



## ANGLE PRISM-BASED LASER INTERFEROMETER FOR HIGH PRECISION MEASUREMENT OF ANGULAR VIBRATION

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**Abstract:** This paper introduces a new ‘angle-prism’-based laser interferometer designed for high-precision measurement of angular vibration. The angle prism, fixed on the centre of the rotating axis, is shown to convert its angular motion into the path difference between two parallel laser beams. Their path difference provides a new way of measuring the angular displacement. This angular displacement measurement method is obviously shown to present a new primary method for the calibration of angular vibration pickups. In comparison to the traditional primary calibration methods in ISO/DIS 16063-15, the suggested one is shown to provide more robustness even for the eccentricity of rotating axis and the unwanted plane motion of the pickup holder. Detailed theoretical and experimental results are presented in this paper.

**Keywords:** Angular vibration, primary calibration, angular vibration measurement, angle-prism interferometer.

### 1. INTRODUCTION

The papers raises technical issues encountered in using the laser interferometers proposed in the standard code of ISO/DIS 16063-15[1] dedicated for the primary calibration of angular vibration transducers. The standard considers two interferometers: the retro-reflector interferometer and diffraction-gating interferometer [2]. The former one has the very limited angle measurement range of  $\pm 3^\circ$  and much difficulty in the alignment of the retro-reflector(s). Whilst the latter enables the measurement of the unlimited range, it has the highest difficulty in manufacturing a precision sine-shaped diffraction-gating device and aligning its centre within 2  $\mu\text{m}$  apart from the rotation axis. Moreover, both interferometers have technical limits in reducing the unwanted transverse motion of the rotation axis systematically. Those issues have will be addressed in details in this conference.

This paper proposes a new ‘angle-prism’-based laser interferometer in Section 2. Its theoretical backgrounds are briefly addressed. A proto-typed model, developed to examine the feasibility of the ‘angle-prism’-based laser interferometer, is shown in Section 3. Experimental results obtained from the proto-typed model are illustrated in Section 4. Finally, main achievements of this work are summarized.

### 2. ANGLE-PRISM LASER ITERFEROMETER

Gary Sommargrem [3,4] was found to study several methods of measuring the rotation angle using the laser interferometer. At the onset of this work, it was apparent that the angle-prism is not prohibited for the use of the primary angular vibration calibration guided by the standard code of ISO/DIS 16063-15. Specifically, it has enabled us to resolve the technical issues of the retro-reflector and diffraction-gating interferometers commented above. It does not have dependency on the unwanted transverse motion of the rotation axis and the eccentricity of the optical device from the centre. It is seen to reduce much work for aligning the angle prism during measurement and machining and fitting the rotating part of an angular vibrator. It is the reason that the angle prism based laser interferometer was chosen in this work.

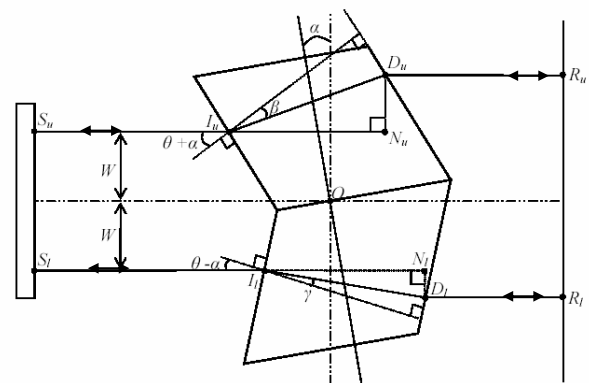


Fig. 1. Path difference between upper and lower laser beams caused by the angular motion of the angle prism

Fig. 1 shows the schematic drawing of illustrating how the path difference can be measured by using the angle prism. When the angle prism is rotated by the rotational angle  $\alpha$ , dual upper and lower beams, denoted by  $S_u$  and  $S_l$  in Fig. 1, are shown to have the different paths whose length is denoted by  $P_u$  and  $P_l$ , respectively. Both beams are reflected from the stationary mirror, denoted by  $R_u$  and  $R_l$  in Fig. 1. Both path length  $P_u$  and  $P_l$  are described as the following equations

$$P_u = \overline{S_u I_u} + \left( \overline{I_u D_u} \right)_{eff} + \overline{D_u R_u} \quad (1)$$

$$P_l = \overline{S_l I_l} + \left( \overline{I_l D_l} \right)_{eff} + \overline{D_l R_l} \quad (2)$$

The subscripts  $u$  and  $l$  denote the upper and lower beams and subscript  $eff$  does the effective length of the beam path in air. Let  $L_0$  the straight path length from the source position ( $S_u$  or  $S_l$ ) to the plane mirror ( $R_u$  or  $R_l$ ). Both path length  $P_u$  or  $P_l$  in equation (1) and (2) are given as

$$P_u = L_0 - \overline{I_u N_u} + \left( \overline{I_u D_u} \right)_{eff} \quad (3)$$

$$P_l = L_0 - \overline{I_l N_l} + \left( \overline{I_l D_l} \right)_{eff} \quad (4)$$

The path difference between the upper and lower beams are defined as

$$\Delta(\alpha) = P_u - P_l = \left( \overline{I_u D_u} \right)_{eff} - \left( \overline{I_l D_l} \right)_{eff} - \overline{I_u N_u} + \overline{I_l N_l} \quad (5)$$

The path difference  $\Delta(\alpha)$  is shown to come from the different incident angle of each beam on the angle prism, which is related to the angular displacement  $\alpha$ . In equation (5), the effective length of the upper and lower beams is give as

$$\left( \overline{I_u D_u} \right)_{eff} = \eta_{g,u} \cdot \frac{h_u}{\cos \beta} \quad (6)$$

$$\left( \overline{I_l D_l} \right)_{eff} = \eta_{g,l} \cdot \frac{h_l}{\cos \gamma} \quad (7)$$

The symbol  $\eta_{g,u}$  and  $\eta_{g,l}$  denote the refraction indices of the upper and lower parts (glasses) of the angle prism and the symbols  $\beta$  and  $\gamma$  do the refraction angle of the upper and lower laser beams. The symbols  $h_u$  and  $h_l$  are the thickness of the upper and lower parts (glasses) of the angle prism. The latter two terms of equation (5), are also given as

$$\overline{I_u N_u} = \frac{h_u}{\cos \beta} \cdot \cos(\theta + \alpha - \beta) \quad (8)$$

$$\overline{I_l N_l} = \frac{h_l}{\cos \gamma} \cdot \cos(\theta - \alpha - \gamma) \quad (9)$$

In equations (8) and (9), the angles  $(\theta + \alpha)$  and  $(\theta - \alpha)$  are the incident angle of the upper and lower laser beams and the symbol  $\theta$  is the incident angle of the upper and lower laser beams at the zero angular displacement ( $\alpha = 0$ ). By substituting the four terms given in equation (6) ~ (9) into equation (5) and rearranging them, the path difference between the upper and lower laser beams are describes as

$$\Delta(\alpha) = h_u \sqrt{n_{g,u}^2 - \sin^2(\theta + \alpha)} - h_l \sqrt{n_{g,l}^2 - \sin^2(\theta - \alpha)} - h_u \cdot \cos(\theta + \alpha) + h_l \cdot \cos(\theta - \alpha) \quad (10)$$

The path difference is actually measured as the following integer-valued equation

$$N(\alpha) = \text{int} \left( M_D \cdot \frac{\Delta(\alpha)}{\lambda} \right) \quad (11)$$

Note that  $\lambda$  denotes the wavelength of the laser and  $M_D$  does the interpolation factor (or division factor) of the wavelength supplied by the Zygo measurement board ( $M_D = 2048$  for Zygo model 4004).

It is interesting to note that the path difference is dependent only on the upper and lower glass paths of the angle prism, not on the air paths. It implies that the measurement of the angular displacement is independent of the air paths of the laser beams. Furthermore, it is obvious that either the eccentricity of the rotation axis or its unwanted transverse motion does not change any beam path difference in the prism. The reason comes from the fact that neither horizontal nor vertical movement of the rotation axis does not give the path difference on the angle prism itself. It is a big advantage to use the angle-prism-based laser interferometer.

### 3. EXPERIMENTAL SETUP

Fig. 2 shows the photograph of the angle-prism-based laser interferometer, which consists of the laser head (Zygo model 7702A), the differential plane mirror interferometer (DPMI, Zygo model 7015), the angle prism (Zygo model 7026-2), and the stationary plane mirror.

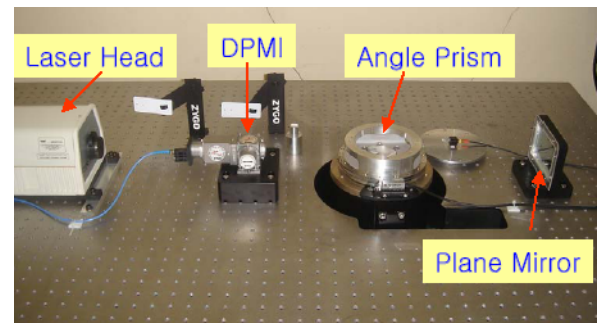
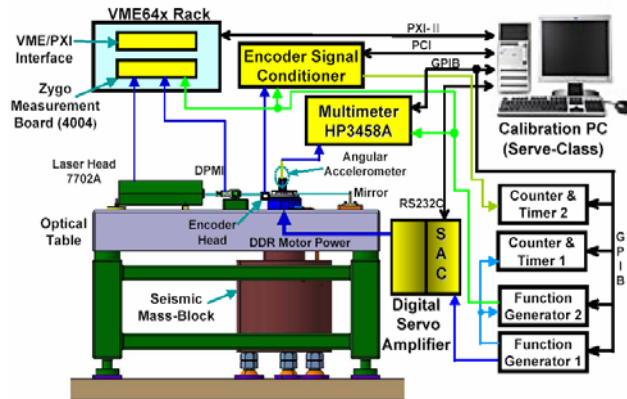


Fig.2 Angle prism-based laser interferometer.

Two laser beams, the reference beam from the laser head and the measurement beam from the DPMI, are delivered through the optical fibers and then inputted to the path difference measurement board (Zygo model 4004). The

measurement board, whose electronics enable high precision heterodyne displacement measurement [5], provides the angle measurement resolution of  $2.55 \times 10^{-8}$  radian and the maximum angular velocity up to 100 radian/s. The interface to the measurement board is based on the VME64x standard bus through which the 37-bit position, 32-bit velocity and 32-bit sampled time data are accessible. This research team has spent much time and effort to complete a high-speed program with the reading rate of 100 k-samples per second for the three measurement data. Such reading rate was achieved by adopting the VME64x single board computer supporting the optimized real-time Linux operating system. Attempts to improve the reading rate up to 256 k-samples per second are still in progress.

Fig. 3 shows the schematic diagram set up for the primary calibration system of rotational vibration pickups.



**Fig.3 Schematic drawing of primary calibration system of rotational vibration pickups**

The major parts of the calibration system consist of the angular vibration generator, the angle-prism-based laser interferometer introduced above, and the instruments for measuring the electrical output of a rotational vibration sensor under calibration. The sinusoidal angular vibration was generated by the direct driven rotary (Kollmorgen model DH0603M-22-1310). The digital servo amplifier (Servostar 610AS) was chosen not only to supply the 3 phased motor power and but also to control the sinusoidal angular motion. The mechanical rotary encoder system is also installed to the calibration system so as to read the large angular displacement of the rotating axis.

The encoder system consists of the following parts: the rotary encoder (Renishaw model RSER20USA150), the encoder head (Renishaw model RGH20BD00A) and the interpolator (Renishaw model GRE50D01A00). This mechanical encoder system with the measurement resolution of  $1.333 \mu\text{-radian}$  ( $4.7123 \times 10^6$  pulses/revolution) was designed to measure the angular displacement larger than the range of  $\pm 30^\circ$  because the measurement range of the angle-prism interferometer was fixed within  $\pm 30^\circ$ . In this year, the encoder system will be replaced into the new one with the five-time

improved resolution. The quadratic output signals of the encoder system were connected to the second counter/timer board (NI 6602, PCI-bus model) with the update rate of 80 MHz (refer to Figure 2).

The first function generator (Wave Factory model WF1493) was chosen to supply the sine function to the analog input of the digital servo motor amplifier. The amplitude and frequency of the function generator was chosen according to the desired vibration level given for each calibration frequency. The second function generator (Agilent model 53120A with the time stability of 1 ppm) was also chosen to generate the sampling clock whose frequency is tuned to  $M \times f$  ( $f$  = the chosen calibration frequency). The frequency multiplication factor  $M$  is designed to control the number of samples per each period such that  $M$  samples are read each period. This sampling clock, as shown in Figure 2, provides the angular displacement instruments: the Zygo measurement board (Zygo model 4004) and the counter/timer (NI 6602). Its frequency is finely controlled by monitoring the current frequency read by the first counter/timer (Agilent model 53131A). It enables the equi-angle sampling of the angular displacement with the  $M$  sample per period regardless of the calibration frequency chosen. This equi-angle sampling method, registered into the Korean patent (pending), is found to present the frequency-independent measurement uncertainty of the amplitude and phase sensitivities of vibration pickups. Furthermore, the equi-angle sampling method enables the realization of the 'digitally integrated' calibration system, not using either DMMs or phase meters. The discrete Fourier transform is sufficient to estimate the amplitude and phase sensitivities of vibration pickups from the equi-angle sampled data. The development of a digitally integrated PC-based system is in progress.

#### 4. EXPERIMENTAL RESULTS AND DISCUSSIONS

Table 1 shows the angular vibration amplitudes measured from the proposed laser interferometer. The first column shows the calibration frequency measured from the counter/timer. The second one shows the mean of counted pulses read by the path difference measurement board (Zygo model 4004). Each pulse is equal to the length of  $\lambda/2048$  ( $\lambda$  = the wavelength of the laser beam) such that the mean values are used to calculate the amplitude of angular vibration from equation (10). The third and fourth columns in Table 1 illustrate the standard uncertainty evaluated from the measurements read from the measurement board. The six right-handed columns demonstrate the evaluated vibration amplitudes. The relative standard uncertainty is shown to be in the range of 0.01 % to 0.09 % over the frequency range of 0.5 to 100 Hz. Specifically, the calibration frequency regions with the relatively high uncertainty were observed to be 5 Hz and 20 Hz where

the drift of zero position of the rotating axis was very slowly varied.

Table 1. Measured angular vibration amplitudes using the angle-prism-based laser interferometer.

Frequency [Hz]	Measured Amplitude Pulse				Angular Vibration Amplitude					
	Mean	Standard Uncertainty			Displacement		Velocity		Acceleration	
	[pulse]	[pulse]	%	%	[rad]	[deg]	[rad/s]	[deg/s]	[rad/s <sup>2</sup> ]	[deg/s <sup>2</sup> ]
0.5	11,959,292.7	1,232.88	0.010%	0.010%	0.30439629	17.441	0.956	54.79	3.004	172.1
0.8	10,820,832.4	2,887.71	0.027%	0.027%	0.27541940	15.780	1.384	79.32	6.959	398.7
1	4,200,809.5	2,287.46	0.054%	0.054%	0.10692194	6.126	0.672	38.49	4.221	241.9
1	13,109,431.6	1,068.92	0.008%	0.008%	0.33367043	19.118	2.097	120.12	13.173	754.7
5	4,076,240.6	3,612.98	0.089%	0.089%	0.10375133	5.945	3.259	186.75	102.398	5,867
8	2,268,973.7	802.07	0.035%	0.035%	0.05775151	3.309	2.903	166.32	145.916	8,360
10	1,387,921.0	812.56	0.059%	0.059%	0.03532634	2.024	2.220	127.17	139.463	7,991
20	425,630.7	358.70	0.084%	0.084%	0.01083345	0.621	1.361	78.00	171.075	9,802
40	90,287.5	50.29	0.056%	0.056%	0.00229806	0.132	0.578	33.09	145.158	8,317
50	47,876.2	15.72	0.033%	0.033%	0.00121858	0.070	0.383	21.93	120.269	6,891
80	20,858.6	8.64	0.041%	0.041%	0.00053091	0.030	0.267	15.29	134.140	7,686
100	70,696.8	13.36	0.019%	0.019%	0.00179942	0.103	1.131	64.78	710.384	40,702

All of the frequency components corresponding to the very slow motion of zero position were found to be less than 0.05 Hz. To remove these effects on the estimation of the vibration amplitude, a specific signal processing technique, known as the trend removal, is under development. It may imply the technical limit encountered in using the direct driven rotary with the finite magnetic poles as the angular motion generator. This research team has started to design a new angular exciter system based on the electromagnetic mechanism well exploited in the linear exciter systems. The electromagnetic angular exciter under development is similar to the electro-dynamic angular vibration exciter suggested in ISO/DIS 16065-15 which is expected to present the minimal effect of such zero position drift and much less unwanted harmonic components.

## 5. CONCLUSIONS

This paper introduces a new ‘angle-prism’-based laser interferometer where the angle prism used to convert its angular motion into the path difference between two parallel laser beams. It provides the angle measurement resolution of  $2.55 \times 10^{-8}$  radian and the angular velocity measurement capability up to 100 radian/s. The theoretical model for the angular displacement measurement is introduced in this paper. The ‘angle-prism’-based laser interferometer is shown to provide that either the eccentricity of the rotation axis or its unwanted transverse motion does not change any beam path difference in the angle-prism-based laser interferometer. It is a big advantage to use the angle-prism-based laser interferometer in comparison to the diffraction-grating interferometer suggested in ISO/DIS 16065-15.

This paper introduces the new equi-angle sampling method that can present the frequency-independent measurement uncertainty of the amplitude and phase sensitivities of vibration pickups. Furthermore, a systematic way of implementing the equi-angle sampling method is presented. This sampling method is shown to enable the realization of the ‘digitally integrated’

calibration system, not using either DMMs or phase meters. The discrete Fourier transform is sufficient to estimate the amplitude and phase sensitivities of vibration pickups from the equi-angle sampled data.

Experimental results are illustrated to confirm that the developed prototype interferometer is quite successful in measuring the angular vibration within the relative standard uncertainty of 0.01 % to 0.09 %. It indicates that the angle prism-based laser interferometer may be sufficient for the primary calibration of angular vibration transducers. I should be noted that the relative high measurement uncertainty observed at 5 and 20 Hz came from the angular motion generator, not the suggested laser interferometer itself. To resolve such high measurement uncertainty, a new angular vibration exciter, based on the electromagnetic mechanism well exploited in the linear exciter systems, is under development. Its mechanism is similar to the electro-dynamic angular vibration exciter suggested in ISO/DIS 16065-15. It is expected to present no effect of such zero position drift and much less unwanted harmonic components.

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