing system. Shown in Fig.3, a table, which is made of marble, is used to mount transducer to be calibrated. Two gratings are fixed on the shift. Grating 1 is a diffraction column grating, which collaborate with two laser interferometers to measure angular movement. Grating 2 is used as angle position measurement component to provide feedback signal for the control system. A brushless DC servomotor is used to drive air-bearing system. The friction is reduced to great extend because the use of brushless motor and the air bearing system.

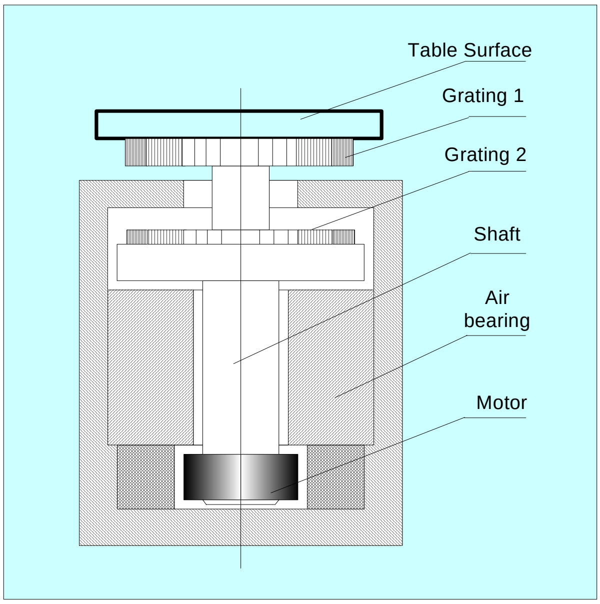

Fig.3.The mechanical system

2.2. The control system

Primary angular acceleration calibration standard mechanical equation can be expressed by

$$J\ddot{\theta}+C_f\dot{\theta}=T_M \tag{1}$$

where J is moment of inertia of the whole rotating system, C_f is friction coefficient, T_M is torque generated by the motor. $\dot{\theta}$ is angular speed and $\ddot{\theta}$ is angular acceleration.

The electromagnetic torque generated by the servomotor is

$$T_M = K_T i \tag{2}$$

where K_T is torque coefficient of the motor and i is current.

The electric equation can be expressed by

$$u_a - E = L_a\frac{di}{dt}+R_a i \tag{3}$$

where u_a is input voltage of the motor, E is induced voltage , L_a is inductance of the motor, and R_a is the resistance of the motor.

The induced voltage can be expressed by

$$E = K_b\dot{\theta} \tag{4}$$

where K_b is a coefficient related with the motor.

From (1) to (4)

$$i=\frac{J\ddot{\theta}+C_f\dot{\theta}}{K_T} \tag{5}$$

$$\frac{di}{dt}=\frac{J\dddot{\theta}+C_f\ddot{\theta}}{K_T} \tag{6}$$

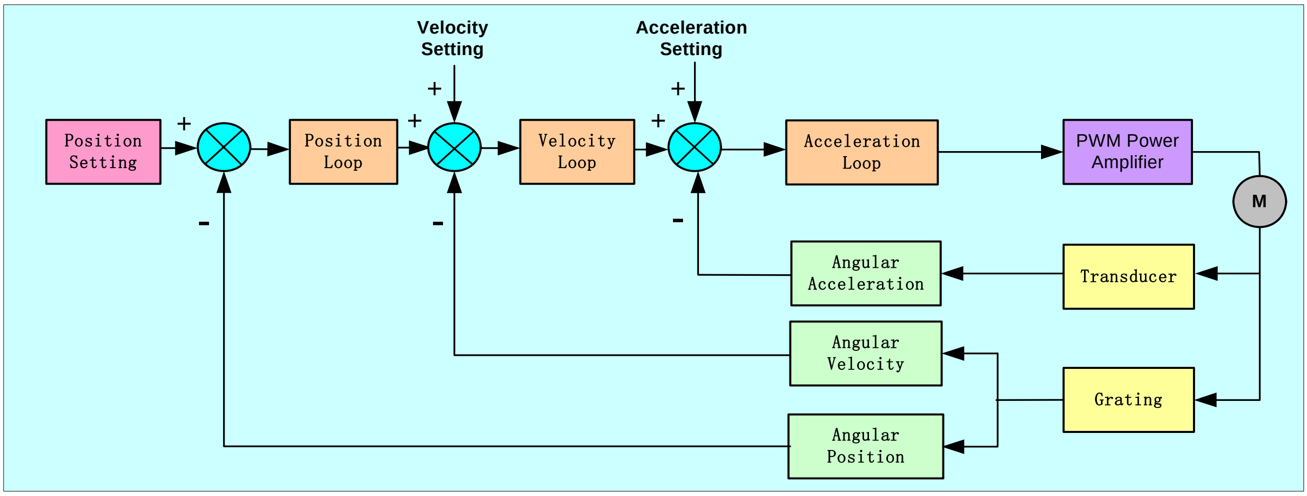

Fig.4. Block diagram of the controller

Finally, the differential equation of the system is

$$(L_a J)\dddot{\theta} + (L_a C_f + R_a J)\ddot{\theta} + (R_a C_f + K_b K_T)\dot{\theta} = K_T u_a \quad (7)$$

The Laplace transform of the system function (7) is

$$G(s) = \frac{\Theta(s)}{U(s)} = \frac{\dfrac{K_T}{L_a J}}{s\left[s^2 + \dfrac{(L_a C_f + R_a J)}{L_a J}s + \dfrac{(R_a C_f + K_b K_T)}{L_a J}\right]} \quad (8)$$

The controller is shown in Fig.4. Triple loops are used to control the system, and parameters of all triple loops can be adjusted digitally by computer command. The arbitrary waveform generator based on PXI bus system is used to generate control voltage signal to the controller. Grating is used for position feedback and the differential result of angle position is used for angular velocity feedback. An angular accelerometer is used for angular acceleration feed back.

The control system works in different mode depend on different applications. High amplitude stability and low harmonic distortion is very important for angular movement calibration. When calibrating rotational angle transducer, the control system works in position control mode to generate standard rotational angle movement. In this mode, three loops work at the same time; the parameters of all three loops must be adjusted to obtain high quality rotational angle movement. Angular velocity loop and angular acceleration loop are used to compensate the characteristics of position loop and increase the responsibility. When calibrate angular velocity transducer, the control system works in three loops or two loops mode, which depend on different calibration frequency. In calibrating angular accelerometer, the control system works in only one loop mode; only angular acceleration loop works in this mode.

2.3. Principle of using laser interferometer and column grating measuring angular movement

A column diffraction grating is located under the air-borne measuring table of the angular vibration exciter, concentrically to the axis of rotation, and transducer(s) are mounted on the surface of the table. The object light beam of the interferometer strikes the grating at the angle, which the first-order beam diffracted by reflection. The diffracted beam returns into the direction of the incident beam. When the grating rotated, the phase of diffracted beam change, which is proportional to the rotation angle of the grating. The angle of diffracted beam is in accordance with the diffraction formula

$$\frac{k\lambda}{d} = \sin\alpha + \sin\beta \quad (9)$$

where k is the order of diffracted beam, λ is wavelength of laser, d is grating distance constant, α is incidence angle, β is diffraction angle.

To reduce the error due to eccentricity mounting of the grating and other error sources, two laser interferometers are used in the measurement at the same time. The angular movement value is calculated using two interferometer's signals separately, and then using the average value as the measurement result.

The heterodyne interferometer and signal processing system are shown in Fig.5. It's a modified Mach-Zehnder interferometer using a helium neon laser with wavelength of 0.6328 μ m, and the frequency relative stability of the laser is in the order of 10^{-9}. A Bragg Cell with relative frequency stability better than 10^{-8} is used in the system. The Bragg cell generates frequency shift f_B (25 MHz) in the reference arm of the interferometer, which is used to determine the sign of the angular movement. The interference signal of the

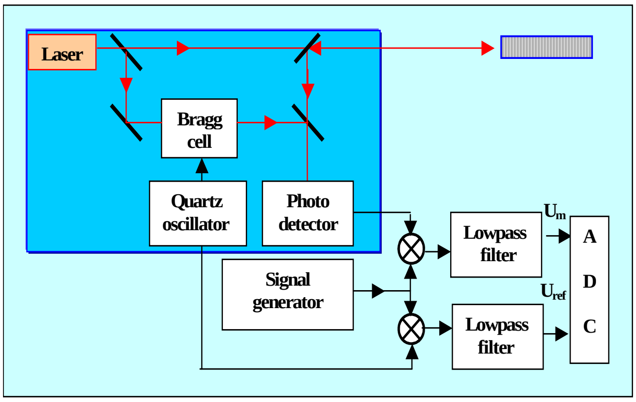

Fig.5. Interferometer and signal processing system

object beam and the reference beam is converted into an electrical signal in the photo detector. When the grating rotated, it will generates a frequency modulated carrier signal in the RF region, whose centre frequency is identical to that of the acousto-optical modulator drive signal (25 MHz). The directionally sensitive Doppler information is thus contained in the RF carrier. The signed object velocity of grating determines sign and amount of frequency deviation with respect to the centre frequency $f_B \pm \Delta f$. The interferometer output signal f_B and $f_B \pm \Delta f$ then mixed with signal generator output signal f_s, after mixing and low-pass filtering the output signal turn to $f_s - f_B \pm \Delta f$ named as U_m and $f_s - f_B$ named as U_{ref}. If f_s is 26MHz, then the output signal frequency after mixing is from around 25MHz down to around 1MHz, therefore, they can be sampled by ADC card and needn't using a very high sample frequency [2].

The sampled data then decoded by the PXI bus instrument and quadrature signals U_1 and U_2 are obtained similar to homodyne interferometer with quadrature outputs [3] [4].

The discrete-time signals U_1 and U_2 can be expressed as $U_1[n]$ and $U_2[n]$. The series of modulation phase $\varphi_{\mathrm{mod}}[n]$ from the movement of grating can be calculated by arctan calculation and phase unwrapping procedure

$$\varphi_{\mathrm{mod}}[n] = \tan^{-1}\frac{U_2[n]}{U_1[n]} + k\pi \tag{10}$$

where n is a simple serial number, and k = 0, 1, 2, 3, …

Choose an integer number k so that discontinuties of $\varphi_{\mathrm{mod}}[n]$ are aviod for the values $k\pi$. From the result of modulation phase versus time series $\varphi_{\mathrm{mod}}[n]$, the rotational angle can be calculated by

$$\Phi[n] = \frac{g}{2\pi}\varphi_{\mathrm{mod}}[n] \tag{11}$$

where g is grating angle constant in radian, which depends on the manufacture with accuracy in the order of serveral arc second.

2.4. Calculating the angular acceleration

The rotation angle of the sinusoidal angular vibration in (11) can be expressed by

$$\Phi[n\Delta t] = \hat{\Phi}\cos(\omega \times n\Delta t + \varphi_{\Phi}) + C \tag{12}$$

where $\hat{\Phi}$ is the amplitude of rotation, φ_{Φ} is the phase of rotation angle, Δt is the sample interval of ADC, ω is the vibration radian frequency, $\omega = 2\pi f$, C is a constant, n = 0, 1, 2, …

For the sinusoidal angular movement, the amplitude and phase of angular velocity and angular acceleration can be calculated by:

$$\hat{\Omega} = 2\pi f \times \hat{\Phi},\ \varphi_{\Omega} = \varphi_{\Phi} - \pi/2 \tag{13}$$

$$\hat{\alpha} = (2\pi f)^2 \times \hat{\Phi},\quad \varphi_{\alpha} = \varphi_{\Phi} - \pi \tag{14}$$

where f is the frequency of angular vibration, $\hat{\Omega}$ and φ_{Ω} is the amplitude and phase of angular velocity, $\hat{\alpha}$ and φ_{α} is the amplitude and phase of angular acceleration.

The output voltage of the transducer is sampled by the ADC and can be expressed in the same way as (12).

$$U[n\Delta t] = \hat{U}\cos(\omega \times n\Delta t + \varphi_u) + C_u \tag{15}$$

where $\hat{U}$ is the amplitude of transducer output voltage, φ_u is the phase of transducer output voltage, C_u is a constant.

In (12) and (15) the amplitude and phase of rotation angle and transducer output voltage $\hat{\Phi}$, φ_{Φ}, $\hat{U}$, φ_u can be calculated using two methods. One method is sine-approximation [5], which uses least square method calculating the amplitude and the phase of the rotational angle. Another method is DFT, which uses the discrete-time Fourier transform method calculating the amplitude and the phase according to the calibration frequency. Digital simulation and experiment showed that the first method could obtain good result if the vibration in low harmonic distortion and high signal-noise ratio; otherwise, DFT method can obtain better result [6].

3. CALIBRATING ANGULAR TRANSDUCERS

Different kinds of angular transducers, including rotational angle transducer, angular velocity transducer and angular accelerometer, are calibrated using the primary angular acceleration calibration standard. It shows the standard is stable and repeatability of the calibration results is very good.

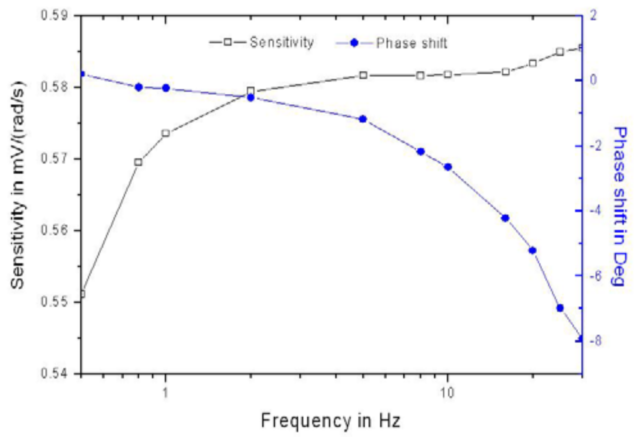

Fig.6. Calibration result of an angular velocity transducer

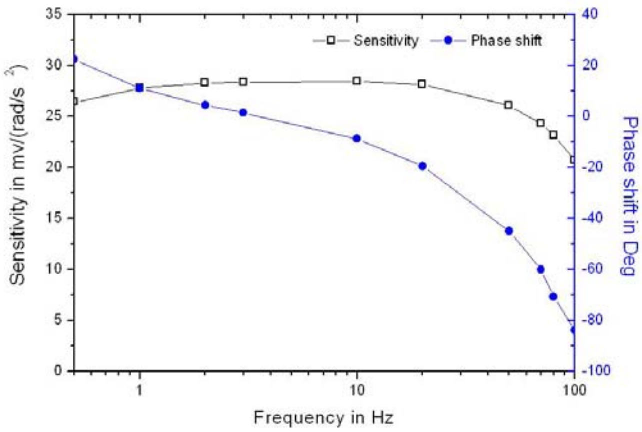

Fig.7. Calibration result of an angular accelerometer

Fig.6 is a calibration of an angular velocity transducer; the calibration frequency range is from 0.5Hz to 30Hz.

Fig.7 is a calibration result of an angular accelerometer; the standard work in constant angular acceleration mode in the calibration process and the frequency range is from 0.5Hz to 100Hz.

4. DISCUSSION

The angular acceleration standard introduced in this paper using laser interferometer and grating measuring angular movement, and PTB uses similar method [1]. The accuracy of this standard can be increased in the future. Because there is no limit of rotation angle and the using of air bearing system, this device shows high performance in low frequency angular movement calibration. For the frequency above 200Hz, another device has been developed and will be introduced in the future.

5. CONCLUSIONS

The primary angular acceleration calibration standard has developed. It uses the brushless motor and the air bearing system to generate angular movement. Triple loops are used in the controller and grating and angular accelerometer are used in the feedback system. The standard can work in constant angle, velocity or acceleration control mode with no limitation of rotation angle. Two laser interferometers and one differential column grating are used in the measurement of angular movement. The standard shows low harmonic distortion and high stability in generating angular movement. It can be used in calibrating complex sensitivity of angular transducers in the frequency range from 0.1Hz to 200Hz with amplitude uncertainty better than 1% and phase uncertainty of 1 degree.

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